## Microeconomics 2



EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání


VŠE - Silvester van Koten

- The presentation is based on the following sources:
- Snyder Christopher M Walter Nicholson and Robert Stewart. 2015. Microeconomic Theory : Basic Principles and Extensions EMEA ed.

Andover: Cengage Learning. $\dagger$

- Varian Hal R. 2020. Intermediate Microeconomics : A Modern Approach Ninth ed. New York: W.W. Norton \& Company. +
- Excerpts from Gravelle H O Hart and R Rees. 2004. Advanced Microeconomics.
- Some more math refinements


## Exponential and natural logarithmic functions - basic relations

$$
\begin{aligned}
& x^{a} \cdot x^{b}=x^{a+b} \\
& \left(x^{a}\right)^{b}=x^{a \cdot b}
\end{aligned}
$$

## Exponential and natural logarithmic functions - basic relations

$$
\begin{aligned}
& \frac{d e^{x}}{d x}=e^{x} \quad y=e^{x} \quad x=\ln [y] \\
& x=\ln \left[e^{x}\right] \quad y=e^{\ln [y]} \\
& e^{0}=1 \quad \lim _{x \rightarrow+\infty} e^{x}=+\infty \quad \lim _{x \rightarrow-\infty} e^{x}=0 \\
& \ln [1]=0 \quad \lim _{x \rightarrow+\infty} \ln [x]=+\infty \quad \lim _{x \downarrow 0} \ln [x]=-\infty
\end{aligned}
$$

For proofs, see: "Proof of Log Properties.pdf" on the google drive

## Exponential and natural logarithmic functions - basic relations

Now we can derive 2 important properties of the In function

$$
\left.\begin{array}{l}
x \cdot y=(x) \cdot(y)=e^{\ln [x]} \cdot e^{\ln [y]}=e^{\ln [x]+\ln [y]} \\
x \cdot y=e^{\ln [x \cdot y]}
\end{array}\right\}
$$

$x^{a}=(x) \cdot(x) \cdot \ldots \cdot(x)\{a$ times $\}$

$$
e^{\ln \left[x^{a}\right]}=e^{\ln [x]} \cdot e^{\ln [x]} \cdot \ldots \cdot e^{\ln [x]}\{a \text { times }\}
$$

$$
e^{\ln \left[x^{a}\right]}=e^{a \cdot \ln [x]}
$$

$$
=e^{\ln [x]+\ln [x]+\ldots+\ln [x]\{a \text { times }\}}
$$

$$
\Leftrightarrow \ln \left[x^{a}\right]=a \cdot \ln [x]
$$

$$
=e^{a \cdot \ln [x]}
$$

## Exponential and natural logarithmic functions - derivatives

$$
\begin{array}{rl}
y=e^{x} & x \\
& =\ln \left[e^{x}\right] \\
y & =e^{\ln [y]}
\end{array}
$$

$$
\frac{d e^{x}}{d x}=e^{x} \quad \frac{d \ln [x]}{d x}=x^{-1}
$$

$$
\frac{d 2^{x}}{d x}=\ldots \quad 2=e^{\ln [2]} \quad 2^{x}=\left(e^{\ln [2]}\right)^{x}=e^{\mathrm{x} \ln [2]}
$$

$$
\frac{d 2^{x}}{d x}=\frac{d e^{\mathrm{x} \ln [2]}}{d x}=e^{\mathrm{x} \ln [2]} \cdot \ln [2]
$$

$$
\text { General: } \frac{d a^{x}}{d x}=\frac{d e^{\mathrm{x} \ln [\mathrm{a}]}}{d x}=e^{\mathrm{x} \ln [a]} \cdot \ln [\mathrm{a}]
$$

Partial derivatives

$$
\begin{aligned}
z & =f(x, y) \\
z & =f(x[t], y[t]) \\
\frac{d z}{d t} & =\frac{d f(x[t], y[t])}{d t} \\
& =\frac{\partial f(x[t], y[t])}{\partial x} \frac{d x[t]}{d t}+\frac{\partial f(x[t], y[t])}{\partial y} \frac{d y[t]}{d t} \\
& \equiv f_{1}(x[t], y[t]) \frac{d x[t]}{d t}+f_{2}(x[t], y[t]) \frac{d y[t]}{d t} \\
& \frac{d z}{d t}=1 \cdot 1+2 \cdot 1=3
\end{aligned}
$$

