Projekt inovace předmětů – Teorie a praxe dluhopisů
Část II

Empirical duration concept

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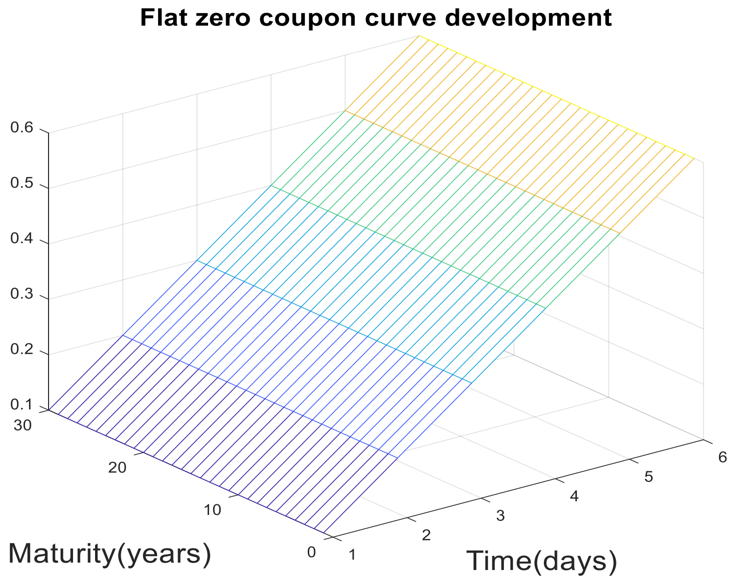
-- Brief ppt summary --

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

Empirical duration concept
Why?

- Macaulay's duration could be roughly financially interpreted as the percentage change of a bond price after interest rate shift of 1% **along the whole zero-coupon curve (along the whole spectrum of maturities)** when initial price of the bond is equaled to100% and the rate shift is 1 p.p.



Source: Author

- In practical life Macaulay's duration has no significant reason as the parallel curve shift is very rare thus we have to find a certain value which respects the realistic empirical zero-coupon curves shifts, so the result will be much closer to the reality. We name this measure as "Empirical duration".

Empirical duration concept, methodology

Methodology could be divided into following steps:

1. We take USD empirical zero-coupon curve rates on daily basis, in this research from USD market (figure1).
2. Based on the empirical data we calculate the daily change of price of fixed coupon bonds with maturities 1 to 30 years (equation 2b). The coupon is, for the demonstration, set to be 3% p.a.
3. Based on the daily price changes we calculate adequate daily empirical duration in the same way how the Macaulay's duration is calculated (equation 9).
4. We define and calculate Empirical duration as mean of daily empirical durations.
5. We calculate the values of Empirical duration for different coupons.

Empirical duration concept, basic theory

$$P(YTM) = \left(1 + YTM\,\frac{l}{T}\right)^{-1}\left[c + \frac{c}{(1+YTM)} + \frac{c}{(1+YTM)^2} + \cdots + \frac{c+100}{(1+YTM)^{n-1}}\right]$$

here *P(YTM)* is the dirty price of a bond determined in the percentage of its face value on purchasing day, *c* is the coupon rate per the coupon period, *YTM is* the yield to maturity per the coupon period, *l* is the number of days to the next coupon payment, *n* is the number of coupon payments till the maturity and *T* is the number of days inside the coupon period.

$$P(YTM) = \frac{c}{(1+YTM)} + \frac{c}{(1+YTM)^2} + \cdots + \frac{c+100}{(1+YTM)^n}$$

in the special case when we purchase a bond on the day with zero accrued interest (could be for example an ex-coupon day) and the clean price equals to the dirty price we can use the formula (2a) for the approximation of the required clean price development.

Empirical duration concept, basic theory

We may consider the total price also to be the sum of total prices of n zero-coupon bonds:

$$P(i_{1,} i_{2,} \dots i_{mat}) = \frac{c}{(1+i_1)} + \frac{c}{(1+i_2)^2} + \dots + \frac{c+100}{(1+i_{mat})^n}$$

where $i_1, i_2, \dots i_{mat}$ are zero-coupon rates for maturities 1,2,….n years.

Based on the Taylor's theorem for a real-valued function f differentiable at the point a, there is a polynomial approximation of a higher degree (quadratic, cubic, quartic…) at the fixed point a. Taylor´s theorem provides this approximation in a sufficiently small neighbourhood (h) of the fixed point a:

$$f(a+h) = f(a) + f'(a)h + \frac{f''(a)h^2}{2!} + \frac{f'''(a)h^3}{3!} + \dots + R$$

With the substitution: h *for* Δi and $f(a)$ for P. Consequently, it can be shown that, the expression holds:

$$\Delta P(YTM) = P'(YTM)\Delta YTM + \frac{P''(YTM)\Delta YTM^2}{2!} + \frac{P'''(YTM)\Delta YTM^3}{3!} + \dots + R$$

Empirical duration concept, basic theory

The general measure of volatility and only up to the second order approximation:

$$\Delta P(YTM) \cong -DUR_{MAC}\frac{P(YTM)}{(1+YTM)}\Delta YTM + \frac{1}{2}P(YTM)CONV\Delta YTM^2$$

Macaulay's duration can be specified using the following shorthand notation:

$$DUR_{MAC} = \frac{\sum_{k=1}^{n}\left[\frac{kc}{(1+YTM)^k}\right] + \frac{100n}{(1+YTM)^n}}{P}$$

Empirical duration concept, definition

$$DUR_{EMP_d}(mat, d, YTM) \cong -\frac{YTM}{P_{mat,d}} \frac{\Delta P_{mat,d}}{\Delta YTM}$$

DUR_{EMP_d} (mat,d,YTM)	**empirical duration on daily basis as function of maturity, day, yield to maturity (figure1)**
YTM	yield to maturity
ΔYTM	change of YTM
$P_{(mat,d,)}$	price of bond with respect to its time to maturity and the structure of zero coupon curve on day d
$\Delta P_{(mat,d,)}$	change of price between d and d-1 (according to figure 1)
d	number of days (from the beginning of time series, figure1)

Source: Author

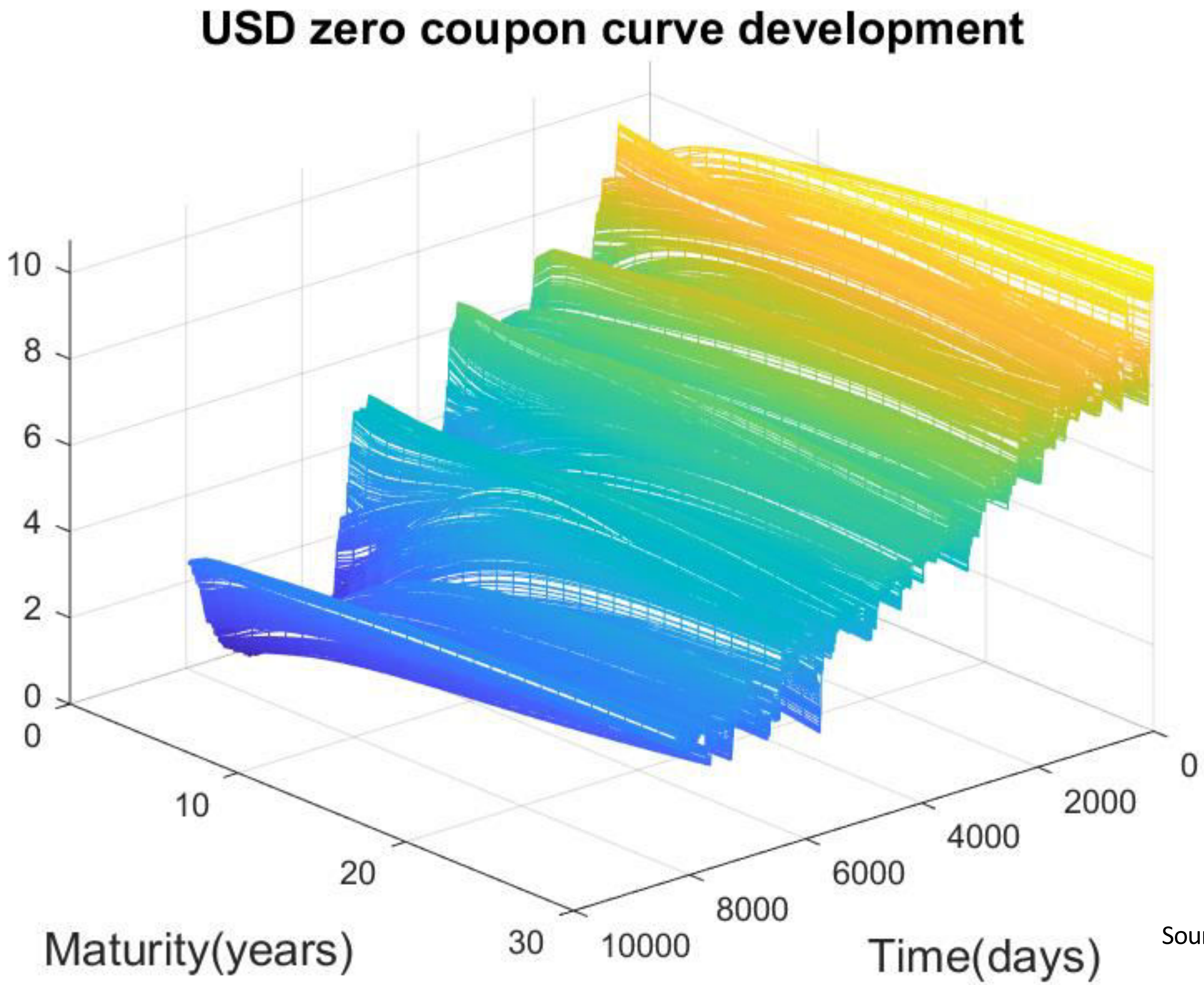
$$DUR_{EMP} = E\left[DUR_{EMP\_d}(mat, d, YTM)\right]$$

Empirical duration concept

$$DUR_{EMP\_PORTF} \cong \sum_{j=1}^{n} w_j DUR_{EMP\ j}$$

Where w_{j} is the weight of j^{th} asset in the portfolio.

