Introduction to Insurers risk management

1BP461



EVROPSKÁ UNIE Evropské strukturální a investiční fondy Operační program Výzkum, vývoj a vzdělávání

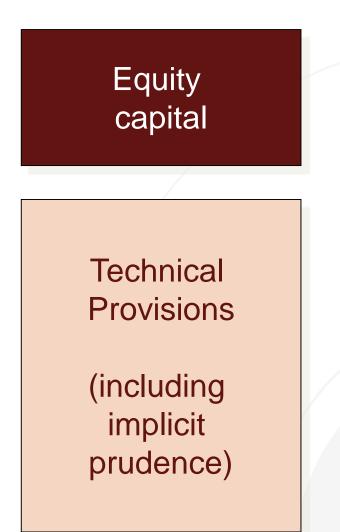


References: DOFF, Rene. Risk Management for Insurers. 2. London: Incisive Media Investments, 2011. ISBN 978 1 906348 61 8.

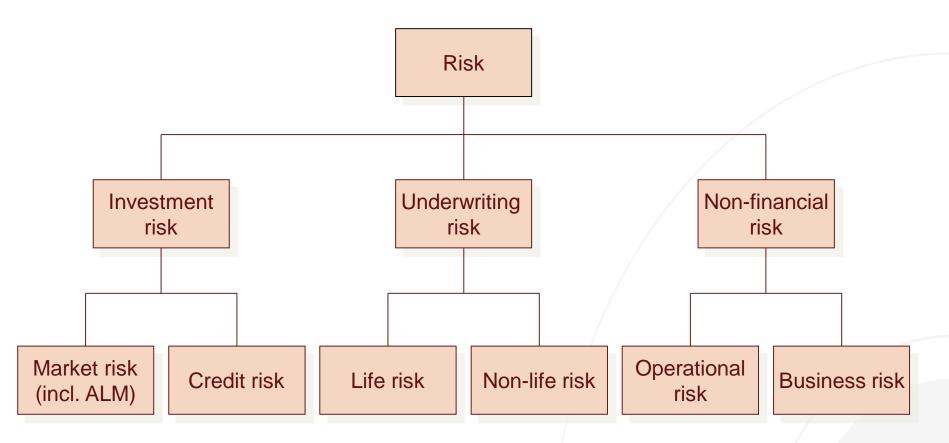
Insurance balance sheet

Investments

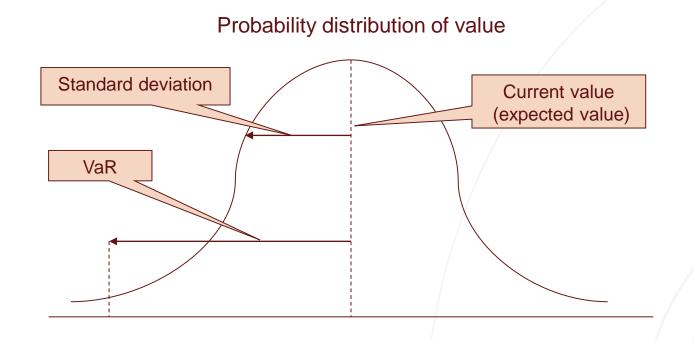
(government) Bonds, stocks, other investments



