Projekt inovace předmětů – Teorie a praxe dluhopisů
Část III

Behaviour of bond's embedded option with regard to credit rating

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-- Brief ppt summary --

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

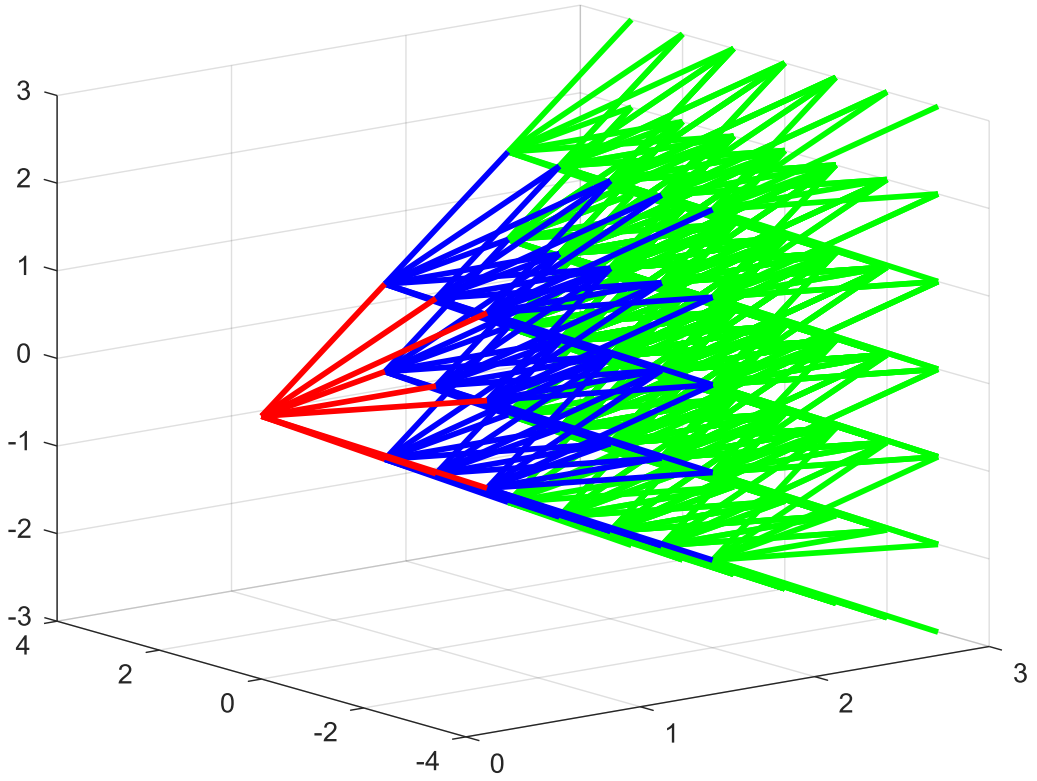
MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

Contribution

1. Conclusions about the **dynamics of changes of embedded option premium**; represented by the **direction and sensitivity**; with respect to the changes of credit rating and also risk-free interest rate development.

1. To simplify the topic for financial practitioners.

3-D Tree



We are about to consider a 3-dimensional process where the dimensions are:

1. dimension - time
2. dimension - risk-free interest rates development (Hull-White model)
3. dimension - rating development proces (derived from transition S&P matrix)

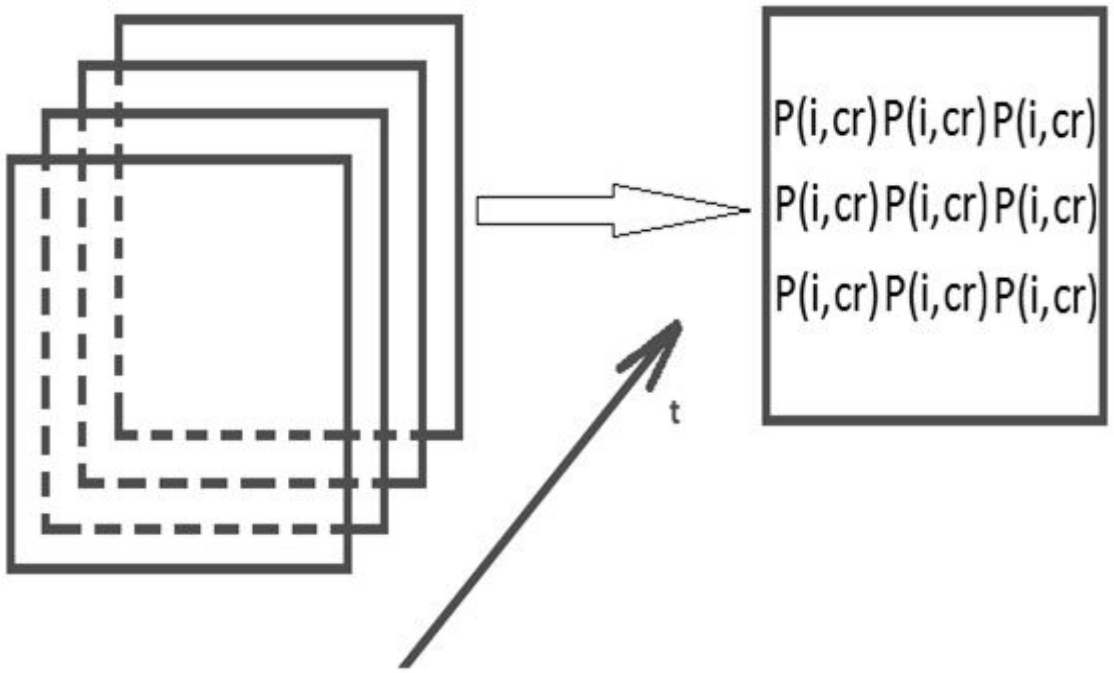
Using risk neutrality concept.

Source: https://journals.vgtu.lt/index.php/BTP/article/view/8140/7057

3-D Tree

The price *P* at each node is given by the formula, where *t* is time, *i* is the value of risk-free rate at the point of time *t*, *cr* is current credit rating value and *q* denotes the probability of each way from the note.

$$P_{t,i,cr} = \sum_{s=1}^{n} q_s \frac{P_{s,t+1}}{(1+i_t)}$$



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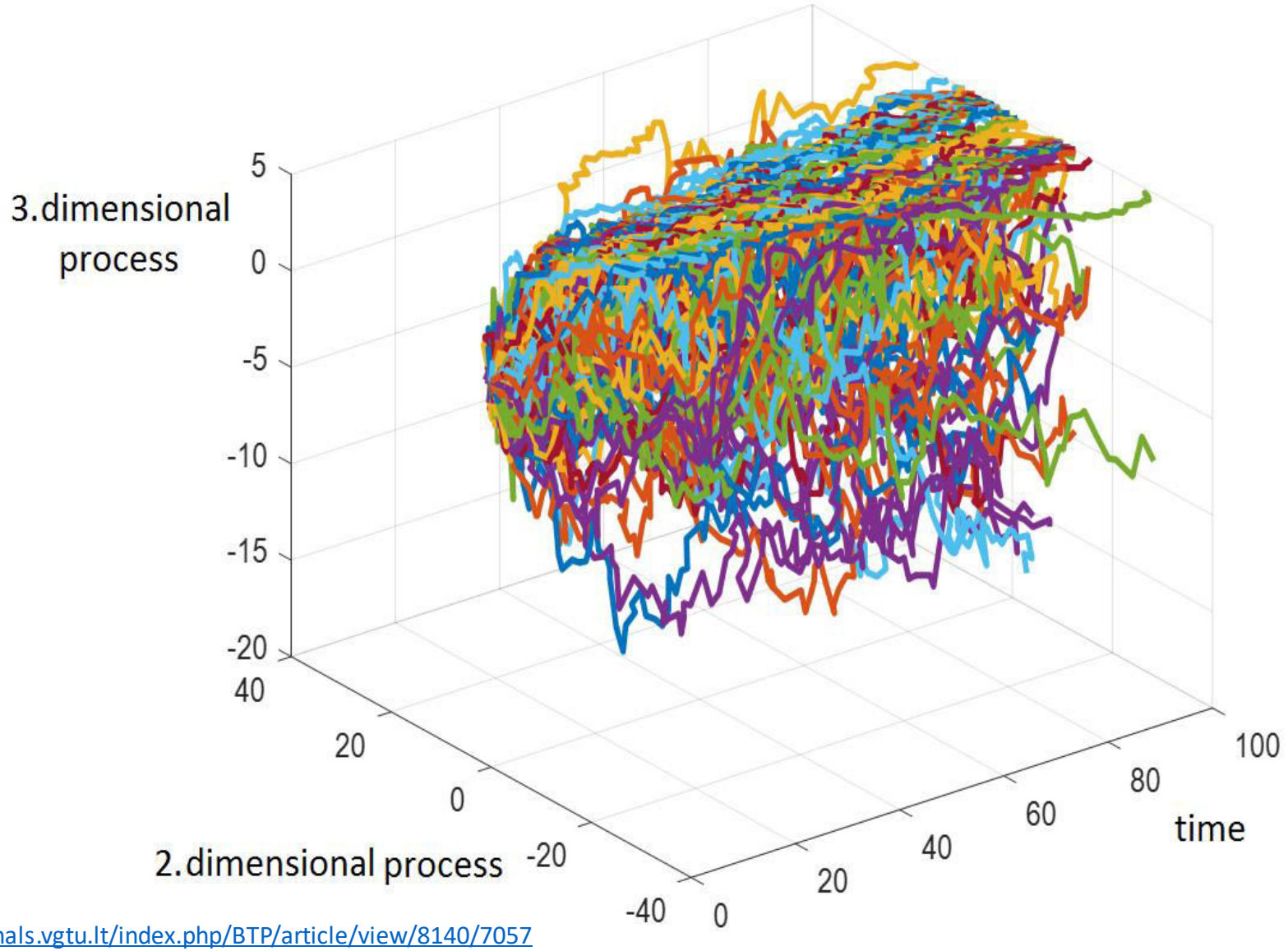
3 dimensional process simulation example