Example 1

$$
\begin{aligned}
& z=f(x, y)=\mathrm{x}+2 \mathrm{y} \\
& x[t]=t, y[t]=t \\
& z=\mathrm{x}+2 \mathrm{y}=\mathrm{t}+2 \mathrm{t}=3 \mathrm{t} \\
& \frac{d z}{d t}=3
\end{aligned}
$$

## Partial derivatives

Z


Partial derivatives

$$
\begin{aligned}
z & =f(x, y) \\
z & =f(x[t], y[t]) \\
\frac{d z}{d t} & =\frac{d f(x[t], y[t])}{t} \\
& =\frac{\partial f(x[t], y[t])}{\partial x} \frac{d x[t]}{d t}+\frac{\partial f(x[t], y[t])}{\partial y} \frac{d y[t]}{d t} \\
& \equiv f_{1}(x[t], y[t]) \frac{d x[t]}{d t}+f_{2}(x[t], y[t]) \frac{d y[t]}{d t}
\end{aligned}
$$

## Example 2

$$
\begin{aligned}
& z=f(x, y)=\mathrm{xy} \\
& x[t]=t, y[t]=t \\
& z=\mathrm{xy}=\mathrm{t} \cdot \mathrm{t}=\mathrm{t}^{2} \\
& \frac{d z}{d t}=2 t
\end{aligned}
$$

$$
\frac{d z}{d t}=y \cdot 1+x \cdot 1=y+x=t+t=2 t
$$

## Partial derivatives

$$
\begin{aligned}
z & =f(x, y[x]) \\
\frac{d z}{d x} & =\frac{\partial f(x, y)}{\partial x}+\frac{\partial f(x, y)}{\partial y} \frac{d y}{d x} \\
& \equiv f_{1}(x, y[\mathrm{x}])+f_{2}(x, y[\mathrm{x}]) \frac{d y[x]}{d x}
\end{aligned}
$$

## Example 3

$$
\begin{aligned}
& z=f(x, y)=\mathrm{xy} \\
& y=20 \cdot x^{-1} \\
& z=\mathrm{xy}=x \cdot 20 \cdot x^{-1}=20 \\
& \frac{d z}{d x}=0
\end{aligned}
$$

$$
\begin{aligned}
\frac{d z}{d x} & =y+x \cdot\left(-1 \cdot 20 \cdot x^{-2}\right)= \\
& =20 \cdot x^{-1}-20 x^{-1}=0
\end{aligned}
$$

## Partial derivatives

$$
\begin{aligned}
z & =f(x, y[x]) \\
\frac{d z}{d x} & =\frac{\partial f(x, y)}{\partial x}+\frac{\partial f(x, y)}{\partial y} \frac{d y}{d x} \\
& \equiv f_{1}(x, y[\mathrm{x}])+f_{2}(x, y[\mathrm{x}]) \frac{d y[x]}{d x}
\end{aligned}
$$

Example 4

$$
\begin{aligned}
& z=f(x, y)=x \cdot y \\
& y=100-x \\
& z=x \cdot y=x \cdot(100-x)=100 x-x^{2} \\
& \frac{d z}{d x}=100-2 x
\end{aligned}
$$

$$
\begin{aligned}
\frac{d z}{d x} & =y+x \cdot(-1)= \\
& =y-x= \\
& =100-x-x \\
& =100-2 x
\end{aligned}
$$

## Exercises

$$
\begin{gathered}
f[x, y]=x+\ln [\mathrm{y}] \\
\frac{\partial f[x, y]}{\partial x}=1 \\
\frac{\partial f[x, y]}{\partial y}=\frac{1}{y}
\end{gathered} \left\lvert\, \begin{aligned}
& f[x, y, z]=z^{2} x y+\ln [y-x] \\
& \frac{\partial f[x, y, z]}{\partial x}=z^{2} y+\frac{1}{y-x} \cdot-1 \\
& \frac{\partial f[x, y, z]}{\partial y}=z^{2} x+\frac{1}{y-x} \\
& \frac{\partial f[x, y, z]}{\partial z}=2 z x y
\end{aligned}\right.
$$