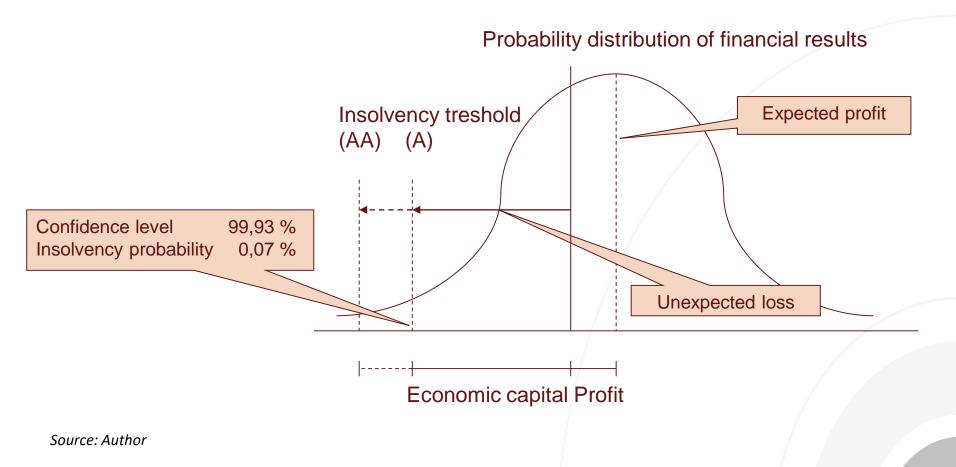
Risk categories



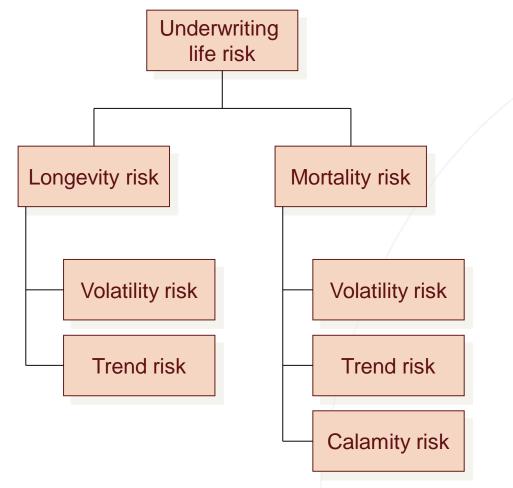
VaR as a multiple of standard deviation



Economic capital rating ambition

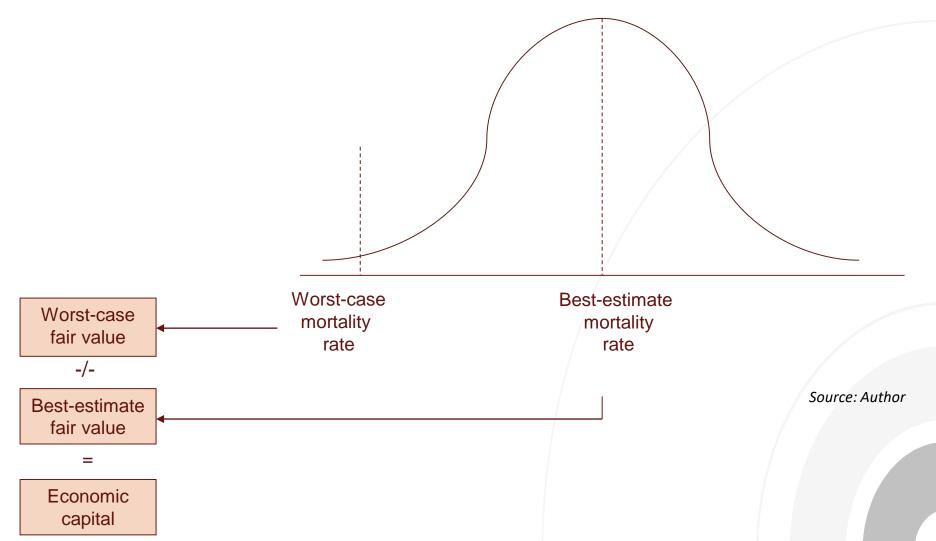


Components of life underwriting risk

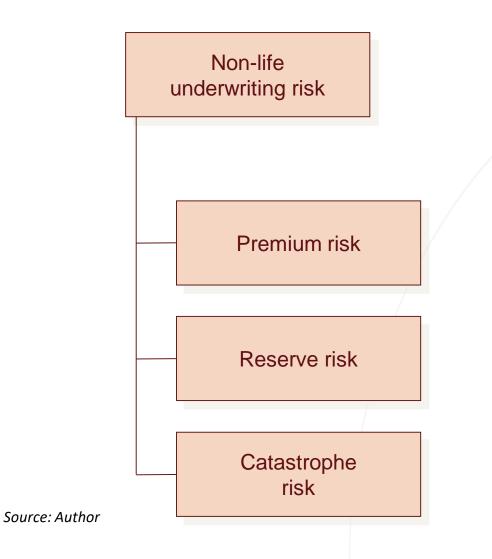


Economic capital for life risk

Probability distribution of mortality rate



Components of non-life underwriting risk



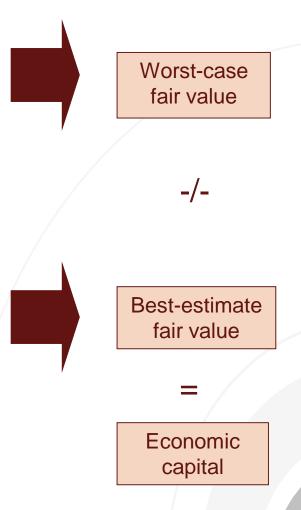
Economic capital for reserve risk

			1		Develo 2		nt year 3		4	ļ	5
	2002	€	100	€	50	€	30	€	10	€	5
year	2003	€	103	€	51	€	31	€	10	€	6
	2004	€	106	€	53	€	32	€	14	€	6
lent	2005	€	73	€	37	€	28	€	9	€	5
ccide	2006	€	149	€	93	€	56	€	19	€	9
Ac	2007	€	193	€	96	€	58	€	19	€	10

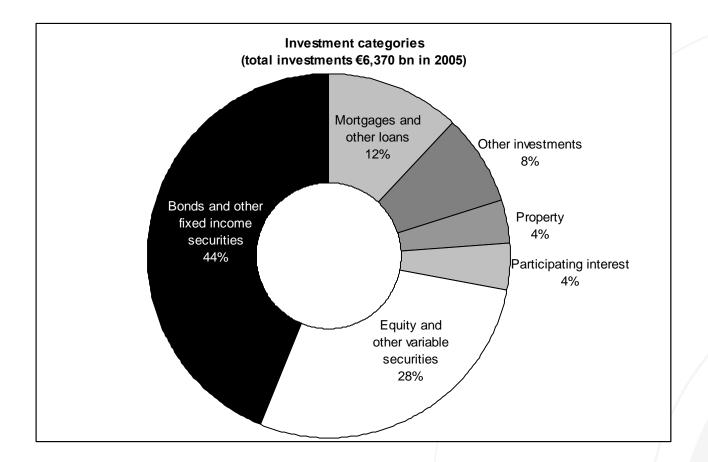
Worst-case (99,95 %) run-off pattern

					Develo	opmer	nt year					
			1	, ,	2	4	3	4	4	Ę	5	
	2002	€	100	€	50	€	30	€	10	€	5	
eal	2003	€	103	€	51	€	31	€	10	€	5	
t y	2004	€	106	€	53	€	32	€	11	€	5	
len	2005	€	73	€	37	€	22	€	7	€	4	
ccid	2006	€	149	€	74	€	45	€	15	€	7	
Ac	2007	€	154	€	77	€	46	€	15	€	8	

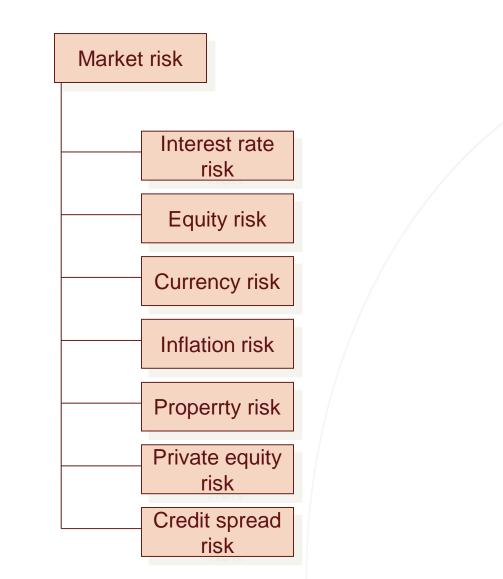
Best-estimate (expected) run-off pattern



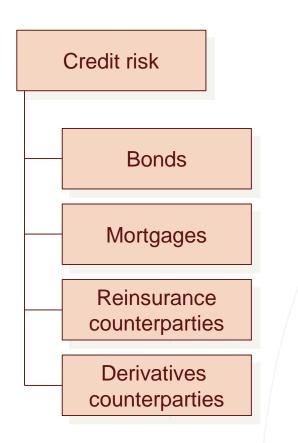
Total investment portfolio of European insurers