Empirical duration concept, **USD** rates market



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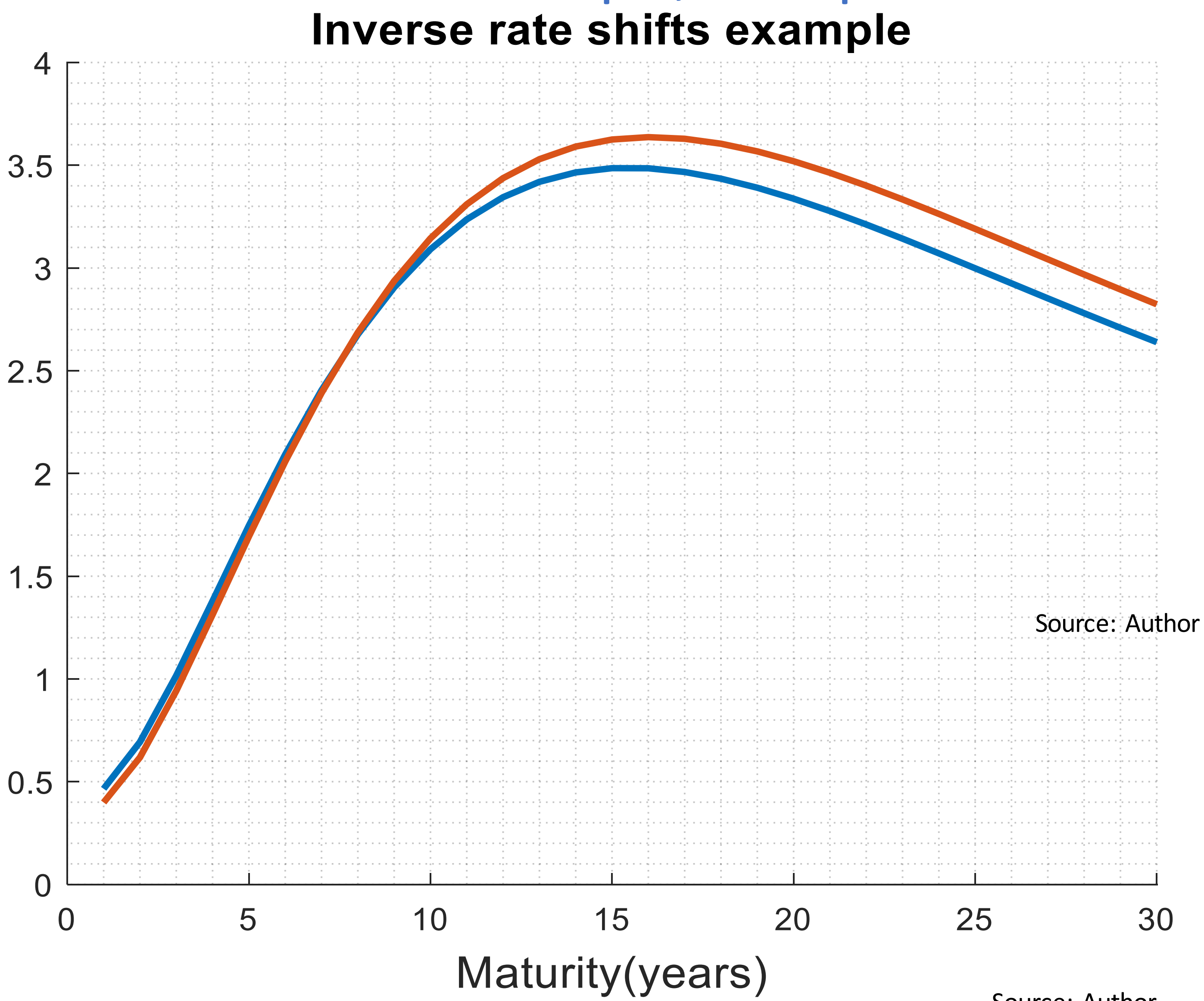
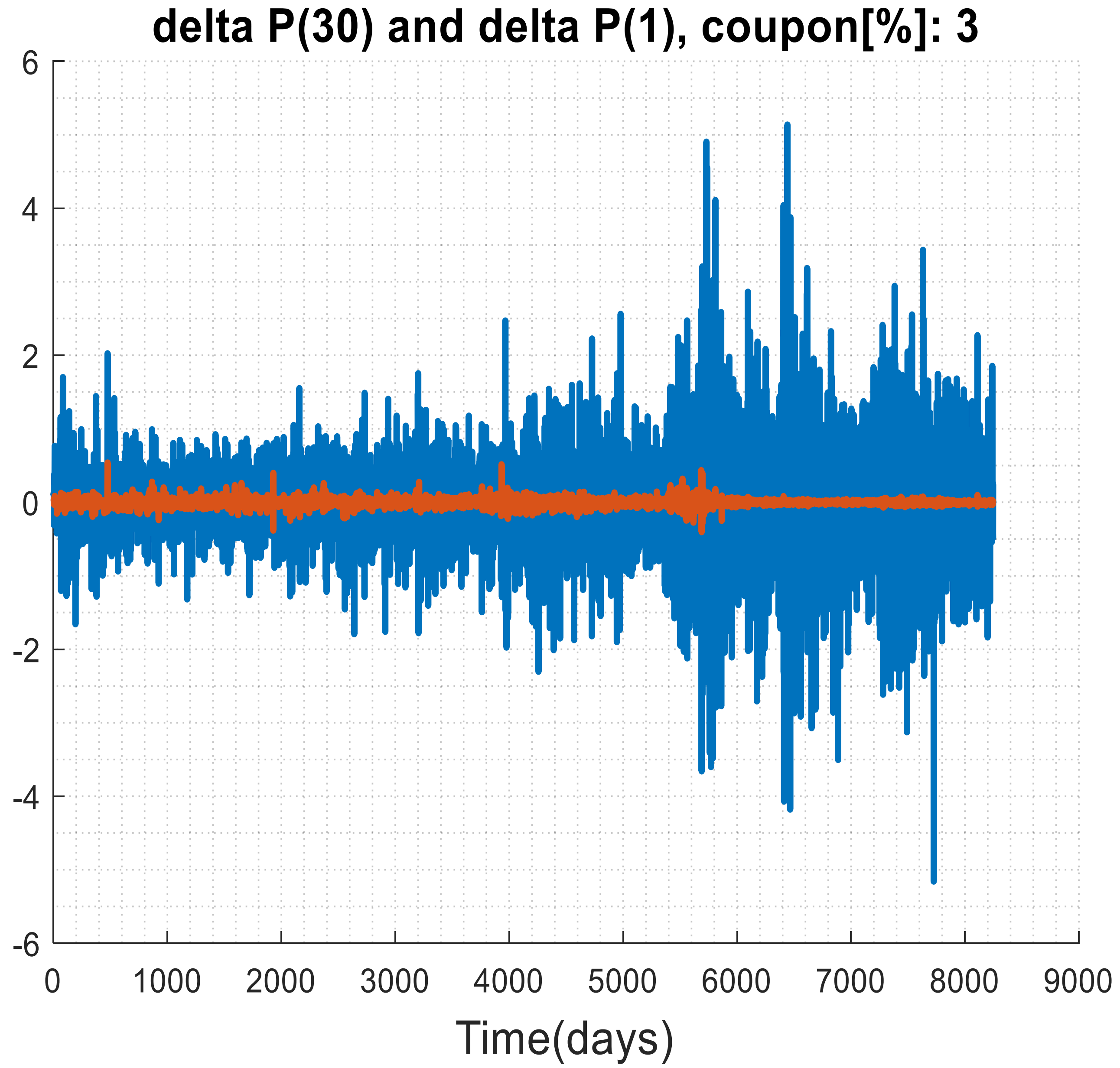
Empirical duration concept, empirical observations