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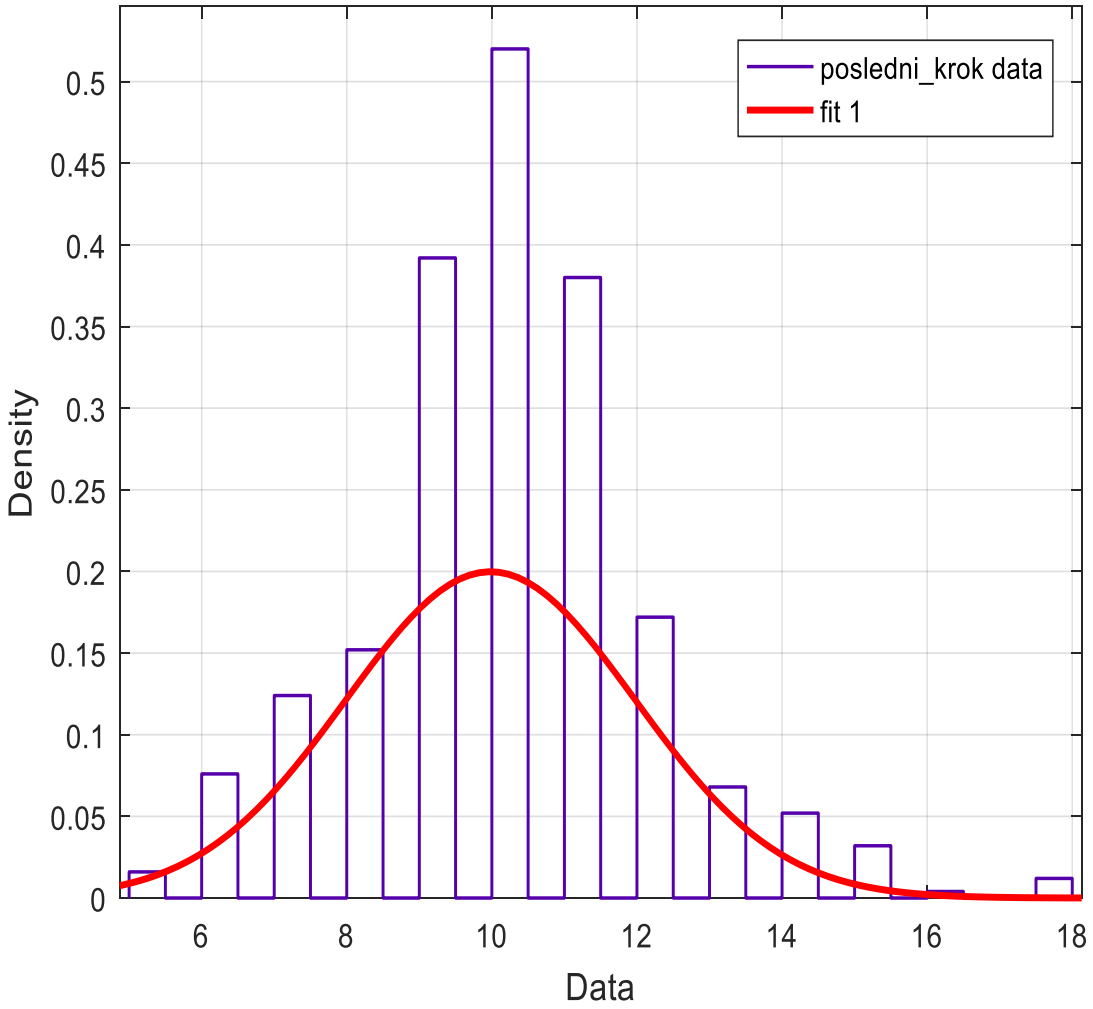
Rating development (rating tree)



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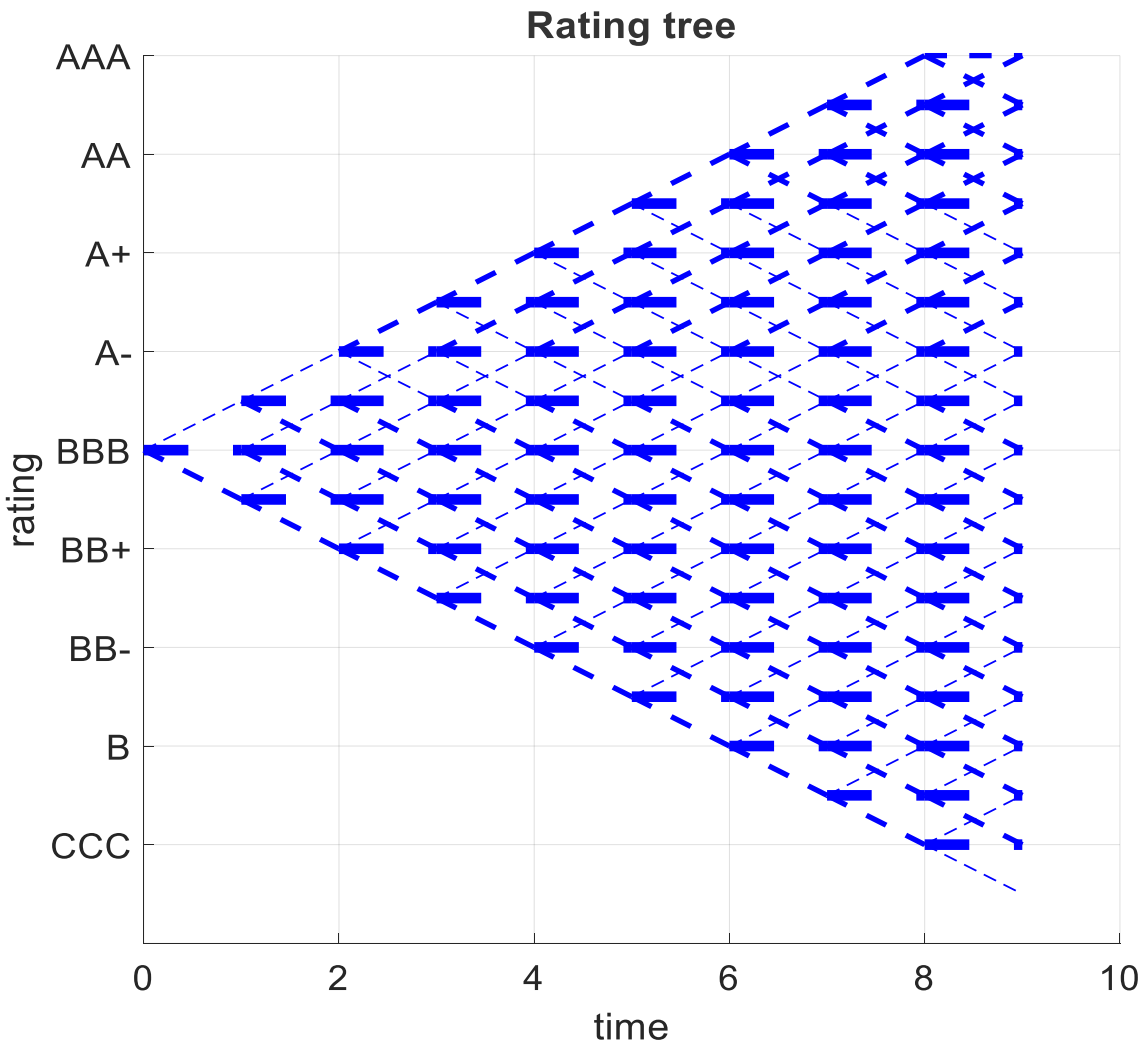
Global Corporate Transition Matrix (%) (1981-2010)