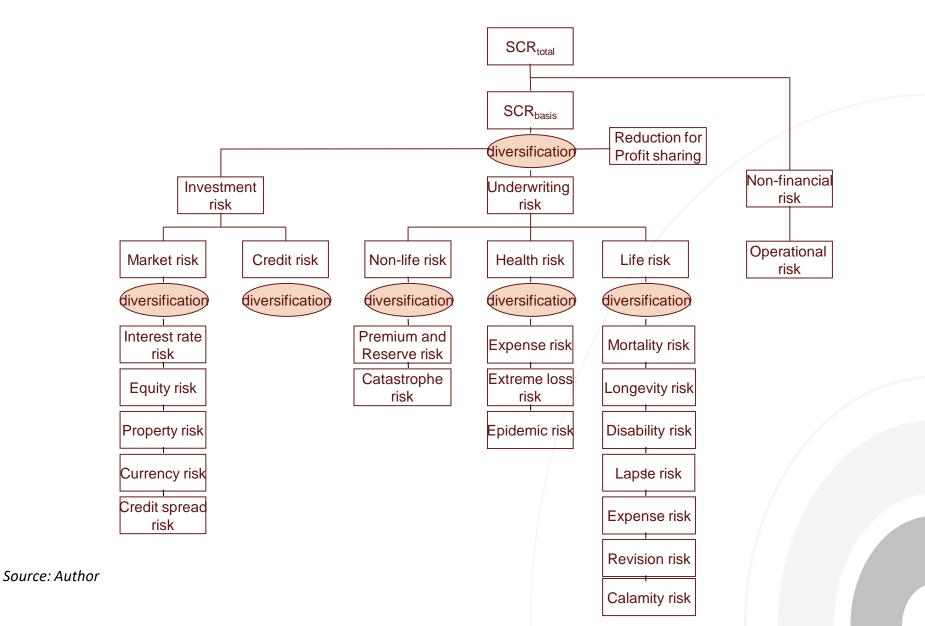
Components of market risk



Components of credit risk



Modular standardised approach to the SCR



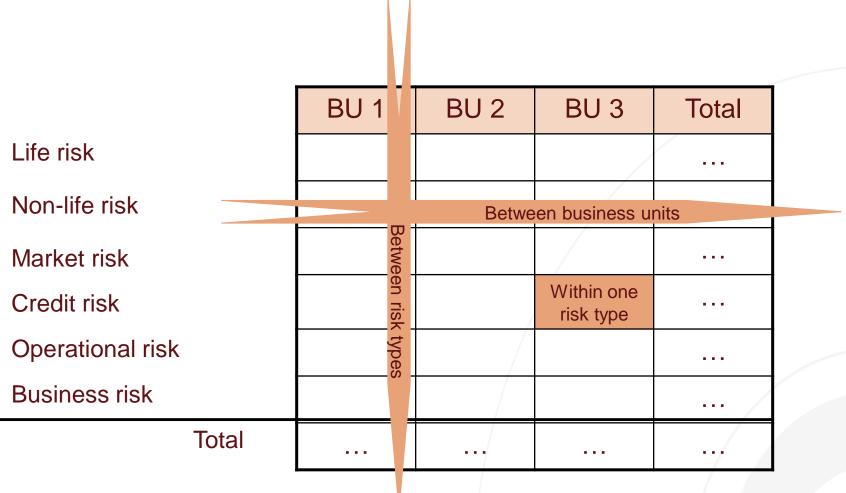
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Calculation methods for each of the SCR modules in QIS3

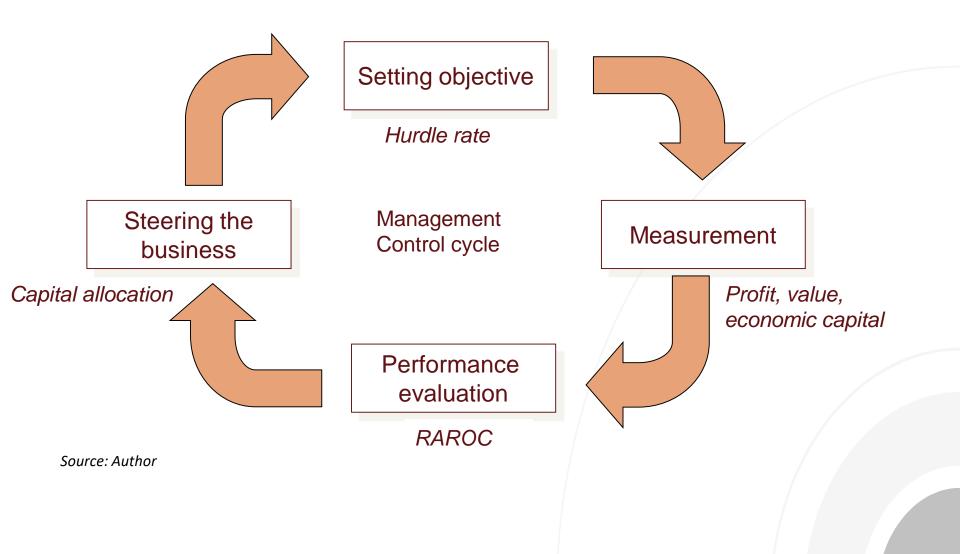
•	Risk type	Component	Methodology for SCR
•	Market risk	Interest rate ri	sk Maximum of upward and downward interest rate shocks (non-parallel)
•	Equity risk		32% decrease for global stock markets and 45% for other markets
•	Propety risk		20% decrease in property markets
•	Currency risk	c 20% change in	foreign exchange rates
•	Credit spread		Market value of the bond times duration times a factor, depending on rating of the bond
•	Concentratio	on An additional	charge for investments with exposures above specified tresholds
•	Aggregation		Correlation matrix and taking into account reductions for profit sharing
•	Credit risk		Replacement costs of a certain credit exposure times a function based on
•			PD per exposure (consistnet with Basel II)
•	Non-life risk	Premium and	Premiums or reserves times function of standard deviation
•	reserve risk		
•	CAT risk		Sum of losses of series of specified CAT scenarios
•	Aggregation		Correlation matrix
•	Health risk	Expense risk	Factor times standard deviation of expenses (10 year historical data) times gross premium
•	Claim risk		Factor times standard deviation of health results (10 year historical data) times gross premium
•	Epidemic risl	K Factor times g	ross premium times market share
•	Aggregation		Correlation matrix and taking into account reductions for profit sharing
•	Life risk Mortality risl	k 10% increase i	in mortality rates in each age class
•	Longevity ris	k 25% decrease	in mortality rates in each age class
•	Disability		35% increase in disability rates fot next year, combined with 25% permanent increase
•	Lapse risk		50% increase in lapse rates on permanent annual increase of 3% of lapse rates
•	Expense risk	10% increase i	in expenses and 1% increase in permanent expense inflation rate
•	Revision risk	3% increase in	annuity benefits for relevant products
•	Calamity risk	A factor for ind	creased mortality rates andfor increased lapse rates in case of a CAT event
•	Aggregation		Correlation matrix and taking into account reductions for profit sharing
•	Operational		Factor times earnings or technical provisions, depending on business line

• risk

Three kinds of diversification



Management control cycle and economic capital



Introduction to ALM

References: HULL, John. Risk management and financial institutions. 5. London: John Wiley & Sons, 2008. ISBN 0-13-613427-0