USD rates

Source: Author

Empirical duration concept, empirical observations



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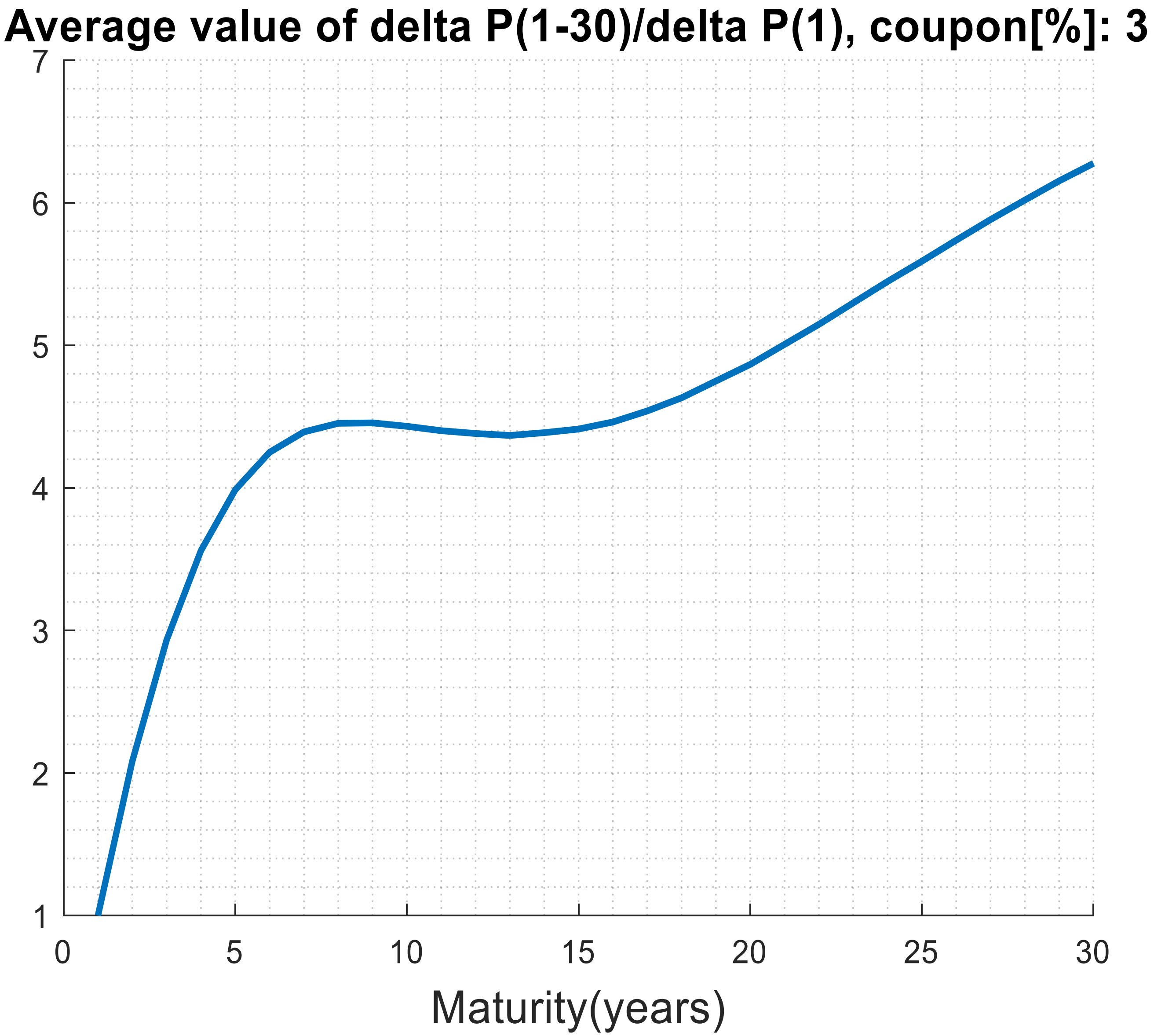
USD rates

Empirical duration concept, empirical observations



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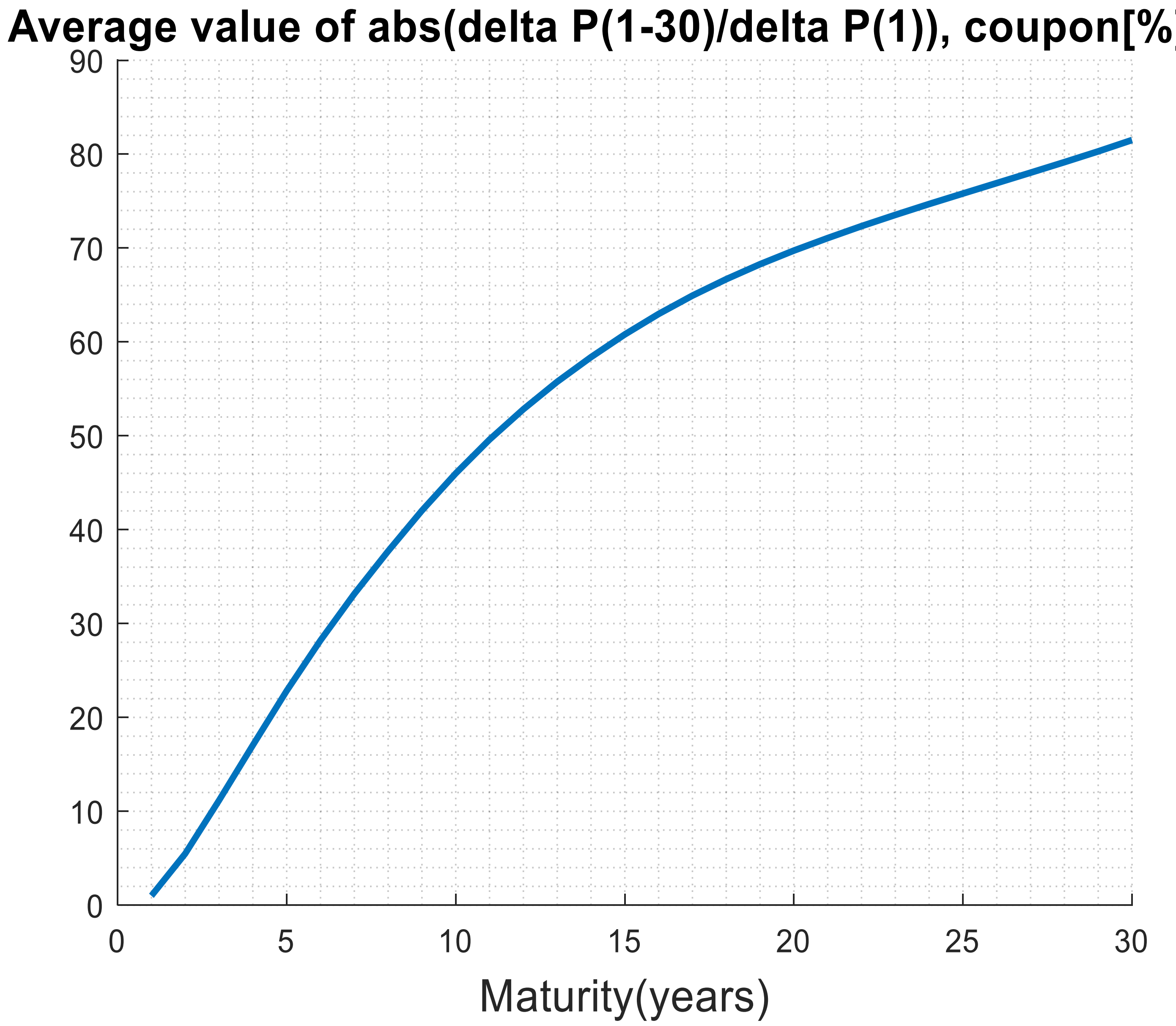
USD rates

Empirical duration concept, empirical observations



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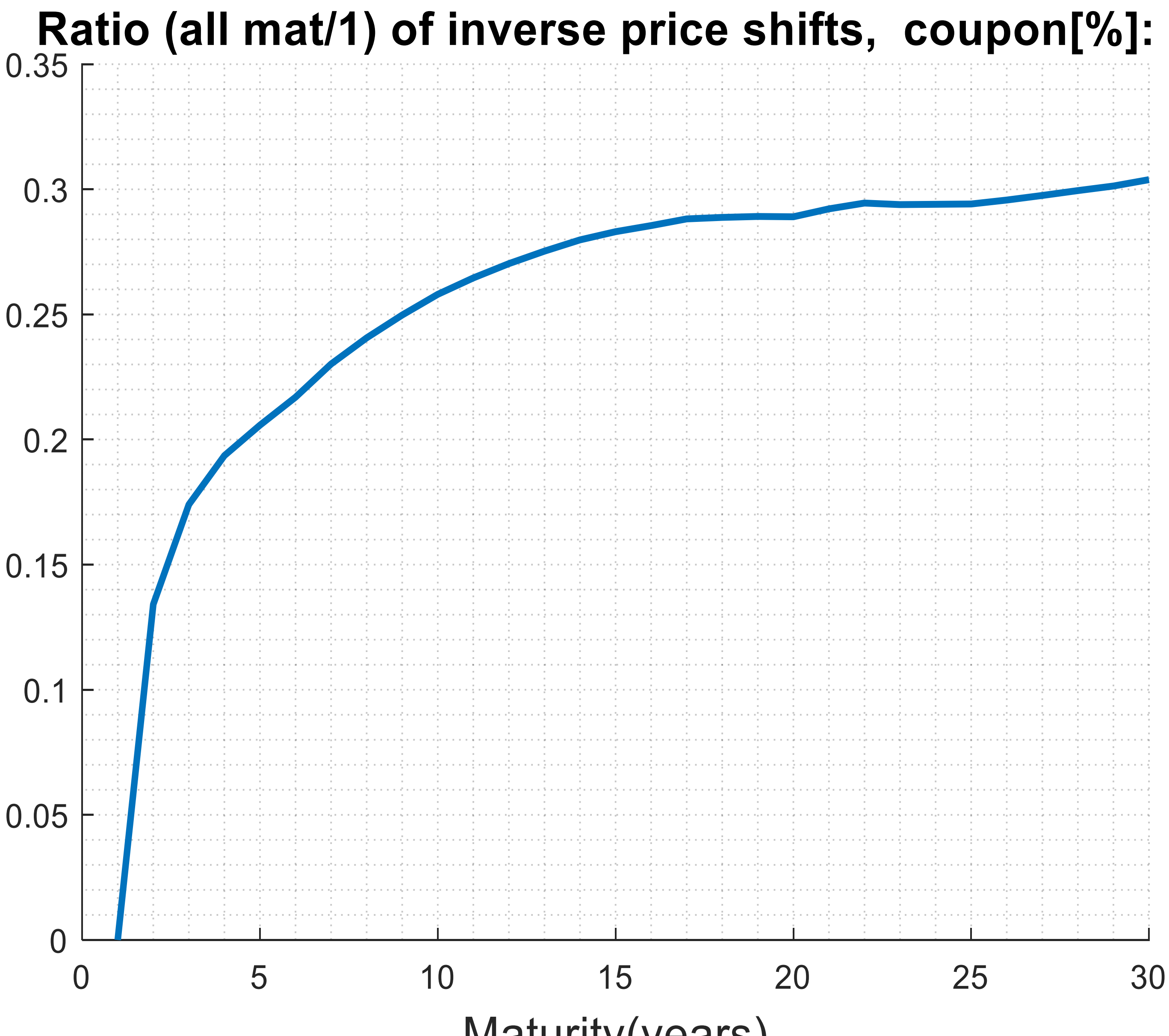
USD rates