Rating	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC/C	D
AAA	87.91	4.72	2.68	0.68	0.16	0.24	0.14	0.00	0.05	0.00	0.03	0.05	0.00	0.00	0.03	0.00	0.05	0.00
AA+	2.62	76.06	11.67	3.93	0.89	0.66	0.30	0.12	0.12	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	0.47	1.32	80.64	8.01	2.89	1.41	0.43	0.42	0.14	0.09	0.05	0.04	0.02	0.00	0.00	0.02	0.05	0.02
AA-	0.05	0.13	4.28	76.93	10.02	2.84	0.71	0.27	0.14	0.07	0.04	0.00	0.00	0.04	0.11	0.02	0.00	0.04
A+	0.00	0.11	0.58	4.46	77.42	8.80	2.57	0.71	0.40	0.09	0.09	0.12	0.01	0.09	0.04	0.01	0.00	0.07
A	0.05	0.06	0.28	0.56	5.01	77.73	6.82	2.69	1.15	0.28	0.15	0.15	0.10	0.12	0.03	0.01	0.02	0.09
A-	0.06	0.01	0.11	0.20	0.61	6.78	75.80	7.51	2.36	0.68	0.16	0.15	0.16	0.14	0.04	0.01	0.05	0.08
BBB+	0.00	0.01	0.07	0.09	0.31	1.05	6.93	73.19	8.85	2.01	0.47	0.40	0.17	0.26	0.15	0.02	0.10	0.16
BBB	0.01	0.01	0.06	0.04	0.17	0.48	1.23	7.04	74.22	6.30	1.62	0.83	0.37	0.31	0.17	0.04	0.09	0.23
BBB-	0.01	0.01	0.01	0.07	0.07	0.24	0.40	1.37	8.56	71.12	5.48	2.59	1.03	0.56	0.34	0.22	0.31	0.38
BB+	0.07	0.00	0.00	0.05	0.02	0.15	0.12	0.63	2.29	11.70	62.56	6.43	3.24	1.27	0.83	0.19	0.51	0.56
BB	0.00	0.00	0.06	0.02	0.00	0.10	0.08	0.23	0.74	2.56	8.51	64.26	7.74	2.69	1.37	0.46	0.74	0.80
BB-	0.00	0.00	0.00	0.01	0.01	0.01	0.07	0.13	0.30	0.48	2.06	8.23	63.76	8.43	3.06	0.97	0.91	1.31
B+	0.00	0.01	0.00	0.04	0.00	0.04	0.09	0.06	0.07	0.10	0.34	1.57	6.92	65.02	7.66	2.62	1.96	2.62
B	0.00	0.00	0.02	0.02	0.00	0.09	0.07	0.04	0.11	0.04	0.23	0.39	1.69	8.39	57.67	7.95	5.42	5.90
B-	0.00	0.00	0.00	0.00	0.04	0.07	0.00	0.14	0.07	0.14	0.18	0.21	0.61	3.13	10.22	51.30	10.82	9.15
CCC/C	0.00	0.00	0.00	0.00	0.05	0.00	0.14	0.09	0.09	0.09	0.05	0.23	0.56	1.39	2.91	8.70	43.80	27.43

Sources: Standard & Poor's Global Fixed Income Research and Standard & Poor's CreditPro®.

Rating development (rating tree)



Source: https://journals.vgtu.lt/index.php/BTP/article/view/8140/7057

Global Corporate Transition Matrix (%) (1981-2010)

Rating	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC/C	D
AAA	87.91	4.72	2.68	0.68	0.16	0.24	0.14	0.00	0.05	0.00	0.03	0.05	0.00	0.00	0.03	0.00	0.05	0.00
AA+	2.62	76.06	11.67	3.93	0.89	0.66	0.30	0.12	0.12	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	0.47	1.32	80.64	8.01	2.89	1.41	0.43	0.42	0.14	0.09	0.05	0.04	0.02	0.00	0.00	0.02	0.05	0.02
AA-	0.05	0.13	4.28	76.93	10.02	2.84	0.71	0.27	0.14	0.07	0.04	0.00	0.00	0.04	0.11	0.02	0.00	0.04
A+	0.00	0.11	0.58	4.46	77.42	8.80	2.57	0.71	0.40	0.09	0.09	0.12	0.01	0.09	0.04	0.01	0.00	0.07
A	0.05	0.06	0.28	0.56	5.01	77.73	6.82	2.69	1.15	0.28	0.15	0.15	0.10	0.12	0.03	0.01	0.02	0.09
A-	0.06	0.01	0.11	0.20	0.61	6.78	75.80	7.51	2.36	0.68	0.16	0.15	0.16	0.14	0.04	0.01	0.05	0.08
BBB+	0.00	0.01	0.07	0.09	0.31	1.05	6.93	73.19	8.85	2.01	0.47	0.40	0.17	0.26	0.15	0.02	0.10	0.16
BBB	0.01	0.01	0.06	0.04	0.17	0.48	1.23	7.04	74.22	6.30	1.62	0.83	0.37	0.31	0.17	0.04	0.09	0.23
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BB	0.00	0.00	0.06	0.02	0.00	0.10	0.08	0.23	0.74	2.56	8.51	64.26	7.74	2.69	1.37	0.46	0.74	0.80
BB-	0.00	0.00	0.00	0.01	0.01	0.01	0.07	0.13	0.30	0.48	2.06	8.23	63.76	8.43	3.06	0.97	0.91	1.31
B+	0.00	0.01	0.00	0.04	0.00	0.04	0.09	0.06	0.07	0.10	0.34	1.57	6.92	65.02	7.66	2.62	1.96	2.62
B	0.00	0.00	0.02	0.02	0.00	0.09	0.07	0.04	0.11	0.04	0.23	0.39	1.69	8.39	57.67	7.95	5.42	5.90
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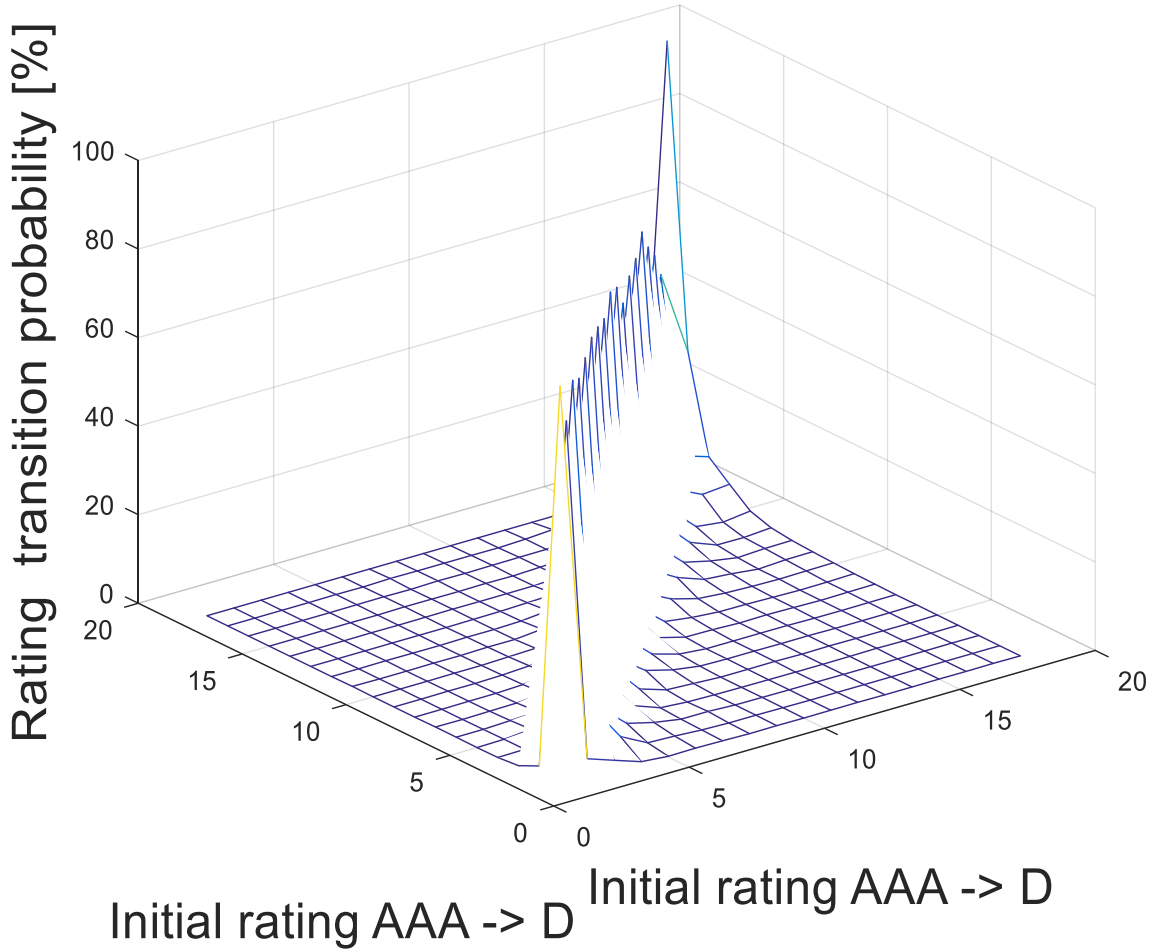
Sources: Standard & Poor's Global Fixed Income Research and Standard & Poor's Credit Pro®.