## Exercises



## Exercises

$y=g(x)=\ln [x]$
$\frac{\partial f(x, g(x))}{\partial x}=1+2 g(x) \cdot g^{\prime}(x)$
$=1+2 \ln [x] \cdot \frac{1}{x}$
$=1+\frac{2 \ln [x]}{x}$

$$
\begin{aligned}
f(x, y) & =f(x, \ln [x]) \\
& =x+\ln [x]^{2}
\end{aligned}
$$

$\frac{\partial f(x, y)}{\partial x}=1+2 \ln [x] \cdot \frac{1}{x}$

## Partial derivatives

In the examples before, $\frac{d y[x]}{d x}$ was given (by announcing $y=f[x]$ ) while $f_{1}$ and $f_{2}$ needed to be calculated.
Often we are in the opposite situation. We can easily find $f_{1}$ and $f_{2}$,

| but need to find, $\frac{d y[x]}{d x}$ with $\frac{d z}{d x}=0$. | $?$ |
| :--- | :--- |
| $\frac{d z}{d x}=f_{1}(x, y[\mathrm{x}])+f_{2}(x, y[\mathrm{x}]) \frac{d y[x]}{d x}$ | $z[x, y]$ could be the <br> indiffence curve (consump. theory) <br> or the iso-cost curve or <br> the iso-quant (product. theory) |

$$
\begin{aligned}
& \Leftrightarrow f_{2}(x, y[\mathrm{x}]) \frac{d y[x]}{d x}=\frac{d z}{d x}-f_{1}(x, y[\mathrm{x}]) \\
& \Leftrightarrow \frac{d y[x]}{d x}=\frac{1}{f_{2}(x, y[\mathrm{x}])} \frac{d z}{d x}-\frac{f_{1}(x, y[\mathrm{x}])}{f_{2}(x, y[\mathrm{x}])} \\
& \text { if } \frac{d z}{d x}=0 \Leftrightarrow \frac{d y[x]}{d x}=-\frac{f_{1}(x, y[\mathrm{x}])}{f_{2}(x, y[\mathrm{x}])}
\end{aligned}
$$

## The Consumer's Budget Constraint



How to find the slope of the budget restriction in the product space

1. Write down the budget restriction as $\mathrm{M}=\mathrm{p} 1$ * $\mathrm{x} 1+\mathrm{p} 2$ * x 2
2. Isolate $x 2$
$\mathrm{p} 2^{*} \mathrm{x} 2=\mathrm{M}-\mathrm{p} 1^{*} \mathrm{x} 1$
$x 2=\left(M-p 1^{*} x 1\right) / p 2$
$x 2=M / p 2-(p 1 / p 2) x 1$
3. Differentiate $\mathbf{x 2}$ to $\mathbf{x 1}$

$$
d x 2 / d x 1=-(p 1 / p 2)
$$

$$
p_{1} x_{1}+p_{2} x_{2}=m
$$

and

$$
p_{1}\left(x_{1}+\Delta x_{1}\right)+p_{2}\left(x_{2}+\Delta x_{2}\right)=m
$$

Subtracting the first equation from the second gives

$$
p_{1} \Delta x_{1}+p_{2} \Delta x_{2}=0
$$

This says that the total value of the change in her consumption must be zero. Solving for $\Delta x_{2} / \Delta x_{1}$, the rate at which good 2 can be substituted for good 1 while still satisfying the budget constraint, gives

$$
\frac{\Delta x_{2}}{\Delta x_{1}}=-\frac{p_{1}}{p_{2}}
$$

## Budget restriction

 implicit function derivation $\bar{M} \equiv B(x, y(x))$


- Consumer preferences: Chap 3


- Consumer can rank bundles of goods, eg $A$ and $B$
- Ranking done by a preference relation ">"
- $\mathrm{A}=\mathrm{B}$
- I like $A$ and $B$ equally much
$-\quad A \geq B$
- I prefer $A$ to $B$ (can include the case that $A=B$ )
$-\quad A>B$
- I strictly prefer $A$ to $B$ (thus excludes the case that $A=B$ )