Empirical duration concept, empirical observations



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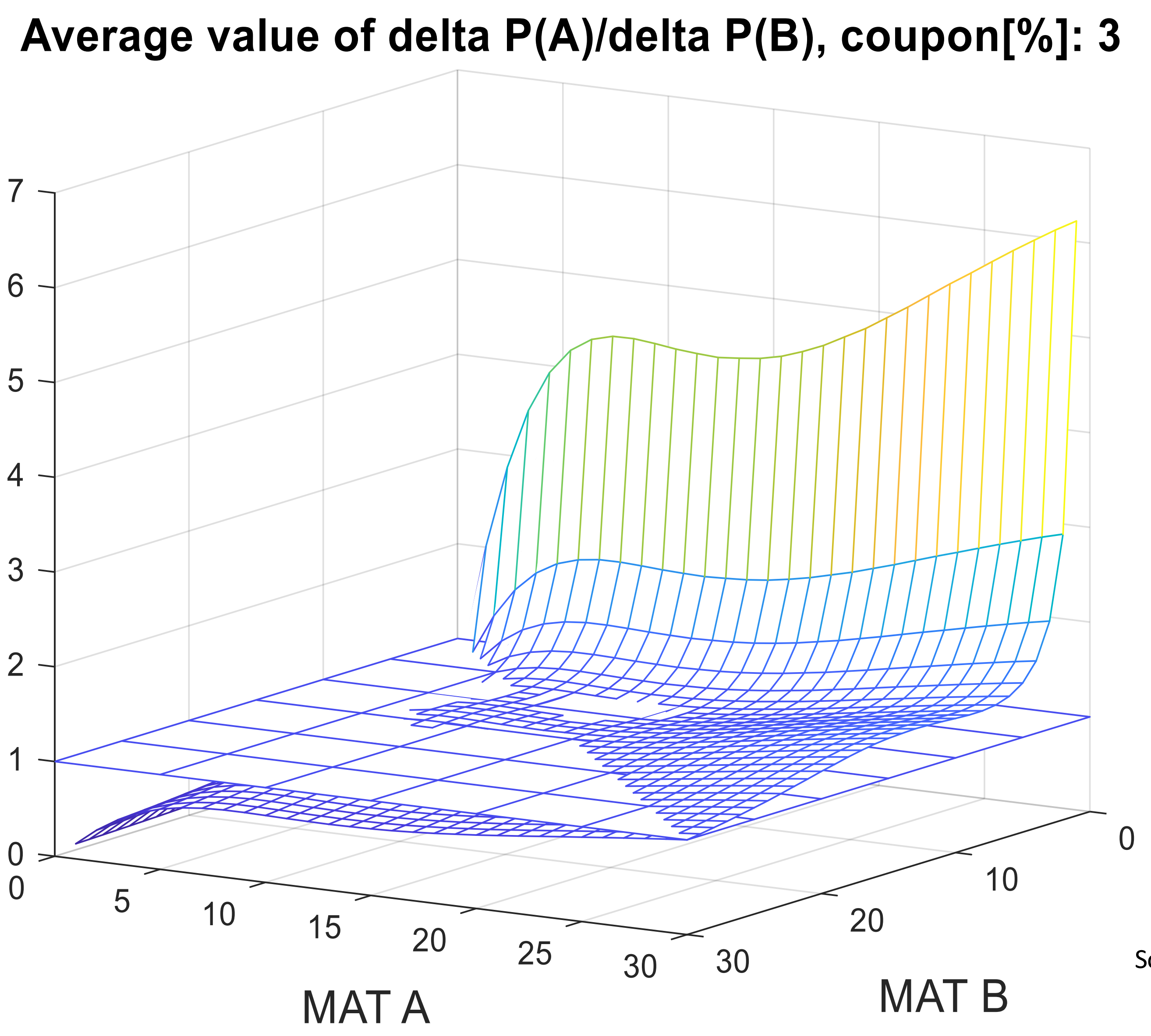
USD rates

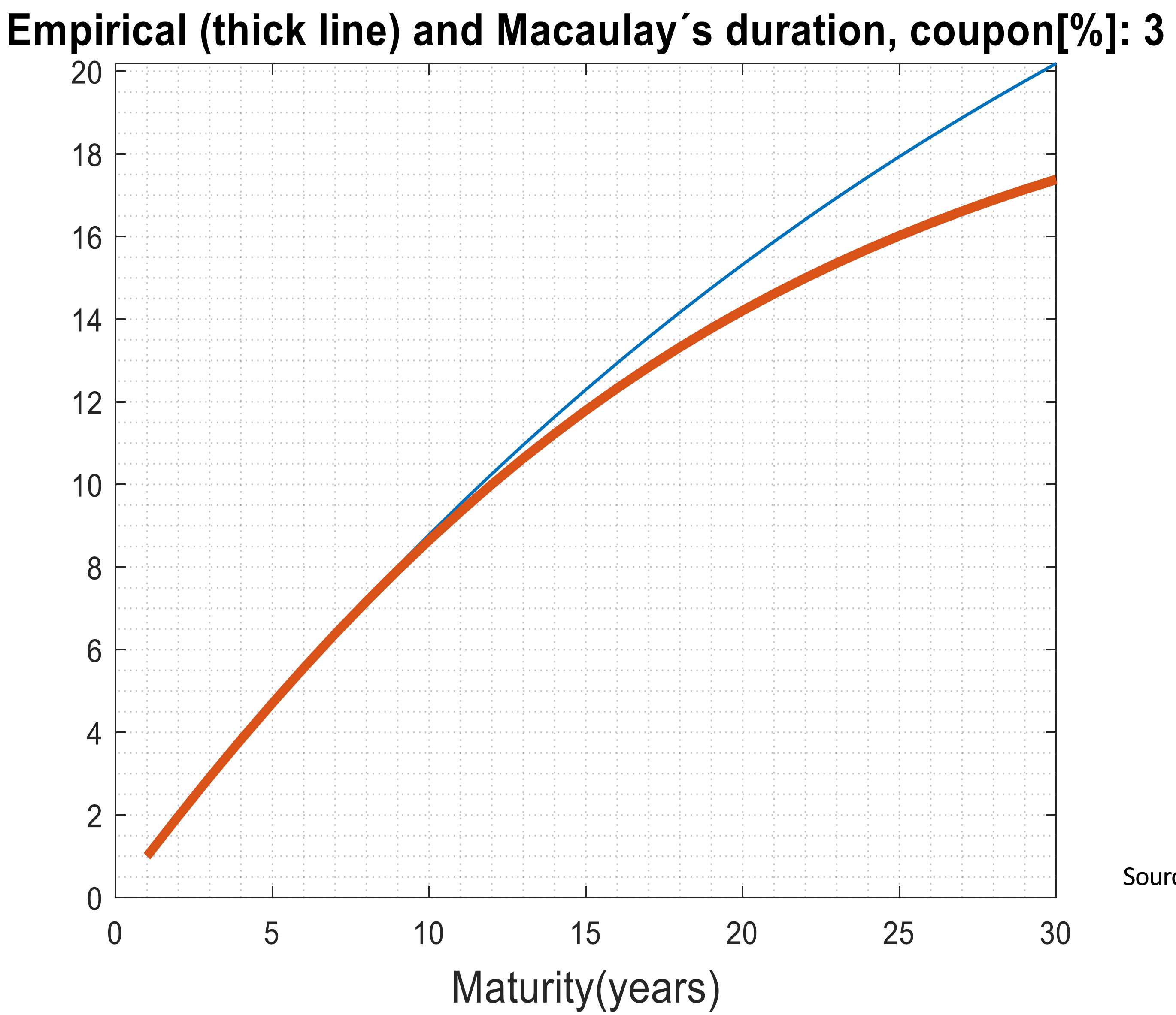
Empirical duration concept, empirical observations