Rating development (rating tree)



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AA	0.47	1.32	80.64	8.01	2.89	1.41	0.43	0.42	0.14	0.09	0.05	0.04	0.02	0.00	0.00	0.02	0.05	0.02
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A	0.05	0.06	0.28	0.56	5.01	77.73	6.82	2.69	1.15	0.28	0.15	0.15	0.10	0.12	0.03	0.01	0.02	0.09
A-	0.06	0.01	0.11	0.20	0.61	6.78	75.80	7.51	2.36	0.68	0.16	0.15	0.16	0.14	0.04	0.01	0.05	0.08
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B	0.00	0.00	0.02	0.02	0.00	0.09	0.07	0.04	0.11	0.04	0.23	0.39	1.69	8.39	57.67	7.95	5.42	5.90
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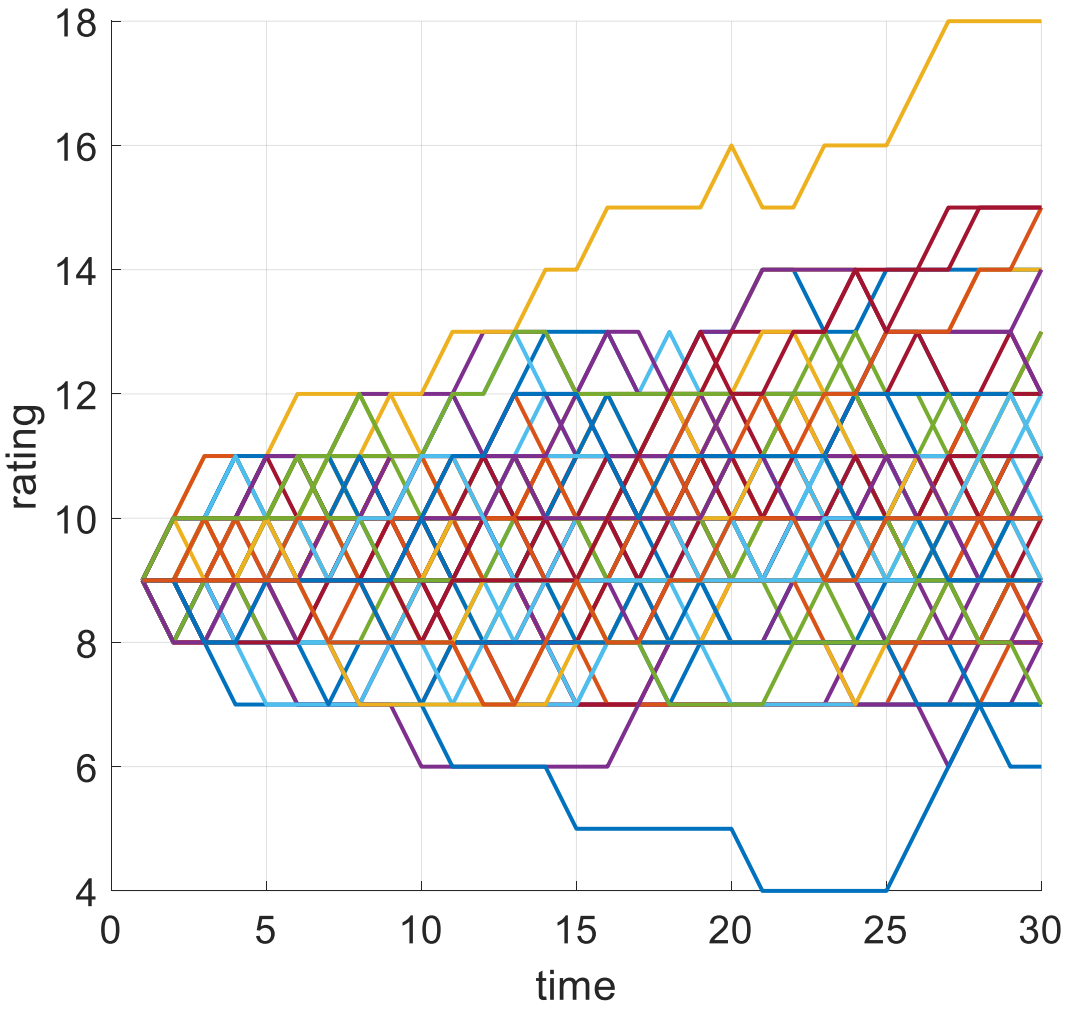
Sources: Standard & Poor's Global Fixed Income Research and Standard & Poor's Credit Pro®.

Rating development (rating tree)



Global Corporate Transition Matrix (%) (1981-2010)

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AA	0.47	1.32	80.64	8.01	2.89	1.41	0.43	0.42	0.14	0.09	0.05	0.04	0.02	0.00	0.00	0.02	0.05	0.02
AA-	0.05	0.13	4.28	76.93	10.02	2.84	0.71	0.27	0.14	0.07	0.04	0.00	0.00	0.04	0.11	0.02	0.00	0.04
A+	0.00	0.11	0.58	4.46	77.42	8.80	2.57	0.71	0.40	0.09	0.09	0.12	0.01	0.09	0.04	0.01	0.00	0.07
A	0.05	0.06	0.28	0.56	5.01	77.73	6.82	2.69	1.15	0.28	0.15	0.15	0.10	0.12	0.03	0.01	0.02	0.09
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BBB+	0.00	0.01	0.07	0.09	0.31	1.05	6.93	73.19	8.85	2.01	0.47	0.40	0.17	0.26	0.15	0.02	0.10	0.16
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BB+	0.07	0.00	0.00	0.05	0.02	0.15	0.12	0.63	2.29	11.70	62.56	6.43	3.24	1.27	0.83	0.19	0.51	0.56
BB	0.00	0.00	0.06	0.02	0.00	0.10	0.08	0.23	0.74	2.56	8.51	64.26	7.74	2.69	1.37	0.46	0.74	0.80
BB-	0.00	0.00	0.00	0.01	0.01	0.01	0.07	0.13	0.30	0.48	2.06	8.23	63.76	8.43	3.06	0.97	0.91	1.31
B+	0.00	0.01	0.00	0.04	0.00	0.04	0.09	0.06	0.07	0.10	0.34	1.57	6.92	65.02	7.66	2.62	1.96	2.62
B	0.00	0.00	0.02	0.02	0.00	0.09	0.07	0.04	0.11	0.04	0.23	0.39	1.69	8.39	57.67	7.95	5.42	5.90
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CCC/C	0.00	0.00	0.00	0.00	0.05	0.00	0.14	0.09	0.09	0.09	0.05	0.23	0.56	1.39	2.91	8.70	43.80	27.43

Sources: Standard & Poor's Global Fixed Income Research and Standard & Poor's Credit Pro®.