Today

- An introduction to the asset/liability management (ALM) process
 - What is the goal of ALM?
- The concepts of duration and convexity
 - Extremely important for insurance enterprises
- Extensions to duration
 - Partial duration or key rate duration

Asset/Liability Management

- As its name suggests, ALM involves the process of analyzing the interaction of assets and liabilities
- In its broadest meaning, ALM refers to the process of maximizing <u>risk-adjusted</u> return
- Risk refers to the variance (or standard deviation) of earnings
- More risk in the surplus position (assets minus liabilities) requires extra capital for protection of policyholders

The ALM Process

- Firms forecast earnings and surplus based on "best estimate" or "most probable" assumptions with respect to:
 - Sales or market share
 - The future level of interest rates or the business activity
 - Lapse rates
 - Loss development
- ALM tests the sensitivity of results for changes in these variables

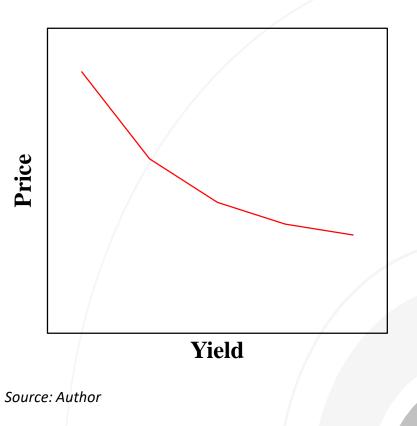


The Goal of ALM

- If assets can be purchased to replicate the liabilities in every potential future state of nature, there would be no risk
- The goal of ALM is to analyze how assets and liabilities move to changes in interest rates and other variables
- We will need tools to quantify the risk in the assets AND liabilities

Price/Yield Relationship

- Recall that bond prices move inversely with interest rates
 - As interest rates increase, present value of fixed cash flows decrease
- For option-free bonds, this curve is not linear but *convex*



Price/yield curve



Simplifications

- Fixed income, non-callable bonds
- Flat yield curve
- Parallel shifts in the yield curve

Examining Interest Rate Sensitivity

- Start with two \$1000 face value zero coupon bonds
- One 5 year bond and one 10 year bond
- Assume current interest rates are 8%

Price Changes on Two Zero Coupon Bonds Initial Interest Rate = 8%

Principal	R	5 year	Change	10 year	Change
1000	0.06	747.2582	9.7967%	558.3948	20.5532%
1000	0.07	712.9862	4.7611%	508.3493	9.7488%
1000	0.0799	680.8984	0.0463%	463.6226	0.0926%
1000	0.08	680.5832	0.0000%	463.1935	0.0000%
1000	0.0801	680.2682	-0.0463%	462.7648	-0.0925%
1000	0.09	649.9314	-4.5038%	422.4108	-8.8047%
1000	0.1	620.9213	-8.7663%	385.5433	-16.7641%

Price Volatility Characteristics of Option-Free Bonds

Properties

- 1 All prices move in opposite direction of change in yield, but the change differs by bond
- 2+3 The percentage price change is <u>not</u> the same for increases and decreases in yields
- 4 Percentage price increases are greater than decreases for a given change in basis points

Characteristics

- 1 For a given term to maturity and initial yield, the lower the coupon rate the greater the price volatility
- 2 For a given coupon rate and intitial yield, the longer the term to maturity, the greater the price volatility



Macaulay Duration

- Developed in 1938 to measure price sensitivity of bonds to interest rate changes
- Macaulay used the weighted average term-to-maturity as a measure of interest sensitivity
- As we will see, this is related to interest rate sensitivity

Macaulay Duration (p.2)

Macaulay Duration = $\sum_{t=1}^{n} \frac{t \times PVCF_{t}}{k \times PVTCF_{t}}$ t = index for period n = total number of periodk = number of coupon payments per year $PVCF_{+}$ = Present value of cash flow in period t $PVTCF_{t}$ = Total present value of cash flows (price) ffu.vse.cz

Applying Macaulay Duration

- For a zero coupon bond, the Macaulay duration is equal to maturity
- For coupon bonds, the duration is less than its maturity
- For two bonds with the same maturity, the bond with the lower coupon has higher duration

Percentage change in price =

 $-\frac{1}{1+\frac{yield}{k}} \times \text{Macaulay duration} \times \text{Yield change} \times 100$

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Modified Duration

- Another measure of price sensitivity is determined by the slope of the price/yield curve
- When we divide the slope by the current price, we get a duration measure called modified duration
- The formula for the predicted price change of a bond using Macaulay duration is based on the first derivative of price with respect to yield (or interest rate)

Modified Duration and Macaulay Duration

$$P = \sum \frac{CF_t}{(1+i)^t}$$

Modified duration $= -\frac{\partial P}{\partial i} \times \frac{1}{P} = \sum \frac{t \times CF_t}{(1+i)^{t+1}} \times \frac{1}{P}$
 $= \frac{1}{(1+i)} \times \text{Macaulay duration}$

i = yield CF = Cash flow

P = price

An Example

Calculate:

What is the modified duration of a 3-year, 3% bond if interest rates are 5%?

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Solution to Example

Period	Cash Flow	PV	t x PV
1	3	2.86	2.86
2	3	2.72	5.44
3	103	88.98	266.93
	Total	94.55	275.23
	aulay duration $=$	94.55	91



Example Continued

 What is the predicted price change of the 3 year, 3% coupon bond if interest rates increase to 6%?



Example Continued

• What is the predicted price change of the 3 year, 3% coupon bond if interest rates increase to 6%?

% Price Change = $-Modified Duration \times Yield Change$ = $-2.77 \times .01 = -2.77\%$

Other Interest Rate Sensitivity Measures

- Instead of expressing duration in percentage terms, <u>dollar</u> <u>duration</u> gives the absolute dollar change in bond value
 - Multiply the modified duration by the yield change <u>and</u> the initial price
- Present Value of a Basis Point (PVBP) is the dollar duration of a bond for a one basis point movement in the interest rate
 - This is also known as the dollar value of an 01 (DV01)



A Different Methodology

- The "Valuation..." book does not use the formulae shown here
- Instead, duration can be computed numerically
 - Calculate the price change given an increase in interest rates of i
 - Numerically calculate the derivative using actual bond prices:

Duration =
$$-\frac{\Delta P}{\Delta i}/P$$

Introduction to VaR

References: HULL, John. Risk management and financial institutions. 5. London: John Wiley & Sons, 2008. ISBN 0-13-613427-0

The Question Being Asked in VaR

"What loss level is such that we are X% confident it will not be exceeded in N business days?"