Source: Author

USD rates

Empirical duration concept, results



Source: Author

USD rates

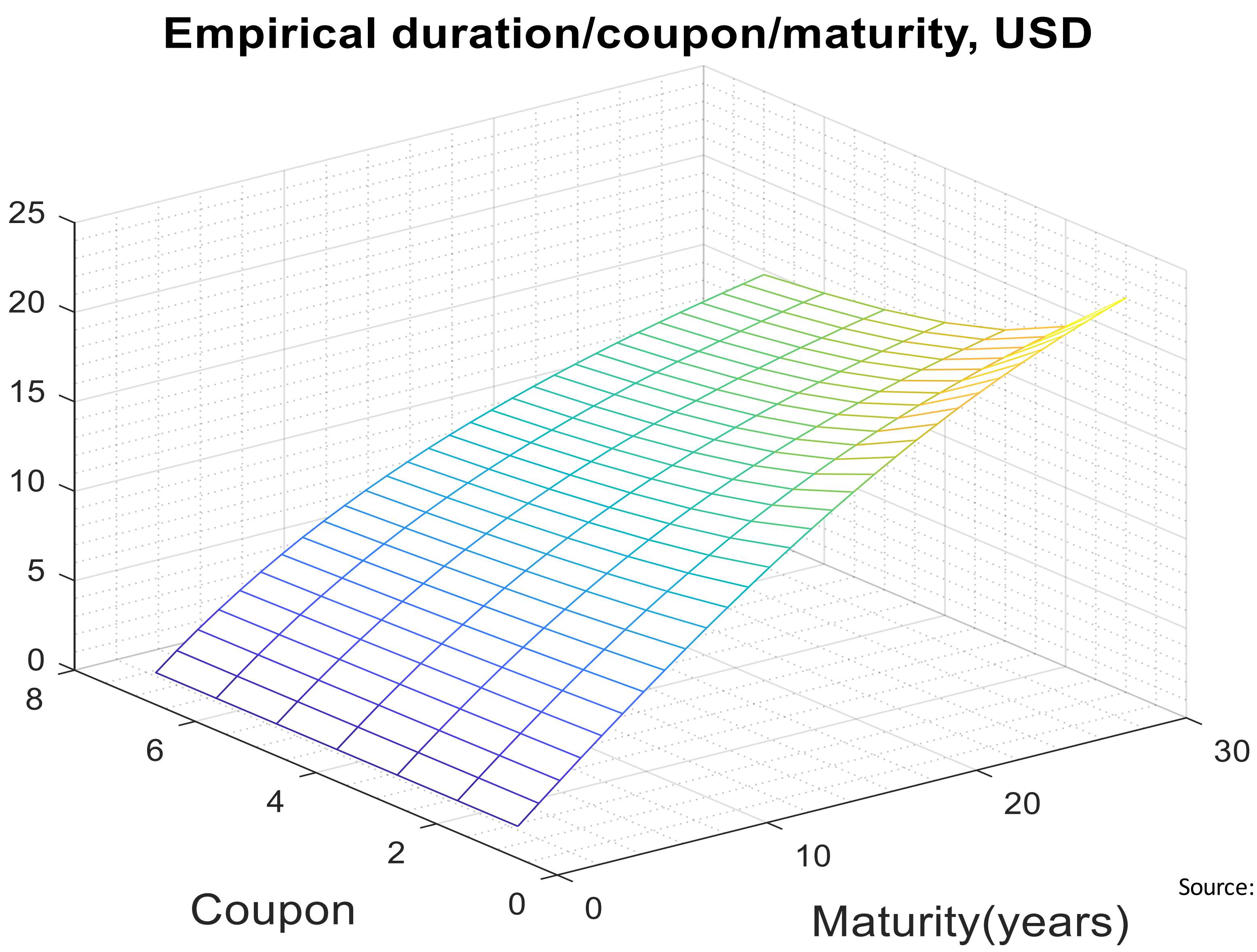
Empirical duration concept, results

MAT	1	2	3	4	5	6	7	8	9	10
E. DUR	1,00	1,97	2,91	3,82	4,71	5,56	6,38	7,17	7,92	8,65
MAT	11	12	13	14	15	16	17	18	19	20
E. DUR	9,34	10,00	10,63	11,23	11,79	12,33	12,84	13,32	13,78	14,21
MAT	21	22	23	24	25	26	27	28	29	30
E. DUR	14,61	15,00	15,36	15,70	16,02	16,33	16,61	16,88	17,14	17,38

Source: Author

USD rates

Empirical duration concept, results



Source: Author

USD rates

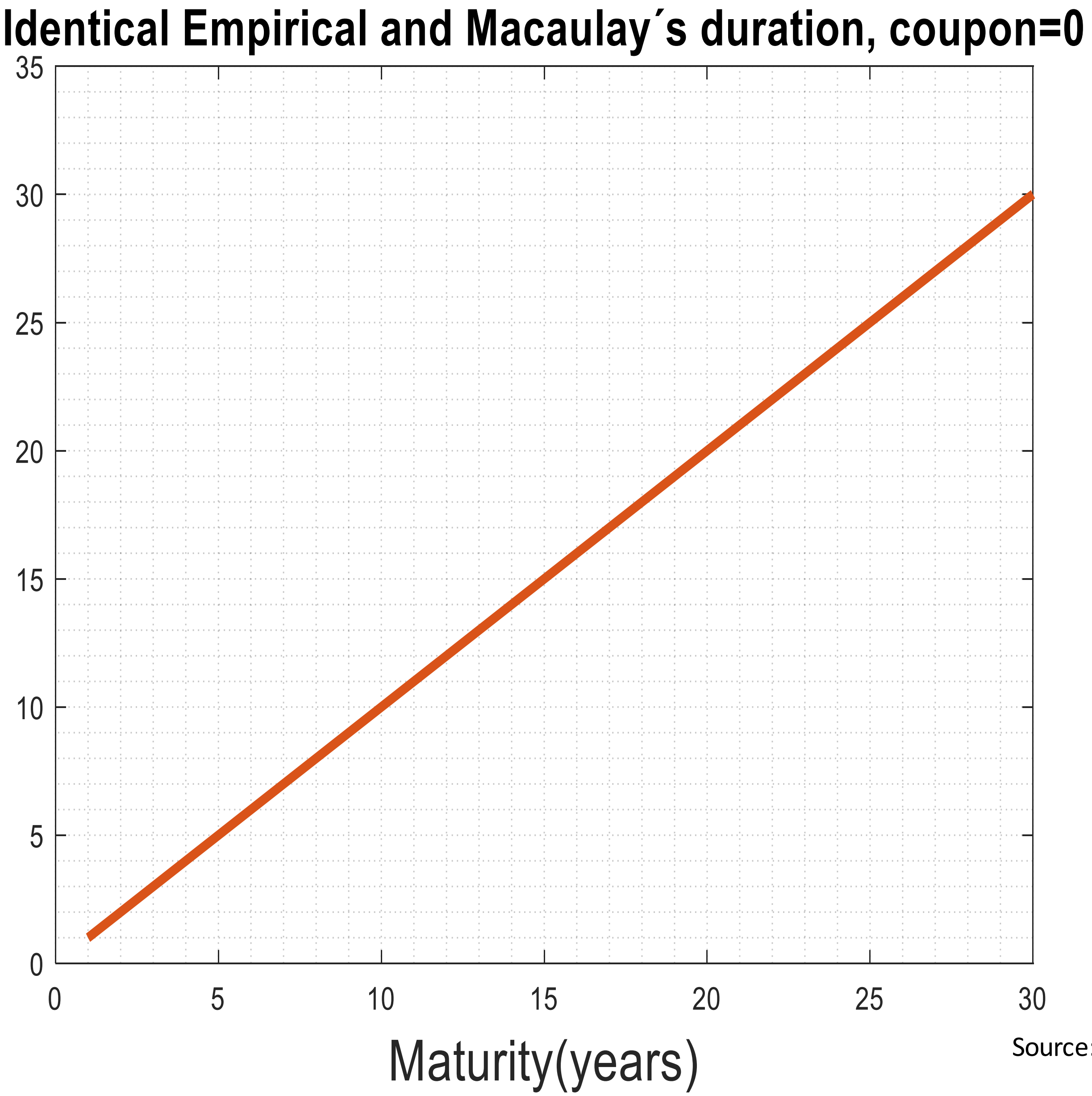
Empirical duration concept, results

MAT\COUPON	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1,99	1,98	1,97	1,96	1,95	1,94	1,94
3	2,97	2,94	2,91	2,89	2,86	2,84	2,82
4	3,94	3,88	3,82	3,77	3,73	3,68	3,64
5	4,89	4,79	4,71	4,63	4,55	4,49	4,42
6	5,83	5,69	5,56	5,44	5,34	5,24	5,16
7	6,76	6,56	6,38	6,22	6,08	5,96	5,85
8	7,67	7,4	7,17	6,97	6,79	6,64	6,51
9	8,57	8,22	7,92	7,68	7,47	7,28	7,12
10	9,45	9,01	8,65	8,35	8,1	7,89	7,7
11	10,3	9,77	9,34	8,99	8,7	8,46	8,25
12	11,14	10,5	10	9,6	9,28	9	8,77
13	11,96	11,2	10,63	10,18	9,81	9,52	9,26
14	12,75	11,88	11,23	10,72	10,32	10	9,73
15	13,52	12,52	11,79	11,24	10,81	10,46	10,17
16	14,27	13,13	12,33	11,73	11,26	10,89	10,59
17	14,99	13,72	12,84	12,19	11,7	11,3	10,98
18	15,69	14,28	13,32	12,63	12,1	11,69	11,36
19	16,36	14,81	13,78	13,04	12,49	12,06	11,72
20	17,01	15,31	14,21	13,43	12,86	12,41	12,06
21	17,62	15,78	14,61	13,8	13,2	12,75	12,38
22	18,22	16,23	15	14,15	13,53	13,06	12,69
23	18,78	16,66	15,36	14,48	13,85	13,37	12,99
24	19,32	17,05	15,7	14,79	14,14	13,65	13,27
25	19,84	17,43	16,02	15,09	14,43	13,93	13,54
26	20,32	17,79	16,33	15,37	14,7	14,19	13,8
27	20,79	18,12	16,61	15,64	14,95	14,45	14,05
28	21,22	18,43	16,88	15,89	15,2	14,69	14,29
29	21,64	18,73	17,14	16,13	15,43	14,92	14,52
30	22,03	19,01	17,38	16,36	15,65	15,14	14,75