## Consumer preferences are:

1. Complete

- $A \geq B$ or $B \geq A$ (or both: then $A=B$ )

2. Reflexive

- $A \geq A$
http://affiliater.hubpages.com/hub/from-chunk-to-hunk-how-one-person-lost-four-hundred-pounds

3. Transitive
$-A \geq B$

- $B \geq C$
- $A \geq C$


Repeat 100.000x


## Consumer preferences are:

4. More is better: Monotonicity (more details to follow)


## Indifference curve



## Indifference curve





## Possible?



Willing to pay for more pizza (in cola) at:


Willing to pay for more pizza (in cola) at:


## Four Properties of Indifference Curves

1. Higher indifference curves are preferred to lower ones.
2. Indifference curves are bowed inward
3. Indifference curves are downward sloping
4. Indifference curves do not cross



Monotonic preferences. More of both goods is a better bundle for this consumer; less of both goods represents a worse bundle.

## Consumer preferences are:

4. More is better: Monotonicity

3 variants:

1. Strong monotonicity

- if $\overrightarrow{\boldsymbol{x}} \geq \vec{y}$ and $\overrightarrow{\boldsymbol{x}} \neq \overrightarrow{\boldsymbol{y}}$, then $\overrightarrow{\boldsymbol{x}} \succ \overrightarrow{\boldsymbol{y}}$

2. Weak monotonicity

- if $\vec{x} \geq \vec{y}$ then $\overrightarrow{\boldsymbol{x}}=\succ \vec{y}$





## Perfect Complements: are they strongly monotonic?

Strong Monotonicity: if $\vec{x} \geq \vec{y}$ and $\vec{x} \neq \vec{y}$, then $\vec{x} \succ \vec{y}$
$\vec{X} \succ \vec{Y}$ ? No! No strong monotonicity here
Weak Monotonicity: if $x \geq y$ then $x=\succ$
$\vec{X} \Rightarrow \vec{Y}$ ? Yes! Weak monotonicity here


- Are indifference curves that are weakly monotonic necessarily strongly monotonic?



## Consumer preferences are:

4. More is better: Monotonicity

3 variants:

1. Strong monotonicity

- if $\overrightarrow{\boldsymbol{x}} \geq \vec{y}$ and $\overrightarrow{\boldsymbol{x}} \neq \overrightarrow{\boldsymbol{y}}$, then $\overrightarrow{\boldsymbol{x}} \succ \overrightarrow{\boldsymbol{y}}$

2. Weak monotonicity

- if $\vec{x} \geq \vec{y}$ then $\overrightarrow{\boldsymbol{x}}=\succ \overrightarrow{\boldsymbol{y}}$

3. Local non-satiation

For any $\overrightarrow{\boldsymbol{x}}$ and any $\boldsymbol{\varepsilon}>\mathbf{0}$, there is always an $\overrightarrow{\boldsymbol{y}}$ with $|\overrightarrow{\boldsymbol{x}}-\overrightarrow{\boldsymbol{y}}|<\varepsilon$ such that $\overrightarrow{\boldsymbol{y}} \succ \overrightarrow{\boldsymbol{x}}$

## Indifference curves




(a) Perfect Substitutes

(b) Perfect Complements


## Perfect Substitutes

Imperfect Substitutes


## Perfect Complements




- CONVEX* preferences
- averages are preferred to extremes.

Second, we are going to assume that averages are preferred to extremes. That is, if we take two bundles of goods $\left(x_{1}, x_{2}\right)$ and $\left(y_{1}, y_{2}\right)$ on the same indifference curve and take a weighted average of the two bundles such as

$$
\left(\frac{1}{2} x_{1}+\frac{1}{2} y_{1}, \frac{1}{2} x_{2}+\frac{1}{2} y_{2}\right)
$$

then the average bundle will be at least as good as or strictly preferred to each of the two extreme bundles. This weighted-average bundle has

$$
\left(t x_{1}+(1-t) y_{1}, t x_{2}+(1-t) y_{2}\right) \succeq\left(x_{1}, x_{2}\right)
$$

for any $t$ such that $0 \leq t \leq 1$.