VaR and Regulatory Capital

- Regulators base the capital they require banks to keep on VaR
- The market-risk capital is k times the 10-day 99% VaR where k is at least 3.0



VaR vs. C-VaR

- VaR is the loss level that will not be exceeded with a specified probability
- C-VaR (or expected shortfall) is the expected loss given that the loss is greater than the VaR level
- Although C-VaR is theoretically more appealing, it is not widely used



Advantages of VaR

- It captures an important aspect of risk in a single number
- It is easy to understand
- It asks the simple question: "How bad can things get?"

Time Horizon

 Instead of calculating the 10-day, 99% VaR directly analysts usually calculate a 1-day 99% VaR and assume

10 - day VaR =
$$\sqrt{10} \times 1$$
 - day VaR

 This is exactly true when portfolio changes on successive days come from independent identically distributed normal distributions



Historical Simulation

- Create a database of the daily movements in all market variables.
- The first simulation trial assumes that the percentage changes in all market variables are as on the first day
- The second simulation trial assumes that the percentage changes in all market variables are as on the second day
- and so on



Historical Simulation continued

- Suppose we use *m* days of historical data
- Let v_i be the value of a variable on day i
- There are *m*-1 simulation trials
- The *i*th trial assumes that the value of the market variable tomorrow (i.e., on day *m*+1) is

$$v_m \frac{v_i}{v_{i-1}}$$

The Model-Building Approach

- The main alternative to historical simulation is to make assumptions about the probability distributions of return on the market variables and calculate the probability distribution of the change in the value of the portfolio analytically
- This is known as the model building approach or the variance-covariance approach



Daily Volatilities

- In option pricing we measure volatility "per year"
- In VaR calculations we measure volatility "per day"

$$\sigma_{day} = \frac{\sigma_{year}}{\sqrt{252}}$$



Daily Volatility continued

- Strictly speaking we should define σ_{day} as the standard deviation of the continuously compounded return in one day
- In practice we assume that it is the standard deviation of the percentage change in one day



Microsoft Example

- We have a position worth \$10 million in Microsoft shares
- The volatility of Microsoft is 2% per day (about 32% per year)
- We use *N*=10 and *X*=99



Microsoft Example continued

- The standard deviation of the change in the portfolio in 1 day is \$200,000
- The standard deviation of the change in 10 days is

$200,000\sqrt{10} = $632,456$

Microsoft Example continued

- We assume that the expected change in the value of the portfolio is zero (This is OK for short time periods)
- We assume that the change in the value of the portfolio is normally distributed
- Since *N*(–2.33)=0.01, the VaR is

2.33 × 632,456 = \$1,473,621



AT&T Example

- Consider a position of \$5 million in AT&T
- The daily volatility of AT&T is 1% (approx 16% per year)
- The S.D per 10 days is
- The VaR is

$$50,000\sqrt{10} = \$158,144$$

158,114 × 2.33 = \$368,405



Portfolio

- Now consider a portfolio consisting of both Microsoft and AT&T
- Suppose that the correlation between the returns is 0.3



S.D. of Portfolio

• A standard result in statistics states that

• In this case $\sigma_X = 200,000$ and $\sigma_Y = 50,000$ and $\rho = 0.3$. The standard deviation of the change in the portfolio value in one day is therefore 220,227

$$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y}$$



VaR for Portfolio

- The 10-day 99% VaR for the portfolio is
- The benefits of diversification are (1,473,621+368,405)–1,622,657=\$219,369
- What is the incremental effect of the AT&T holding on VaR?

$220,227 \times \sqrt{10} \times 2.33 = \$1,622,657$

The Linear Model

We assume

- The daily change in the value of a portfolio is linearly related to the daily returns from market variables
- The returns from the market variables are normally distributed

The General Linear Model continued

$$\Delta P = \sum_{i=1}^{n} \alpha_i \Delta x_i$$
$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j \sigma_i \sigma_j \rho_{ij}$$

1/1

$$\sigma_P^2 = \sum_{i=1}^n \alpha_i^2 \sigma_i^2 + 2 \sum_{i < j} \alpha_i \alpha_j \sigma_i \sigma_j \rho_{ij}$$

where σ_i is the volatility of variable *i* and σ_p is the portfolio's standard deviation

Handling Interest Rates: Cash Flow Mapping

- We choose as market variables bond prices with standard maturities (1mth, 3mth, 6mth, 1yr, 2yr, 5yr, 7yr, 10yr, 30yr)
- Suppose that the 5yr rate is 6% and the 7yr rate is 7% and we will receive a cash flow of \$10,000 in 6.5 years.
- The volatilities per day of the 5yr and 7yr bonds are 0.50% and 0.58% respectively

- We interpolate between the 5yr rate of 6% and the 7yr rate of 7% to get a 6.5yr rate of 6.75%
- The PV of the \$10,000 cash flow is

$$\frac{10,000}{1.0675^{\,6.5}} = 6,540$$



- We interpolate between the 0.5% volatility for the 5yr bond price and the 0.58% volatility for the 7yr bond price to get 0.56% as the volatility for the 6.5yr bond
- We allocate α of the PV to the 5yr bond and (1- α) of the PV to the 7yr bond



- Suppose that the correlation between movement in the 5yr and 7yr bond prices is 0.6
- To match variances
- This gives α =0.074

 $0.56^{2} = 0.5^{2}\alpha^{2} + 0.58^{2}(1-\alpha)^{2} + 2 \times 0.6 \times 0.5 \times 0.58 \times \alpha(1-\alpha)$

The value of 6,540 received in 6.5 years

in 5 years and by

$6,540 \times 0.074 = 484

in 7 years. $6,540 \times 0.926 = $6,056$

This cash flow mapping preserves value and variance

When Linear Model Can be Used

- Portfolio of stocks
- Portfolio of bonds
- Forward contract on foreign currency
- Interest-rate swap

The Linear Model and Options

Consider a portfolio of options dependent on a single stock price, S. Define

and
$$\delta = \frac{\Delta P}{\Delta S}$$

$$\Delta x = \frac{\Delta S}{S}$$

Linear Model and Options continued

- As an approximation
- Similarly when there are many underlying market variables $\Delta P = \delta \Delta S = S \delta \Delta x$

where δ_i is the delta of the portfolio with respect to the *i*th asset $\Delta P = \sum S_i \delta_i \Delta x_i$



Example

- Consider an investment in options on Microsoft and AT&T. Suppose the stock prices are 120 and 30 respectively and the deltas of the portfolio with respect to the two stock prices are 1,000 and 20,000 respectively
- As an approximation

$$\Delta P = 120 \times 1,000 \Delta x_1 + 30 \times 20,000 \Delta x_2$$

where Δx_1 and Δx_2 are the percentage changes in the two stock prices



Skewness

The linear model fails to capture skewness in the probability distribution of the portfolio value.