Source: Author

USD rates

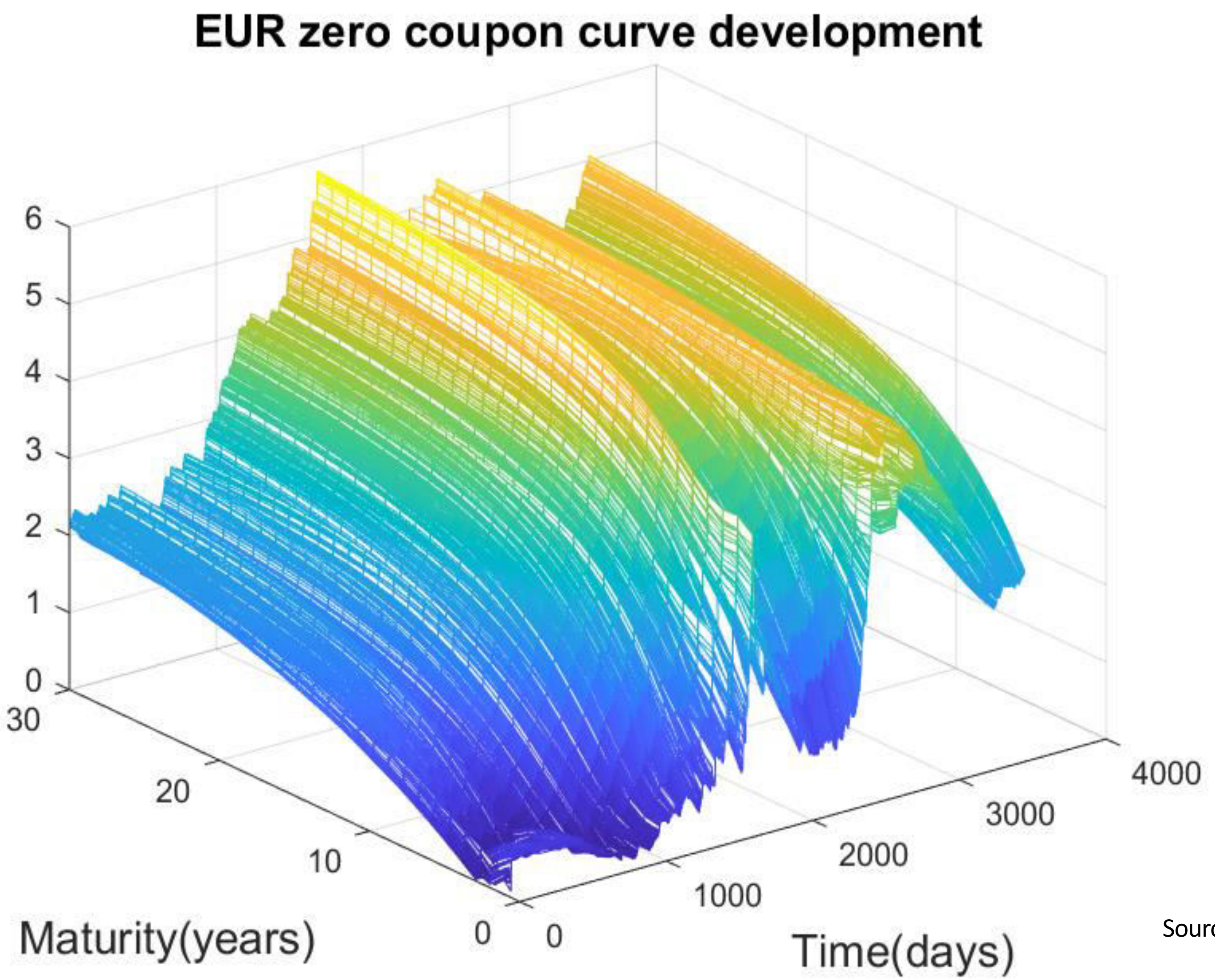
Empirical duration concept, results



USD rates

Source: Author

Empirical duration concept, **EUR** rates market



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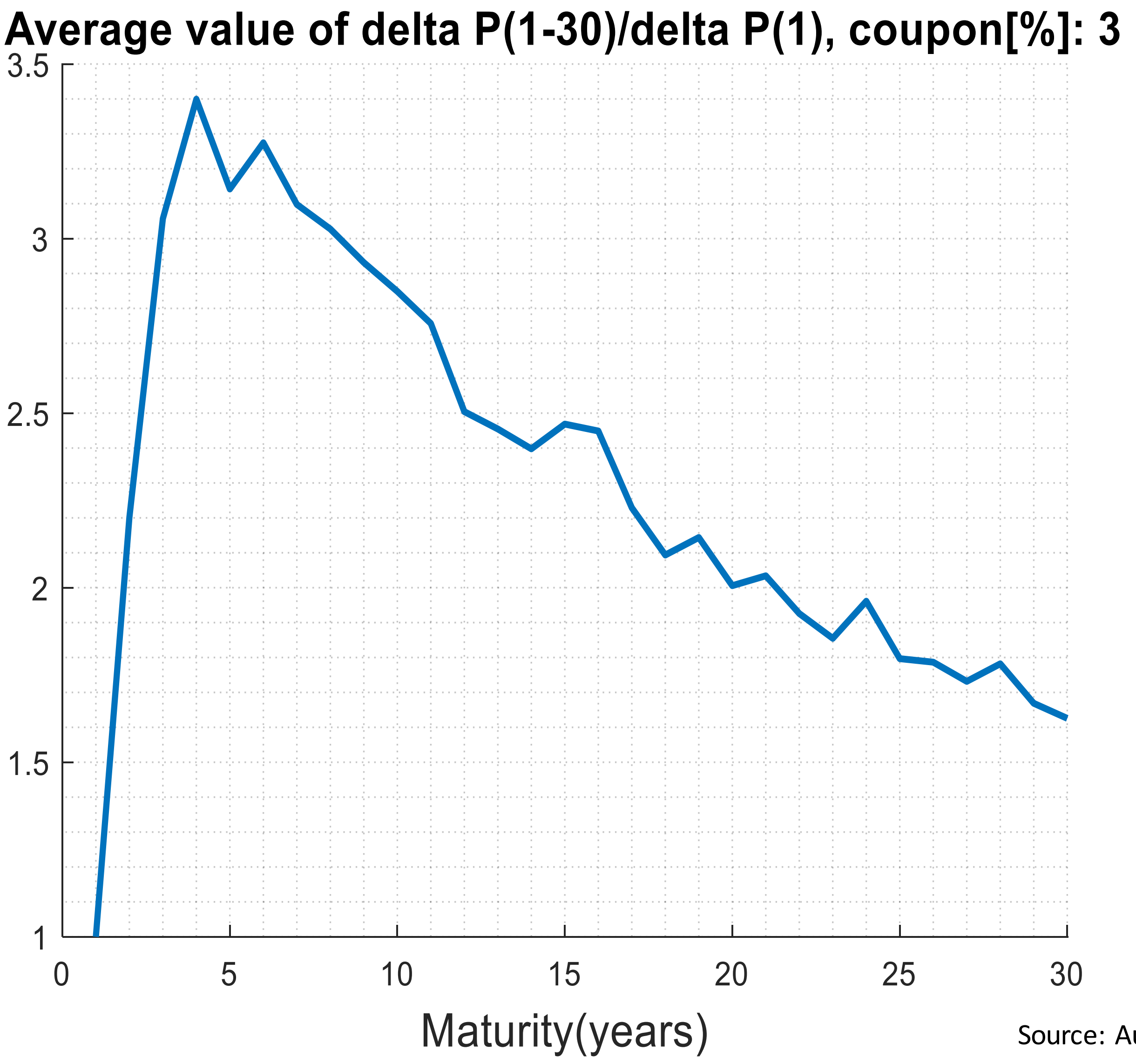
EUR rates

Empirical duration concept, empirical observations

Source: Author

EUR rates

Empirical duration concept, empirical observations



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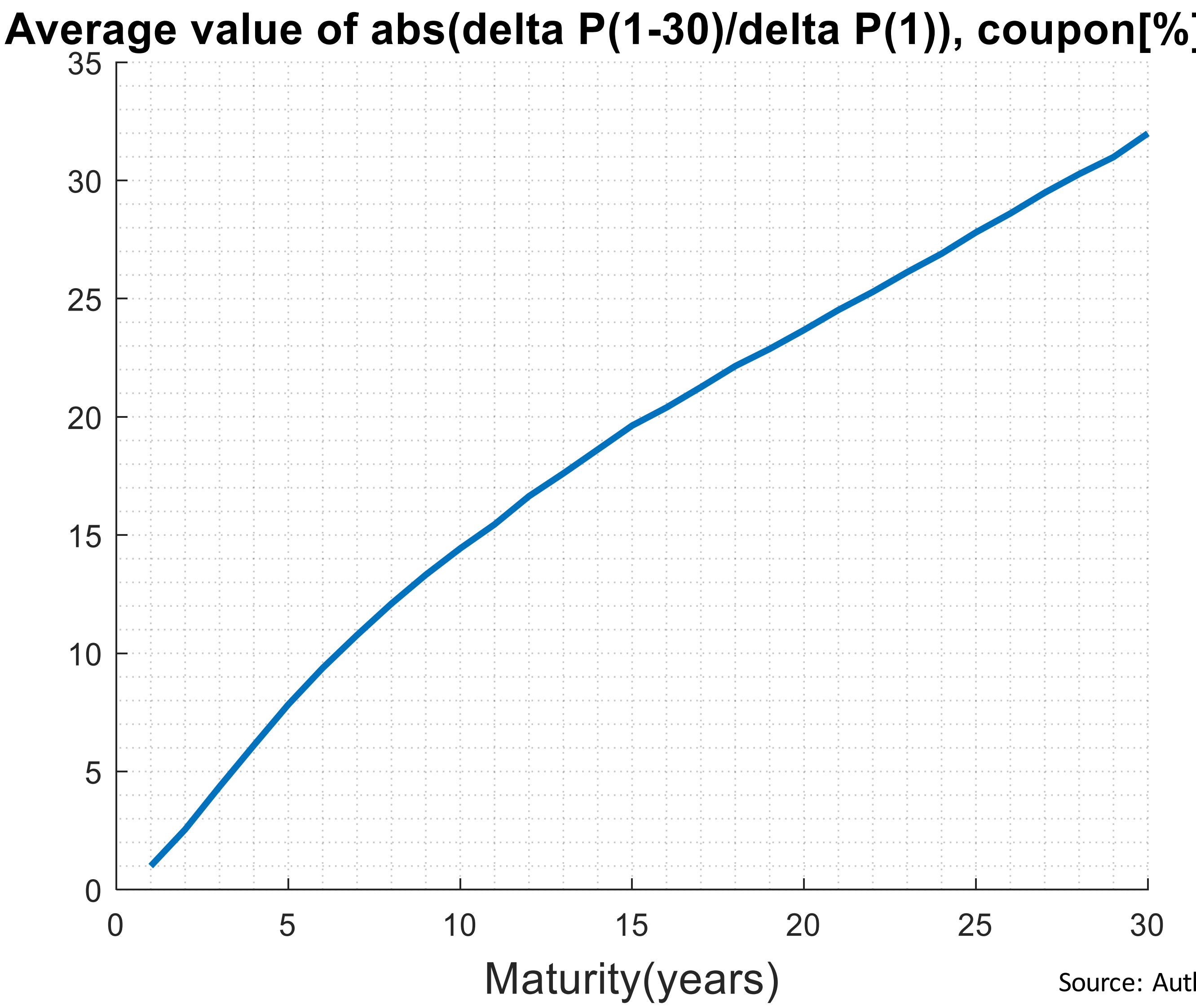
EUR rates