A Convex preferences


B Nonconvex preferences


C Concave preferences

What does this assumption about preferences mean geometrically? It means that the set of bundles weakly preferred to $\left(x_{1}, x_{2}\right)$ is a convex set. For suppose that $\left(y_{1}, y_{2}\right)$ and $\left(x_{1}, x_{2}\right)$ are indifferent bundles. Then, if averages are preferred to extremes, all of the weighted averages of $\left(x_{1}, x_{2}\right)$ and $\left(y_{1}, y_{2}\right)$ are weakly preferred to $\left(x_{1}, x_{2}\right)$ and $\left(y_{1}, y_{2}\right)$. A convex set has the property that if you take any two points in the set and draw the line segment connecting those two points, that line segment lies entirely in the set.

## Define

## convexity

## CONVEXITY

- $\mathrm{V}(\mathrm{y})$ is a convex set when:
$-x^{\prime}, x^{\prime \prime} \in V(y)$, then $\left(t x^{\prime}+(1-t) x^{\prime \prime}\right) \in V(y)$ for all 0 $\leq \mathrm{t} \leq 1$
- Generally, define $x_{t}=t x^{\prime}+(1-t) x^{\prime \prime}$
- Then $x_{t}$ is the convex combination of $x^{\prime}$ and $x^{\prime \prime}$
- More on the convex combo




## CONVEXITY

- $\mathrm{V}(\mathrm{y})$ is a convex set when:
$-x^{\prime}, x^{\prime \prime} \in V(y)$, then $\mathbf{X}_{\mathbf{t}} \in \mathrm{V}(\mathrm{y})$ for all $0 \leq \mathrm{t} \leq 1$

Convex
Not convex


$$
\left(t x_{1}+(1-t) y_{1}, t x_{2}+(1-t) y_{2}\right) \succeq\left(x_{1}, x_{2}\right)
$$

for any $t$ such that $0 \leq t \leq 1$.


A Convex preferences


B Nonconvex preferences


C Concave preferences

- Can you think of preferences that are not convex?
- preferences for ice cream and olives?
- strict convexity versus weak convexity


The marginal rate of substitution (MRS). The marginal rate of substitution measures the slope of the indifference curve.

The marginal rate of substitution measures an interesting aspect of the consumer's behavior. Suppose that the consumer has well-behaved preferences, that is, preferences that are monotonic and convex, and that he is currently consuming some bundle ( $\mathrm{x} 1, \mathrm{x} 2$ ).

We now will offer him a trade: he can exchange good 1 for 2 , or good 2 for 1 , in any amount at a "rate of exchange" of $E$.

That is, if the consumer gives up $\Delta x 1$ units of good 1 , he can get $E \Delta x 1$ units of good 2 in exchange. Or, conversely, if he gives up $\Delta x 2$ units of good 2 , he can get $\Delta \times 2 / E$ units of good 1 . Geometrically, we are offering the consumer an opportunity to move to any point along a line with slope $-E$ that passes through ( $\mathrm{x} 1, \mathrm{x} 2$ ), as depicted in Figure 3.12.

Moving up and to the left from ( $\mathrm{x} 1, \mathrm{x} 2$ ) involves exchanging good 1 for good 2, and moving down and to the right involves exchanging good 2 for good 1. In either movement, the exchange rate is $E$. Since exchange always involves giving up one good in exchange for another, the exchange rate $E$ corresponds to a slope of $-E$.

We can now ask what would the rate of exchange have to be in order for the consumer to want to stay put at ( $\mathrm{x} 1, \mathrm{x} 2$ ) ?

To answer this question, we simply note that any time the exchange line crosses the indifference curve, there will be some points on that line that are preferred to ( $\mathrm{x} 1, \mathrm{x} 2$ ) -that lie above the indifference curve. Thus, if there is to be no movement from ( $\mathrm{x} 1, \mathrm{x} 2$ ), the exchange line must be tangent to the indifference curve.