Quadratic Model

For a portfolio dependent on a single stock price it is approximately true that this becomes

$$\Delta P = \delta \Delta S + \frac{1}{2} \gamma (\Delta S)^2$$

$$\Delta P = S\delta \,\Delta x + \frac{1}{2} \,S^2 \gamma \,(\Delta x)^2$$

Quadratic Model continued

With many market variables we get an expression of the form

$$\Delta P = \sum_{i=1}^{n} S_i \delta_i \Delta x_i + \sum_{i=1}^{n} \frac{1}{2} S_i S_j \gamma_{ij} \Delta x_i \Delta x_j$$

where

$$\delta_i = \frac{\partial P}{\partial S_i} \qquad \qquad \gamma_{ij} = \frac{\partial^2 P}{\partial S_i S_j}$$

This is not as easy to work with as the linear model

Monte Carlo Simulation

To calculate VaR using M.C. simulation we

- Value portfolio today
- Sample once from the multivariate distributions of the Δx_i
- Use the Δx_i to determine market variables at end of one day
- Revalue the portfolio at the end of day

Monte Carlo Simulation

- Calculate ΔP
- Repeat many times to build up a probability distribution for ΔP
- VaR is the appropriate fractile of the distribution times square root of *N*
- For example, with 1,000 trial the 1 percentile is the 10th worst case.

Speeding Up Monte Carlo

Use the quadratic approximation to calculate ΔP

Comparison of Approaches

- Model building approach assumes normal distributions for market variables. It tends to give poor results for low delta portfolios
- Historical simulation lets historical data determine distributions, but is computationally slower



Stress Testing

 This involves testing how well a portfolio performs under some of the most extreme market moves seen in the last 10 to 20 years



Back-Testing

- Tests how well VaR estimates would have performed in the past
- We could ask the question: How often was the actual 10-day loss greater than the 99%/10 day VaR?

Principal Components Analysis for Interest Rates

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- The first factor is a roughly parallel shift (83.1% of variation explained)
- The second factor is a twist (10% of variation explained)
- The third factor is a bowing (2.8% of variation explained)

Using PCA to calculate VaR

Example: Sensitivity of portfolio to rates (\$m)

1 yr	2 yr	3 yr	4 yr	5 yr
+10	+4	-8	-7	+2

Source: Author

Sensitivity to first factor :

 $10 \times 0.32 + 4 \times 0.35 - 8 \times 0.36 - 7 \times 0.36 + 2 \times 0.36 = -0.08$

Similarly sensitivity to second factor = -4.40

Using PCA to calculate VaR continued

- As an approximation
- The f_1 and f_2 are independent
- The standard deviation of ΔP is

$$\Delta P = -0.08f_1 - 4.40f_2$$

 $\sqrt{0.08^2 \times 17.49^2 + 4.40^2 \times 6.05^2} = 26.66$

• The 1 day 99% VaR is 26.66 × 2.33 = 62.12

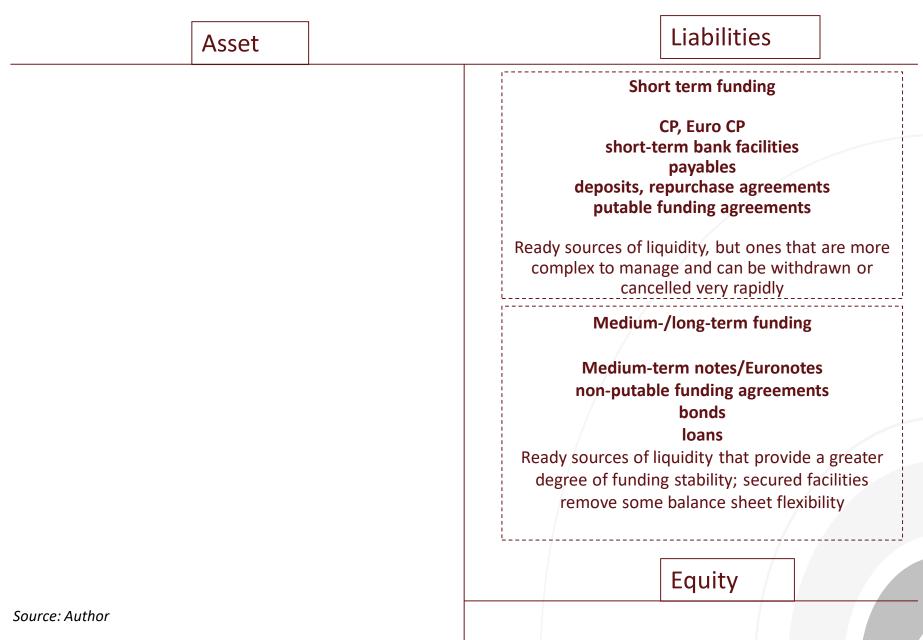
Introduction to liquidity risk

References: Banks, E. (2014). Sources of Liquidity. In: Liquidity Risk. ISBN: 978-1-137-37440-0, [on-line],Global Financial Markets Series. Palgrave Macmillan, London. https://doi.org/10.1057/9781137374400_3

Common sources of asset liquidity

Assets	Liabilities
Liquid assets	
 Cash and marketable securities A ready source of liquidity, either through outright sale or pledge of unencumbered securities for cash 	
 Receivables A ready source of liquidity, either through outright sale (factoring) or pledge of unencumbered receivables for cash 	Equity
 Inventories An accaptable source of liquidity, either through outright sale or pledge of unencumbered inventories; most effective for standard, durable inventories 	
Fixed assets and intangibles	
Fixed assets A possible source of liquidity, primarilly through pledge of unencumbered plant and equipment for cash	
Intangibles Not a source of liquidity	Source: Author

Common sources of funding liquidity





Common sources of off-balance sheet liquidity

Off-balance sheet

Securitization

An acceptable source of liquidity, primarily through transfer of securities or receivables to conduit in exchange for cash

Contingent financing

A good source of liquidity, releasing cash to be used to meet other obligations

Leases

A good source of liquidity, releasing cash to be used to meet other obligations

Derivates

A limited source of liqudity, primarily through offmarket, synthetic, or leveraged structures that provide upfront cash or relieve funding requirements