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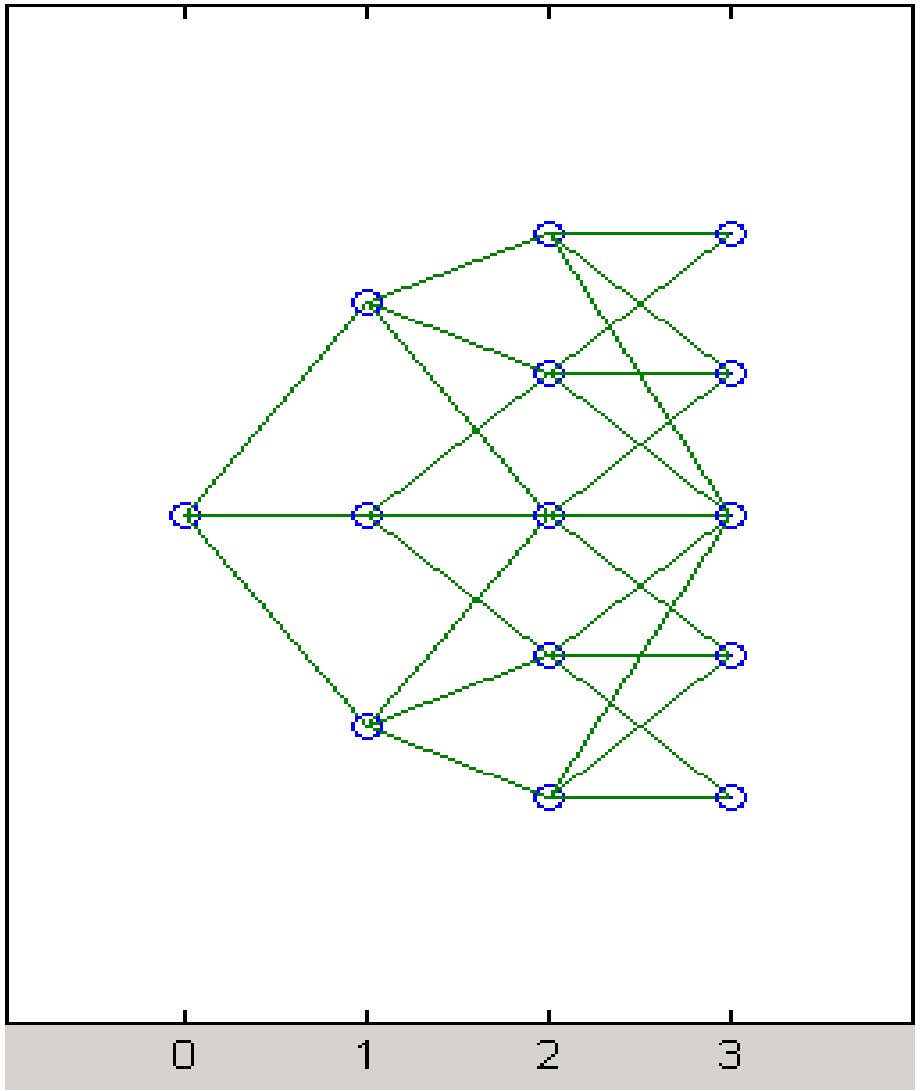
Risk-free interest rate development (interest rate tree)



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Hull-White model

Two-factor model [edit]

The two-factor Hull–White model contains an additional disturbance term whose mean reverts to zero, and is of the form:

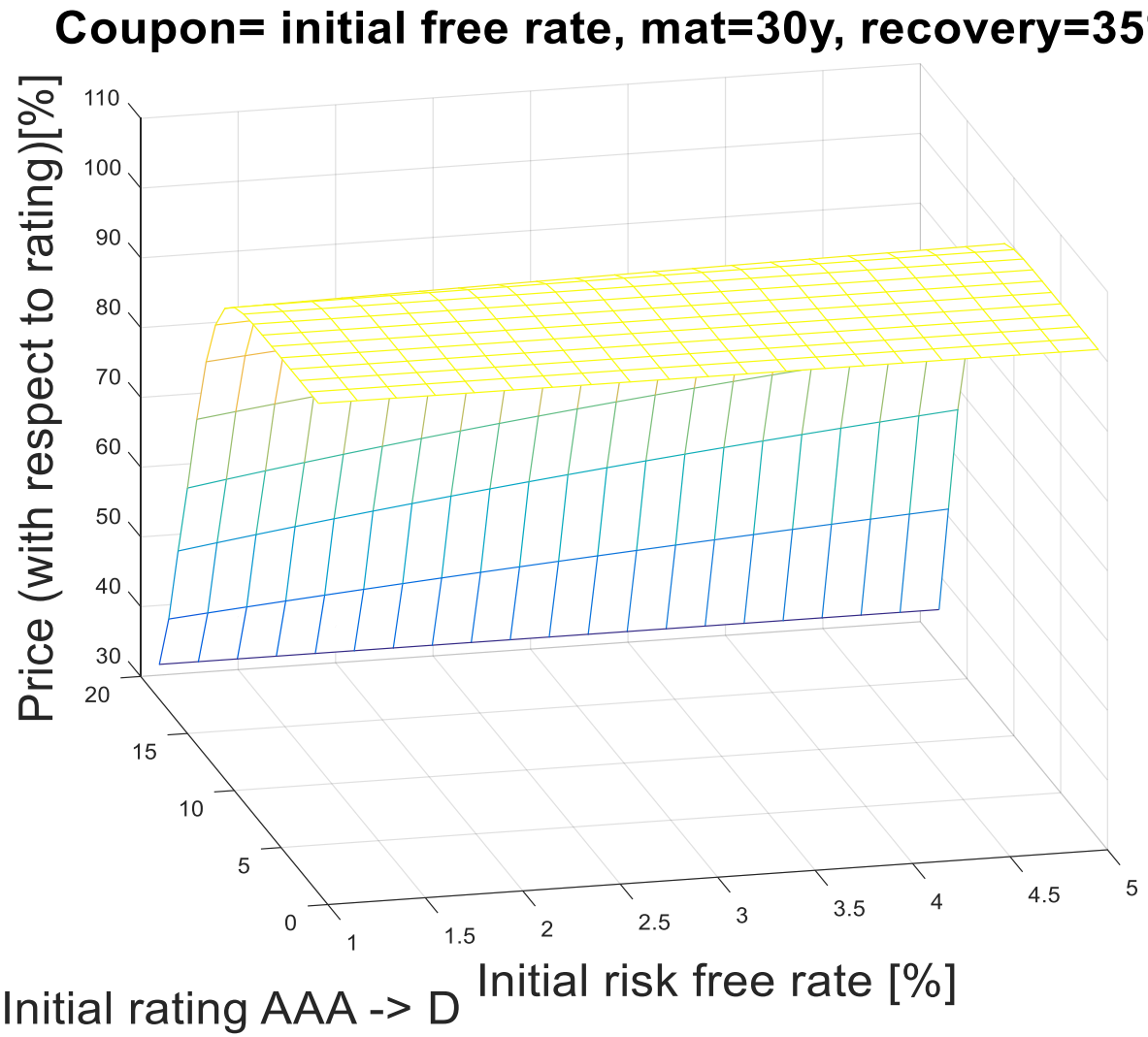
$$d\,f(r(t)) = [\theta(t) + u - \alpha(t)\,f(r(t))]\,dt + \sigma_1(t)\,dW_1(t)$$

where u has an initial value of 0 and follows the process:

$$du = -bu\,dt + \sigma_2\,dW_2(t)$$

NumericalResults
(example: Typical bond, 30 years, fixed coupon rate, recovery rate 0/35)
Matlab implementation