Empirical duration concept, empirical observations



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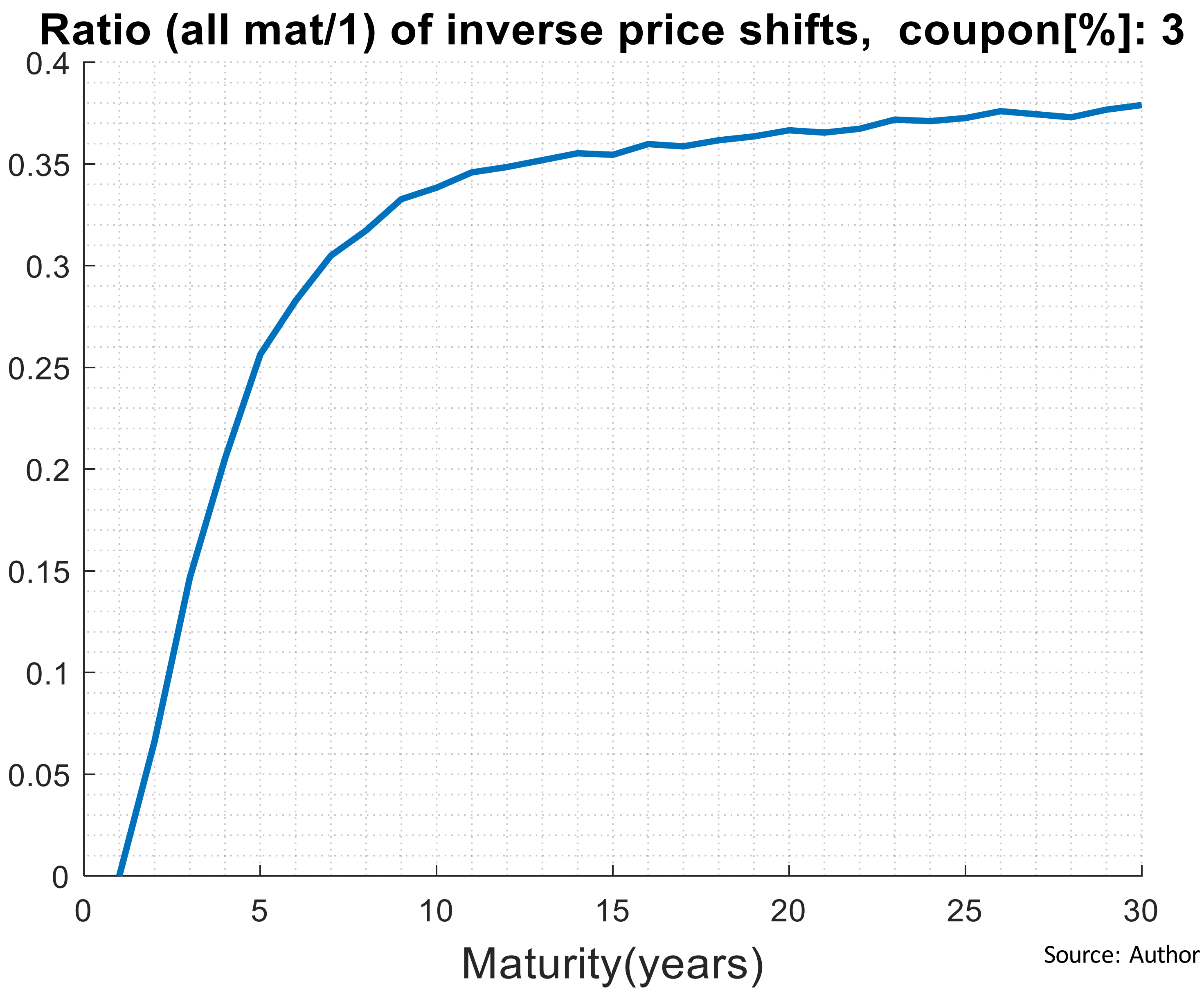
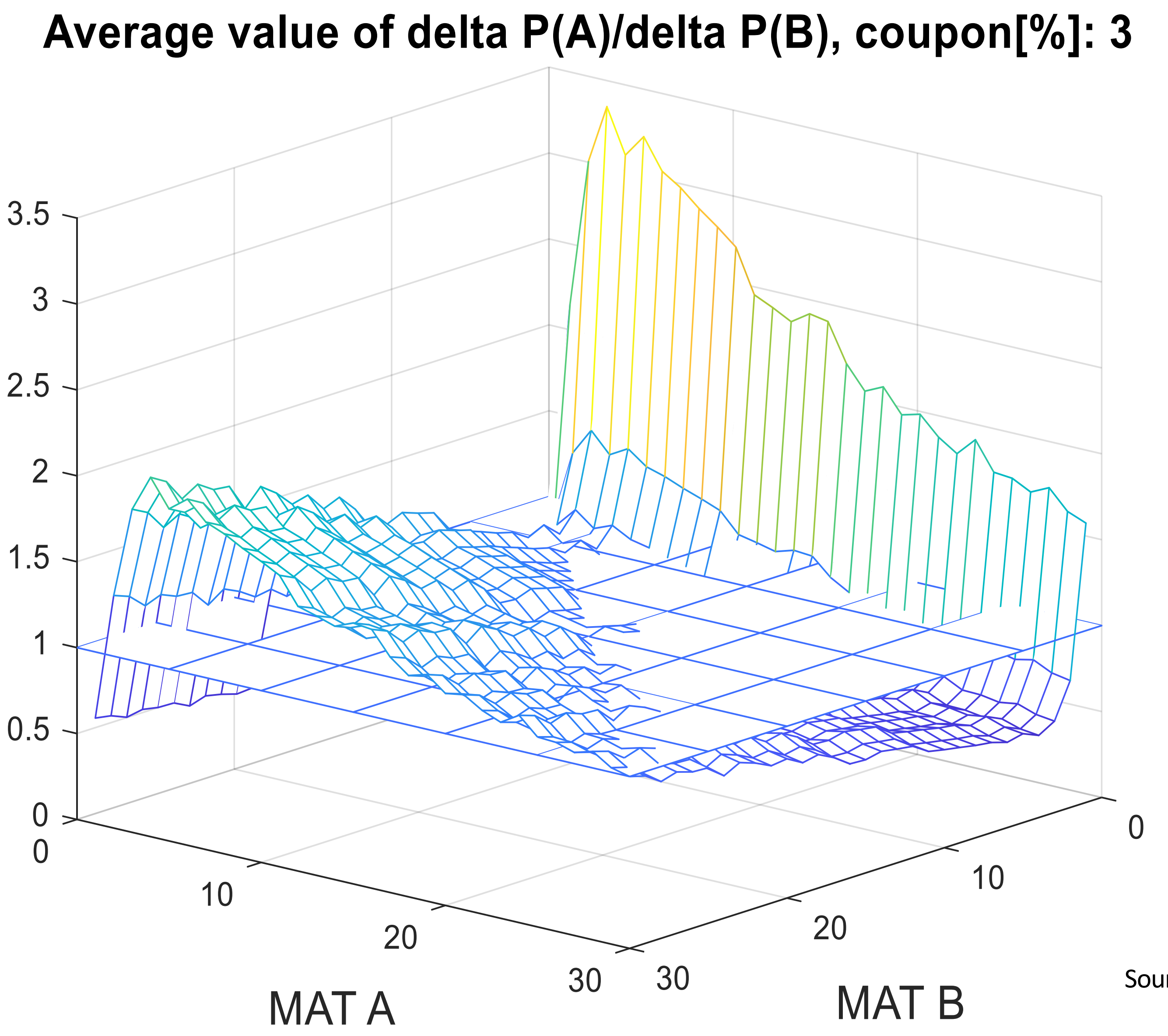
EUR rates