Source: Author

Key sources OF on- and off-balance sheet liquidity

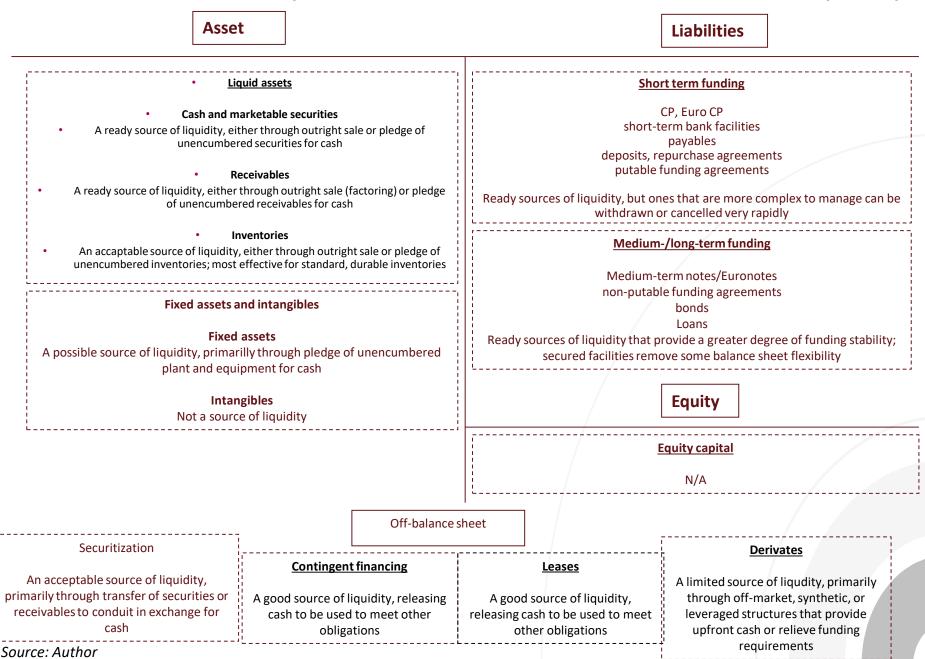
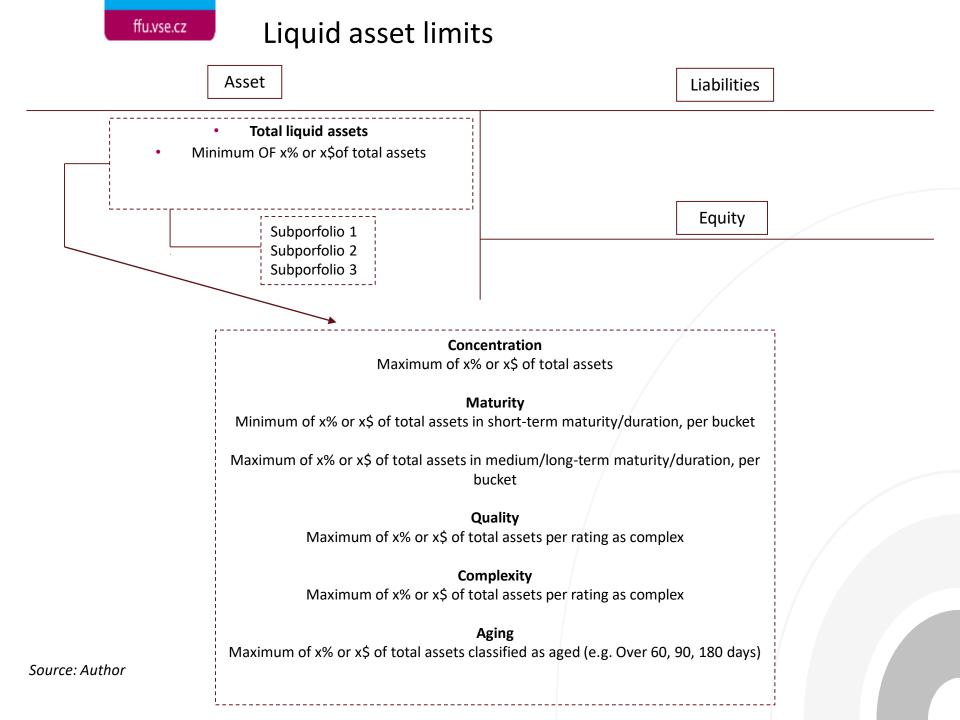


Table: Corporate liquidity ratios

- Gross Working Capital = Current Assets + Current Liabilities
- Net Working Capital = Current Assets Current Liabilities
- Current Assets = Cash + Markatable Securities + Receivables + Inventories
- Current Liabilities = Short-Term Debt Obligations + Current Portion OF Long-Term Debt + Payables
- Working Capital Ratio = Net Working Capital/Total Assets
- Current Ratio = Current Assets/Current Liabilities
- Quick Ratio = (Current Assets + Markatable Securities)/Current Liabilities
- Cash Ratio = (Cash + Markatable Securities)/Current Liabilities
- Liquidity coverage Ratio = (Current Assets Inventories)/Avarage Daily Operating Expenses
- Current Liability Ratio 1 = Current Liabilities/Equity
- Current Liability Ratio 2 = Current Liabilities/Total Assets
- Current Liability Ratio 3 = Current Liabilities/Total Debt
- Avarage Payables Maturity (days) = (365* Avarage Payables)/Purchases
- Payables Turnover = Purchases/ Avarage Annual Payables
- Avarage Receivables Maturity (days) = (365* Avarage Receivables)/Sales
- Receivables Turnover = Sales/ Avarage Annual Receivables
- Capital Expanditure Coverage = Operating Cash Flow/ Capital Expanditure

Fixed/liquid asset limits

Asset	Liabilities
 Asset Total liquid assets Minimum of x% or x\$ of total assets Cash and marketable securities Minimum of x% or x\$ of total assets Receivables Minimum of x% or x\$ of total assets Inventories Maximum of x% or x\$ of total assets Total fixed assets Maximum of x% or x\$ of total assets 	Equity
Source: Author	



Collateral/pledging limits

Asset

Liabilities

Total liquid assets Maximum of x% or x\$ of total liquid assets pledged as collateral