Price of bond with respect to credit rating



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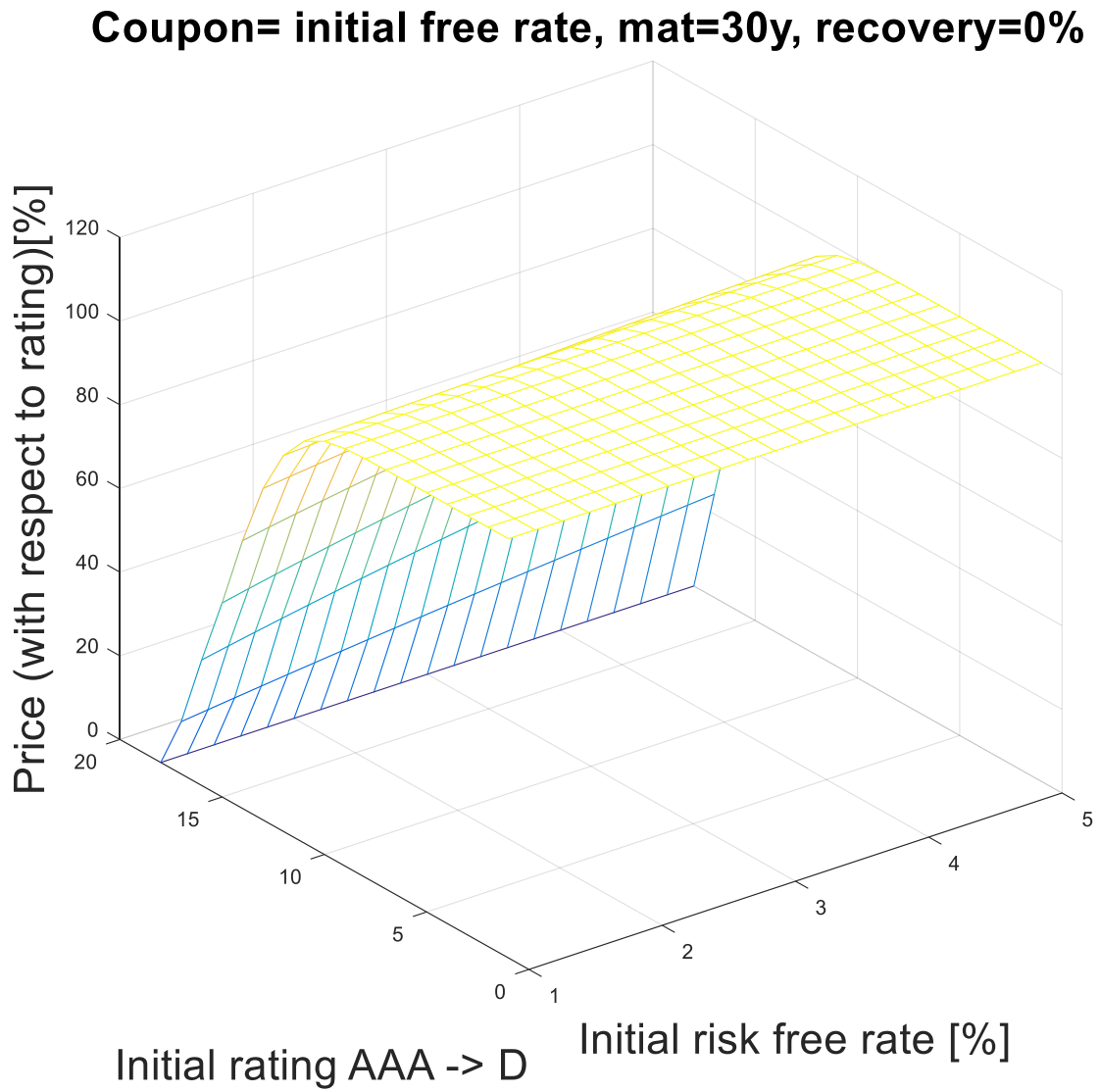
Price of bond with respect to credit rating



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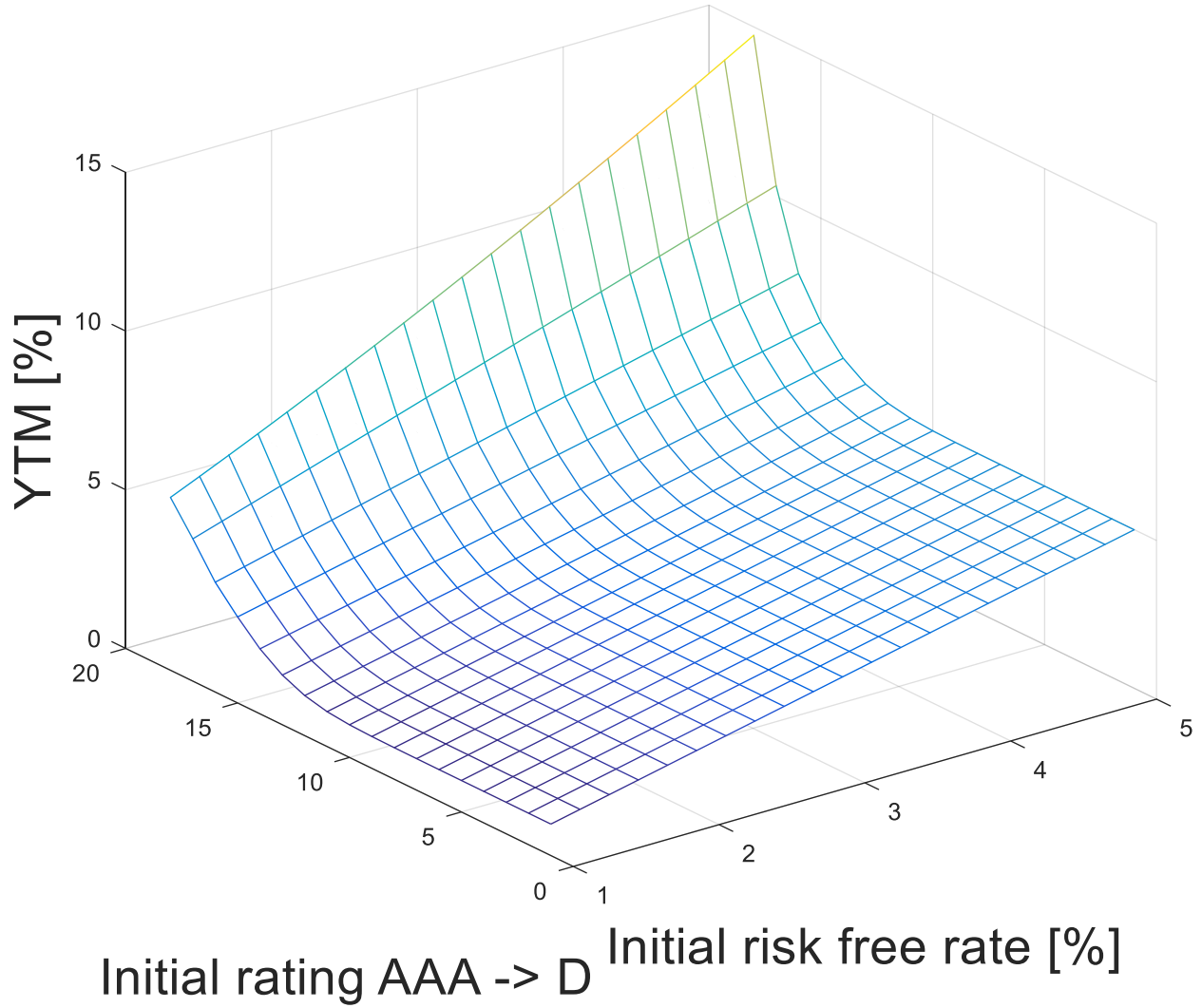
Price of bond with respect to credit rating