Empirical duration concept, empirical observations



EUR rates

Empirical duration concept, empirical observations



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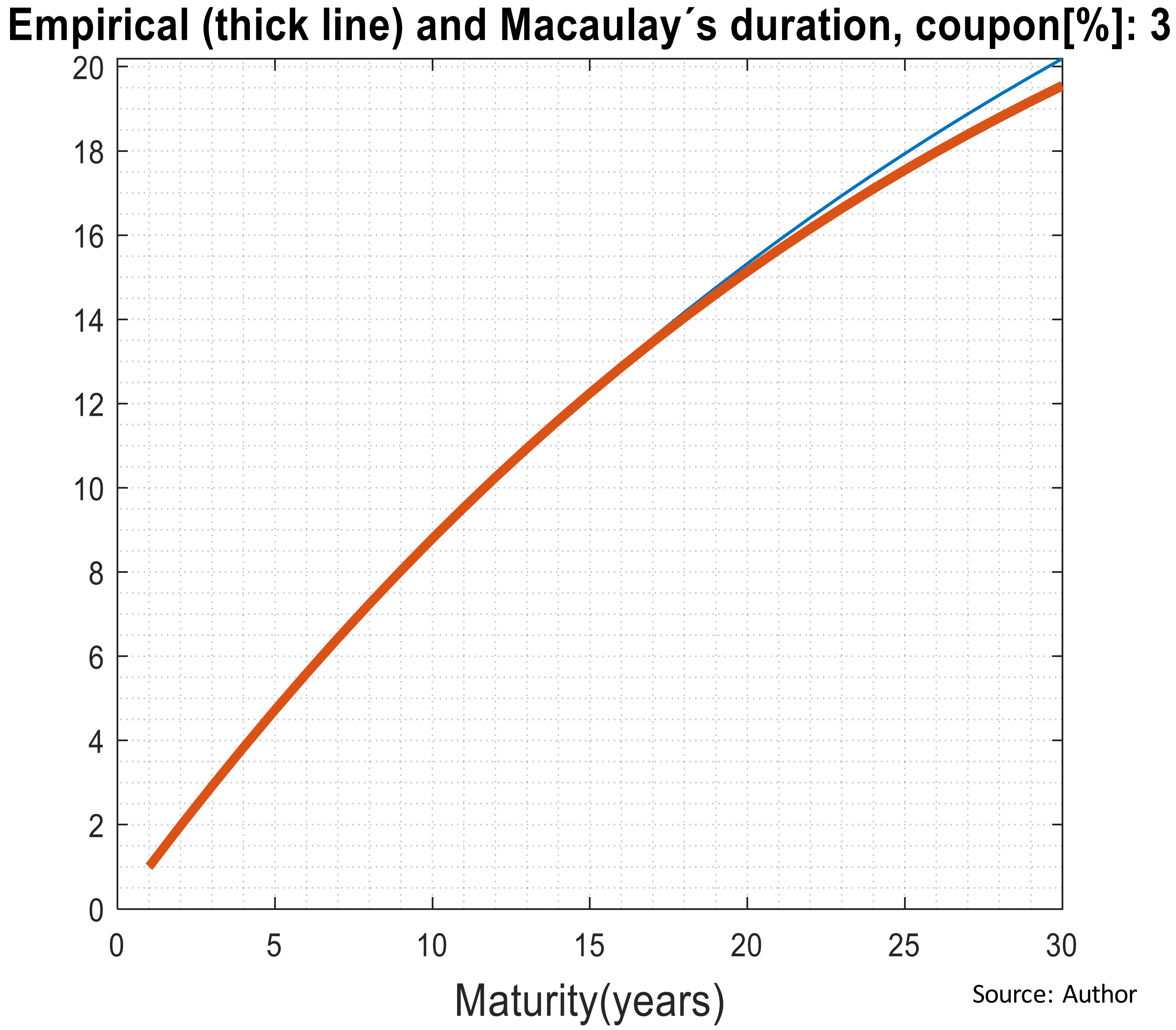
EUR rates

Empirical duration concept, results



Source: Author

EUR rates

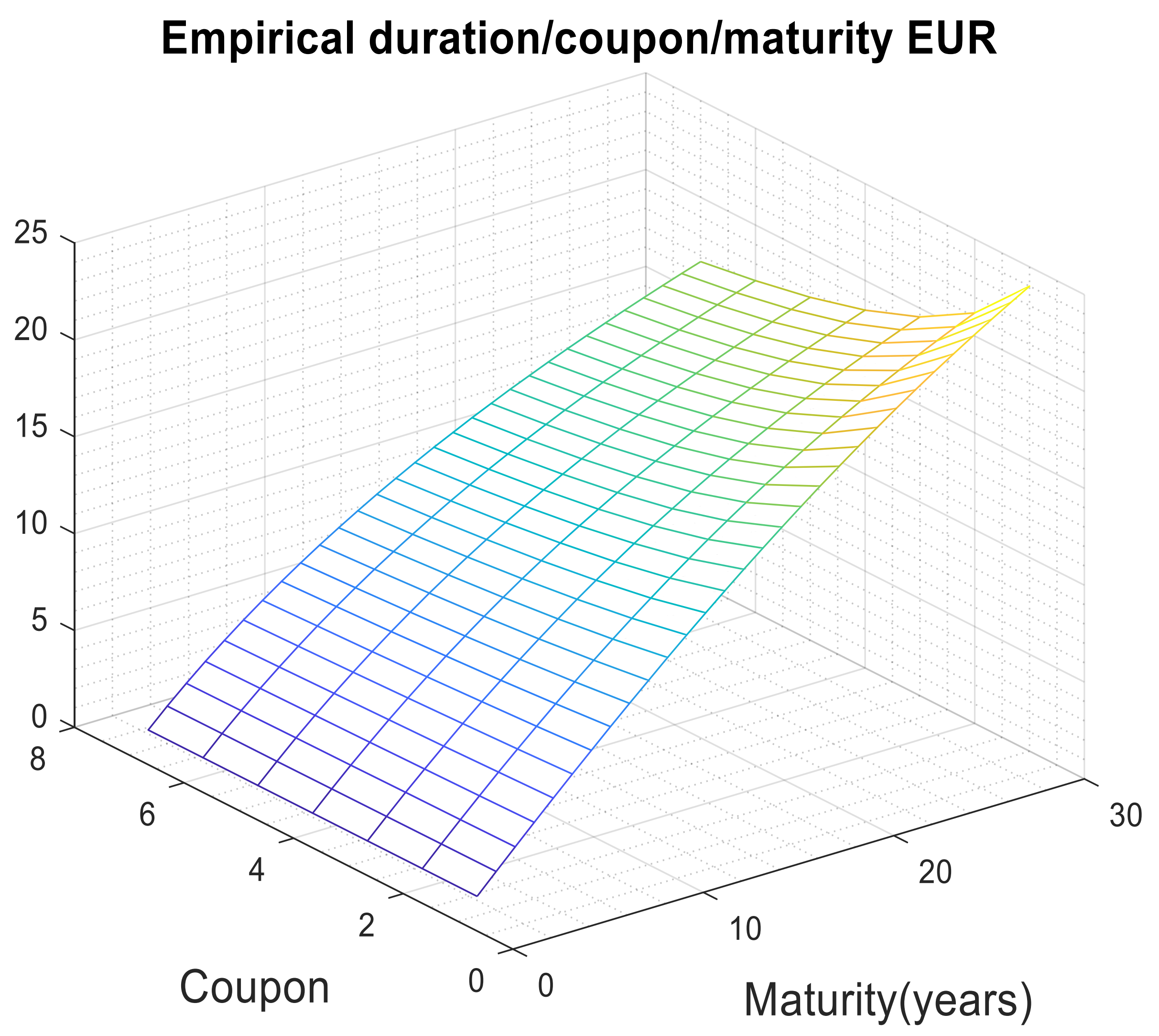
Empirical duration concept, results

MAT.	1	2	3	4	5	6	7	8	9	10
E. DUR	1,00	2,00	2,91	3,83	4,72	5,60	6,43	7,24	8,03	8,80
MAT.	11	12	13	14	15	16	17	18	19	20
E. DUR	9,53	10,25	10,94	11,60	12,25	12,85	13,47	14,04	14,60	15,14
MAT.	21	22	23	24	25	26	27	28	29	30
E. DUR	15,66	16,15	16,64	17,10	17,55	17,98	18,39	18,79	19,18	19,55

Source: Author

EUR rates

Empirical duration concept, results



Source: Author

EUR rates

Empirical duration concept, results

MAT\COUPON	1	2	3	4	5	6	7
1	1,00	1,00	1,00	1,00	1,00	1,00	1,00
2	1,99	1,98	1,97	1,96	1,95	1,95	1,94
3	2,97	2,94	2,92	2,89	2,87	2,84	2,82
4	3,94	3,88	3,83	3,78	3,74	3,70	3,66
5	4,90	4,81	4,72	4,65	4,58	4,51	4,45
6	5,85	5,71	5,59	5,48	5,38	5,29	5,21
7	6,78	6,59	6,43	6,28	6,15	6,04	5,93
8	7,71	7,46	7,24	7,06	6,89	6,75	6,62
9	8,62	8,30	8,03	7,80	7,60	7,43	7,28
10	9,51	9,12	8,79	8,52	8,29	8,09	7,91
11	10,40	9,92	9,53	9,21	8,94	8,71	8,51
12	11,27	10,70	10,25	9,88	9,57	9,31	9,09
13	12,12	11,45	10,94	10,52	10,18	9,89	9,65
14	12,96	12,19	11,60	11,14	10,76	10,45	10,18
15	13,78	12,90	12,25	11,73	11,32	10,98	10,70
16	14,58	13,60	12,87	12,30	11,86	11,50	11,20
17	15,37	14,27	13,46	12,86	12,38	11,99	11,67
18	16,14	14,92	14,04	13,39	12,88	12,47	12,14
19	16,90	15,55	14,60	13,90	13,36	12,93	12,58
20	17,63	16,15	15,14	14,39	13,83	13,38	13,02
21	18,35	16,74	15,65	14,87	14,28	13,81	13,43
22	19,05	17,31	16,15	15,33	14,71	14,23	13,84
23	19,74	17,86	16,64	15,77	15,13	14,63	14,23
24	20,40	18,39	17,10	16,20	15,53	15,02	14,61
25	21,05	18,90	17,55	16,61	15,92	15,40	14,98
26	21,67	19,39	17,98	17,01	16,30	15,76	15,34
27	22,28	19,87	18,39	17,39	16,67	16,12	15,69
28	22,87	20,33	18,79	17,76	17,02	16,47	16,03
29	23,44	20,77	19,18	18,12	17,37	16,80	16,36
30	24,00	21,19	19,55	18,47	17,70	17,13	16,68

Source: Author

EUR rates

Conclusion

Long bonds are not so risky as they should be according Macaulya's duration.

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