Cash and marketable securities

Maximum of x% or x\$ of total cash/ securities pledged as collateral

Receivables

Maximum of x% or x\$ of total receivables pledged as collateral

Inventories

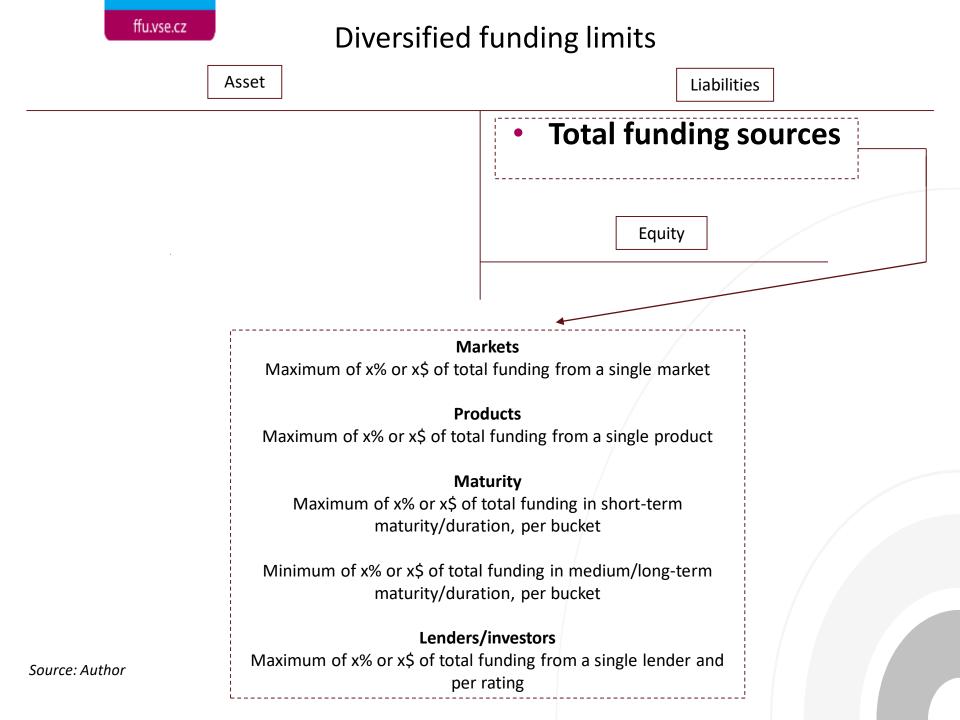
Maximum of x% or x\$ of total inventories pledged as collateral

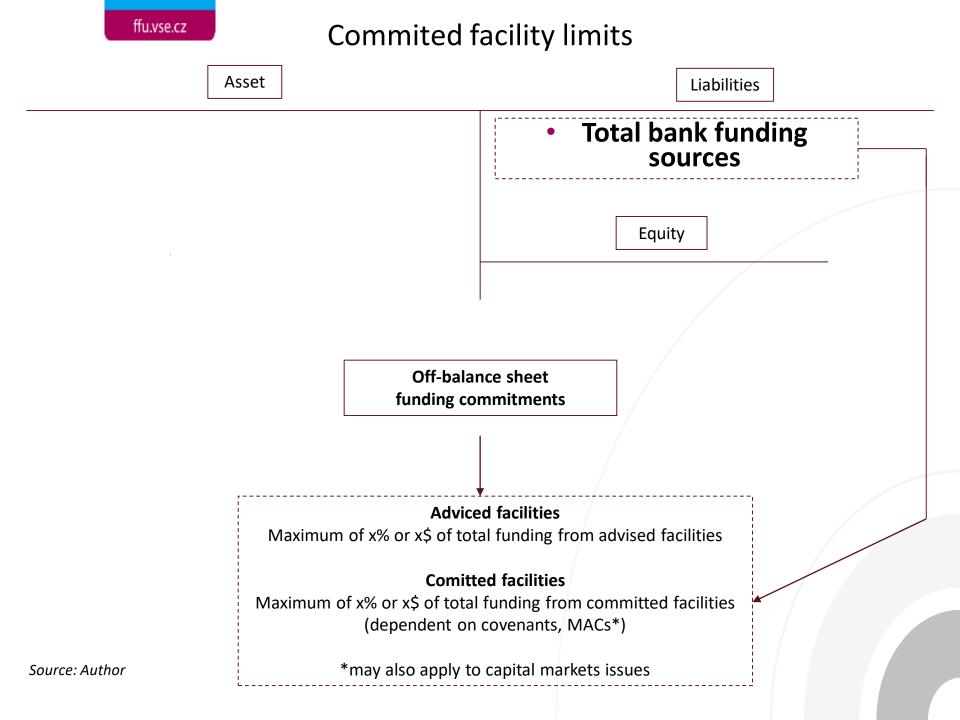
Total fixed assets

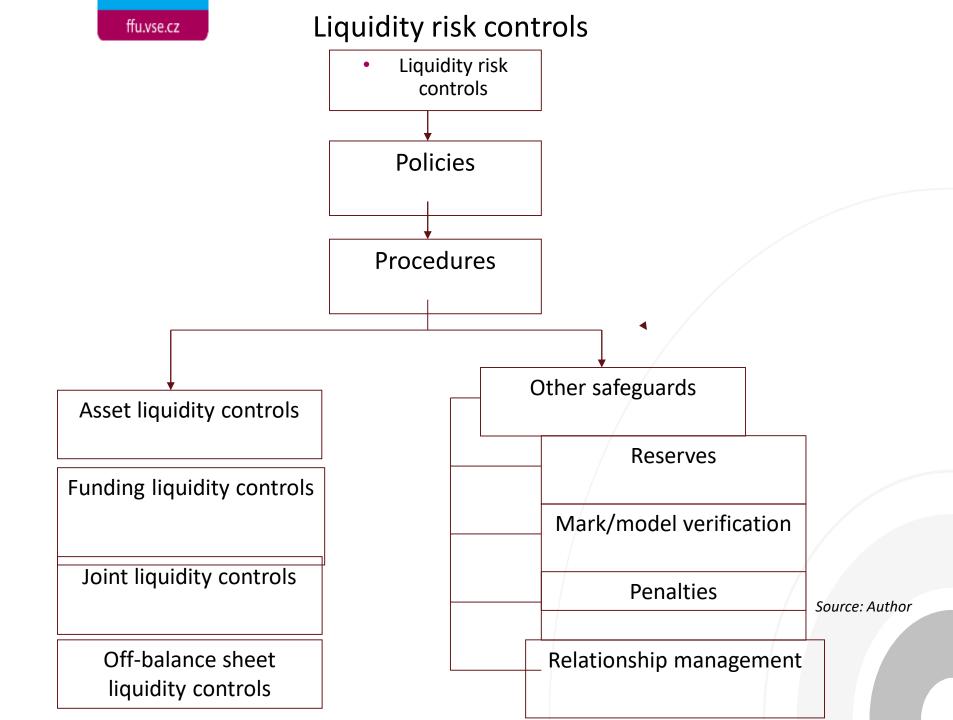
Maximum of x% or x\$ of total assets pledged as collateral

Equity

Source: Author









EVROPSKÁ UNIE Evropské strukturální a investiční fondy Operační program Výzkum, vývoj a vzdělávání



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