Coupon= initial free rate, mat=30y, recovery=35%



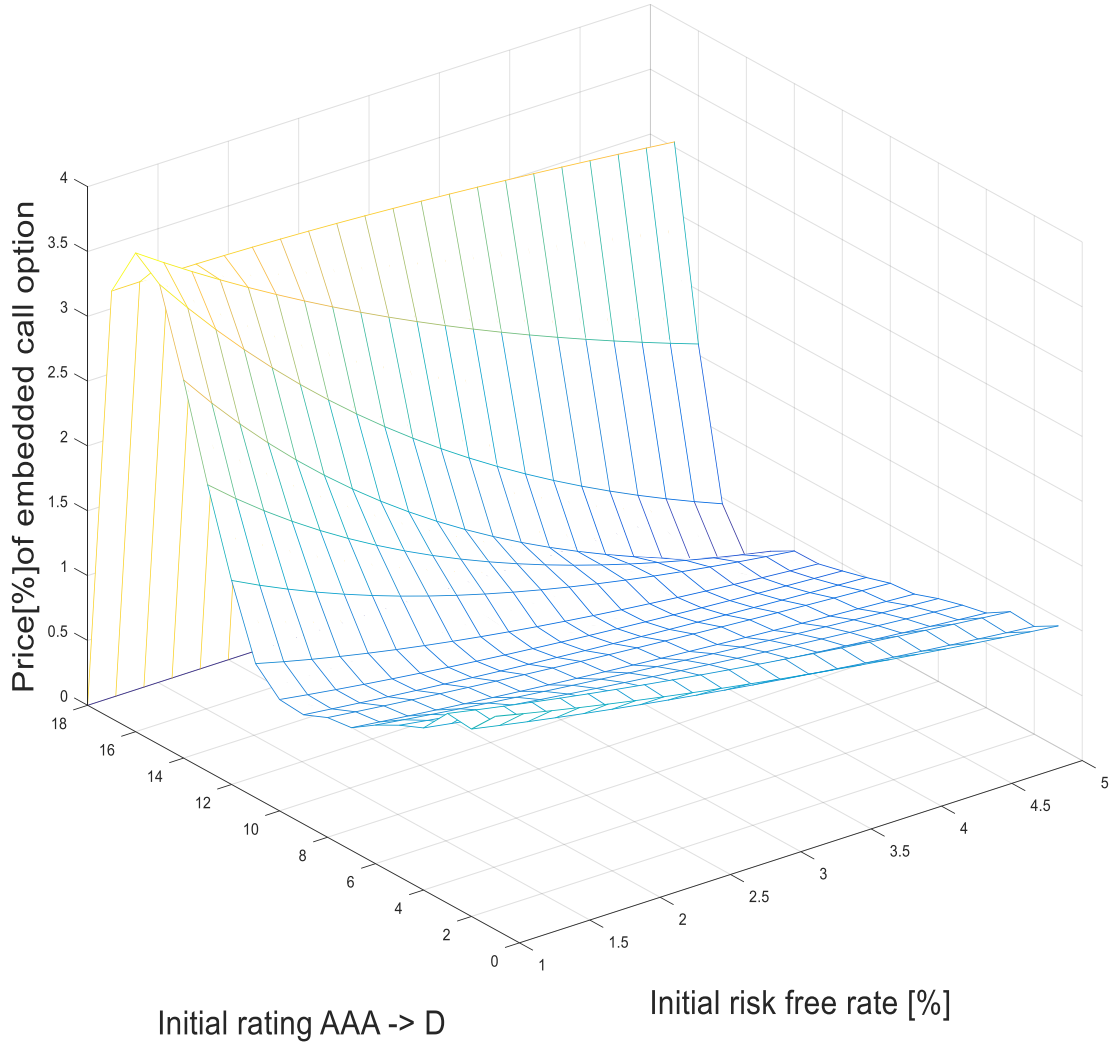
Source: https://journals.vgtu.lt/index.php/BTP/article/view/8140/7057

Price of bond with respect to credit rating

Rate:	1.0	1.2	1.4	1.6	1.8	2.0
AAA	100,2	100,2	100,2	100,2	100,2	100,2
AA+	100,2	100,2	100,2	100,2	100,2	100,2
AA	100,2	100,2	100,2	100,2	100,2	100,2
AA-	100,2	100,2	100,2	100,2	100,2	100,2
A+	100,2	100,2	100,2	100,2	100,2	100,2
A	100,2	100,2	100,2	100,2	100,2	100,2
A-	100,2	100,2	100,2	100,2	100,1	100,1
BBB+	100,1	100,1	100,1	100,1	100,1	100,1
BBB	99,8	99,8	99,8	99,8	99,8	99,8
BBB-	98,7	98,8	98,9	98,9	99,0	99,0
BB+	96,0	96,2	96,3	96,5	96,6	96,8
BB	89,9	90,3	90,6	91,0	91,4	91,7
BB-	79,2	80,0	80,7	81,4	82,0	82,6
B+	64,0	65,1	66,2	67,3	68,2	69,2
B	46,4	47,7	49,0	50,2	51,4	52,6
B-	30,0	31,2	32,4	33,6	34,7	35,8
CCC	12,5	13,2	14,0	14,7	15,4	16,0
D	0,0	0,0	0,0	0,0	0,0	0,0

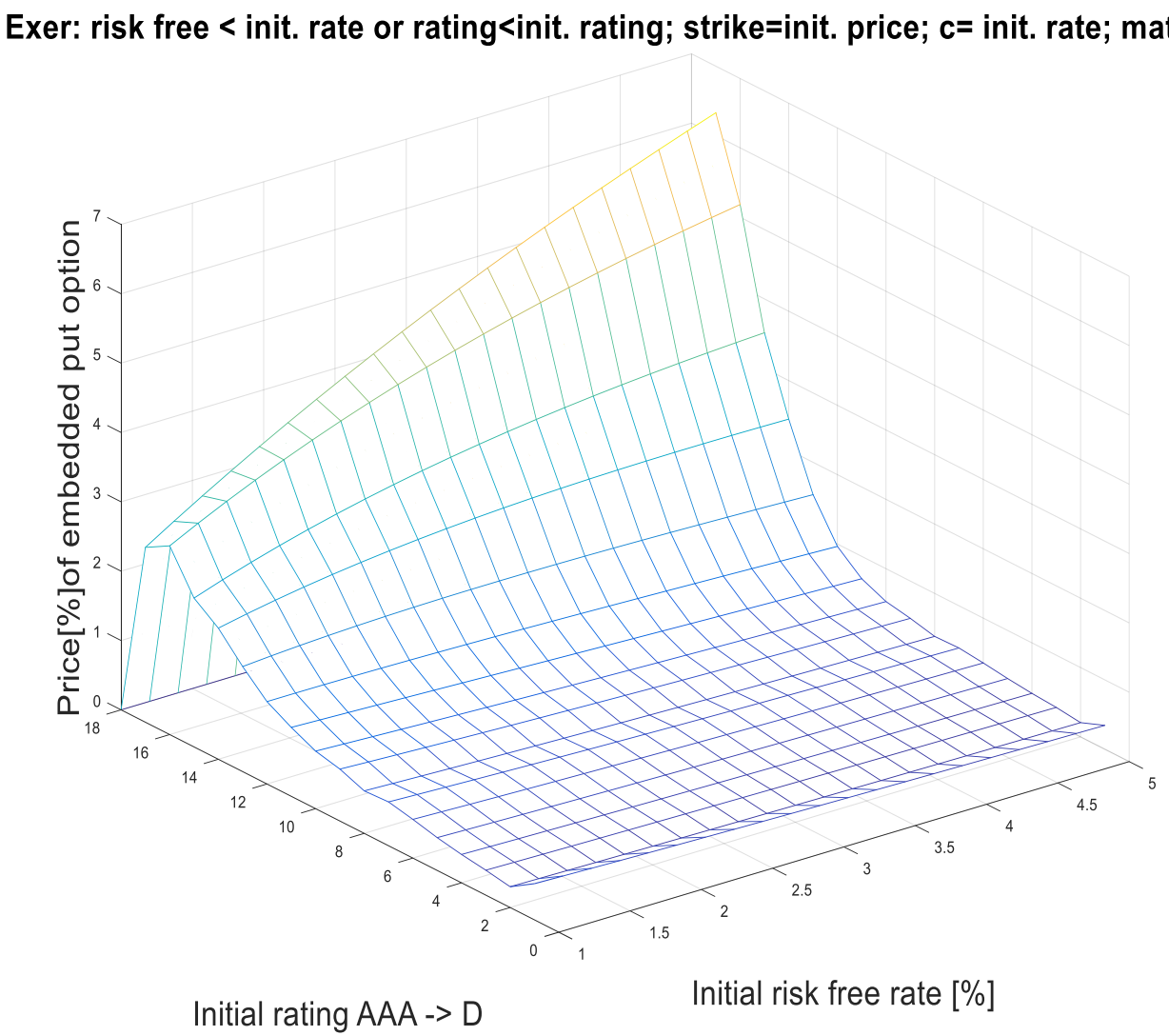
Source: https://journals.vgtu.lt/index.php/BTP/article/view/8140/7057

Price of embedded call/put option



Source: https://journals.vgtu.lt/index.php/BTP/article/view/8140/7057

Price of embedded call/put option



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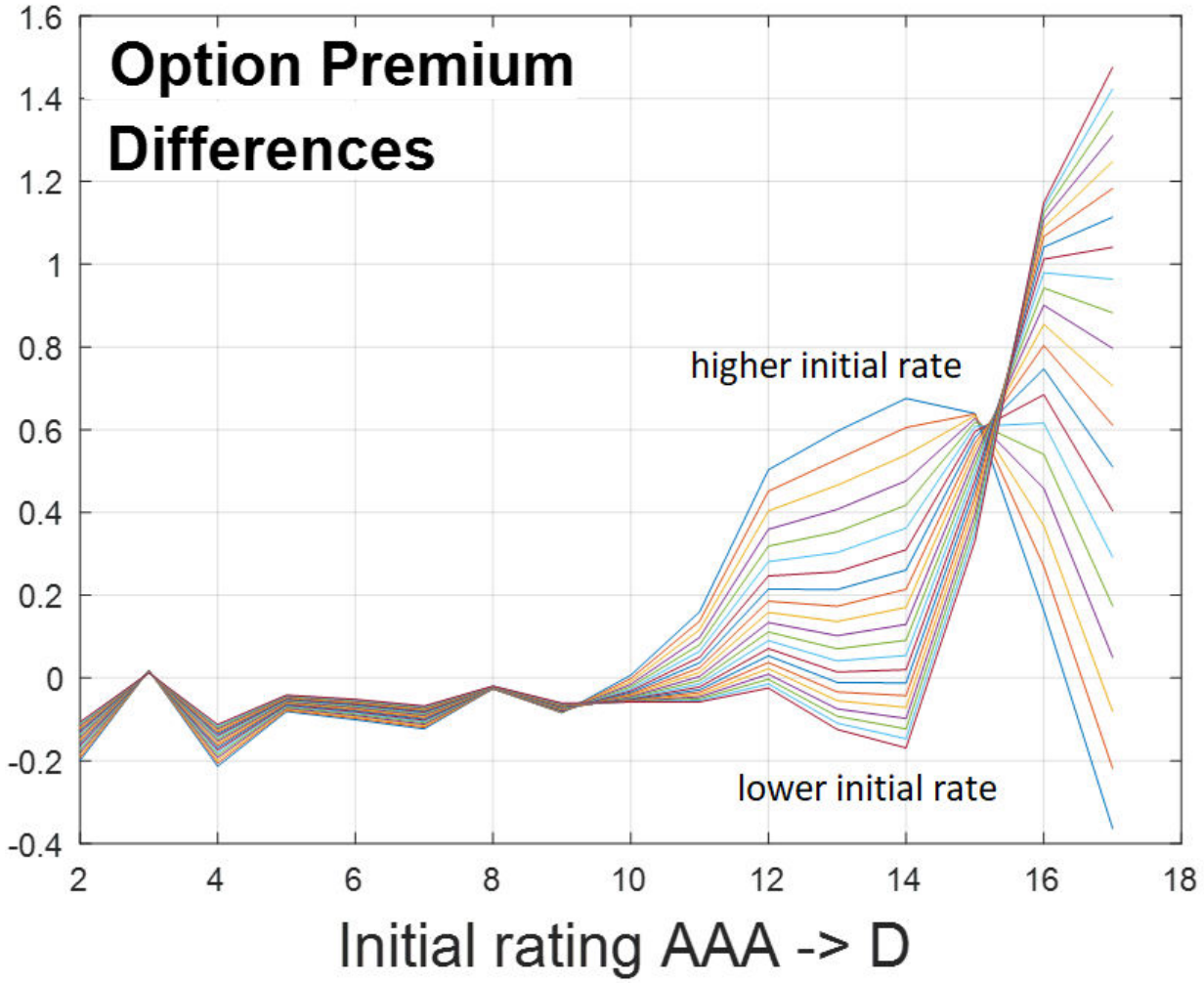
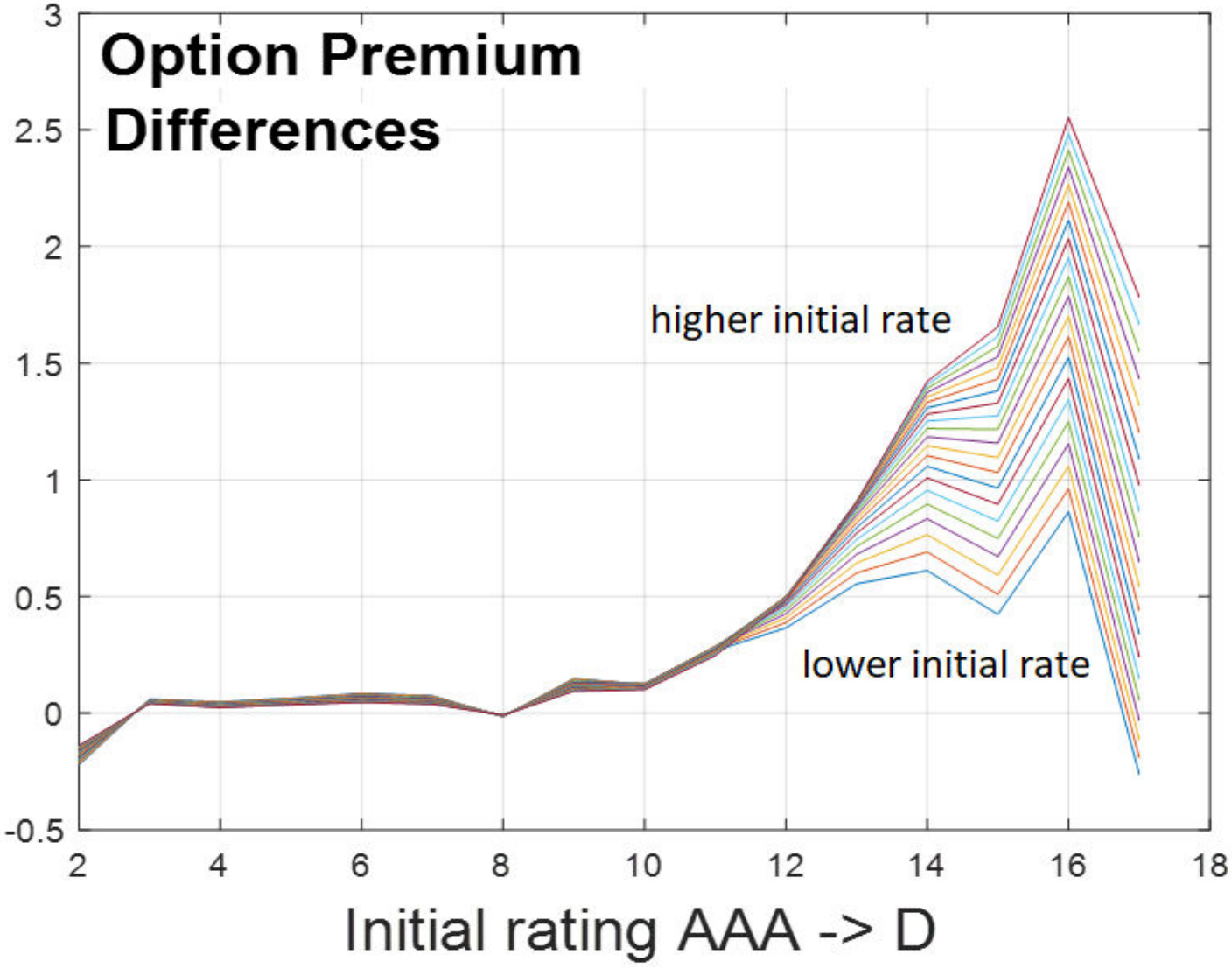
Call/Put option premium sensitivity with respect to the development of credit risk



Source: https://journals.vgtu.lt/index.php/BTP/article/view/8140/7057

Call/Put option premium sensitivity with respect to the development of credit risk



Source: https://journals.vgtu.lt/index.php/BTP/article/view/8140/7057

Conclusions

1. The value of option premium of embedded call/put option increases with the worsening of credit rating. It could be well explained by the higher volatility of underlying asset price in the area of worse rating.

2. Based on the parameters of rating transition matrix, the sensitivity may not increase continuously (it increases in average); also the surface is not smooth because of parameters of the rating transition matrix.

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