That is, the slope of the exchange line, $-E$, must be the slope of the indifference curve at ( $\mathrm{x} 1, \mathrm{x} 2$ ). At any other rate of exchange, the exchange line would cut the indifference curve and thus allow the consumer to move to a more preferred point.


Trading at an exchange rate. Here we are allowing the consumer to trade the goods at an exchange rate $E$, which implies that the consumer can move along a line with slope $-E$.

We have said that the MRS measures the rate at which the consumer is just on the margin of being willing to substitute good 1 for good 2 . We could also say that the consumer is just on the margin of being willing to "pay" some of good 1 in order to buy some more of good 2 . So sometimes

- Other way of looking at it (not looking at the slope)


## Consumer Choice



We've already pointed out that the assumption of monotonicity implies that indifference curves must have a negative slope, so the MRS always involves reducing the consumption of one good in order to get more of another for monotonic preferences.

The case of convex indifference curves exhibits yet another kind of behavior for the MRS. For strictly convex indifference curves, the MRS-the slope of the indifference curve-decreases (in absolute value) as we increase $\times 1$.

Thus the indifference curves exhibit a diminishing marginal rate of substitution. This means that the amount of good 1 that the person is willing to give up for an additional amount of good 2 increases the amount of good 1 increases.

Stated in this way, convexity of indifference curves seems very natural: it says that the more you have of one good, the more willing you are to give some of it up in exchange for the other good. (But remember the ice cream and olives example-for some pairs of goods this assumption might not hold!)

## - Exercises

1. If we observe a consumer choosing $\left(x_{1}, x_{2}\right)$ when $\left(y_{1}, y_{2}\right)$ is available one time, are we justified in concluding that $\left(x_{1}, x_{2}\right) \succ\left(y_{1}, y_{2}\right)$ ?
3.1. No. It might be that the consumer was indifferent between the twc bundles. All we are justified in concluding is that $\left(x_{1}, x_{2}\right) \succeq\left(y_{1}, y_{2}\right)$.
2. Consider a group of people A, B, C and the relation "at least as tall as," as in "A is at least as tall as B." Is this relation transitive? Is it complete?
3.2. Yes to both.

## - Exercises

3. Take the same group of people and consider the relation "strictly taller than." Is this relation transitive? Is it reflexive? Is it complete?
3.3. It is transitive, but it is not complete - two people might be the same height. It is not reflexive since it is false that a person is strictly taller than himself.
4. A college football coach says that given any two linemen A and B, he always prefers the one who is bigger and faster. Is this preference relation transitive? Is it complete?
3.4. It is transitive, but not complete. What if A were bigger but slower than B? Which one would he prefer?
5. Can an indifference curve cross itself? For example, could Figure 3.2 depict a single indifference curve?
3.5. Yes. An indifference curve can cross itself, it just can't cross another distinct indifference curve.
6. Could Figure 3.2 be a single indifference curve if preferences are monotonic?
3.6. No, because there are bundles on the indifference curve that have strictly more of both goods than other bundles on the (alleged) indifference curve.
7. If both pepperoni and anchovies are bads, will the indifference curve have a positive or a negative slope?
3.7. A negative slope. If you give the consumer more anchovies, you've made him worse off, so you have to take away some pepperoni to get him back on his indifference curve. In this case the direction of increasing utility is toward the origin.
8. Explain why convex preferences means that "averages are preferred to extremes."
3.8. Because the consumer weakly prefers the weighted average of two bundles to either bundle.
9. What is your marginal rate of substitution of $\$ 1$ bills for $\$ 5$ bills?
3.9. If you give up one $\$ 5$ bill, how many $\$ 1$ bills do you need to compensate you? Five $\$ 1$ bills will do nicely. Hence the answer is -5 or $-1 / 5$, depending on which good you put on the horizontal axis.
10. If good 1 is a "neutral," what is its marginal rate of substitution for good 2 ?
3.10. Zero-if you take away some of good 1 , the consumer needs zero units of good 2 to compensate him for his loss.
11. Think of some other goods for which your preferences might be concave. 3.11. Anchovies and peanut butter, scotch and Kool Aid, and other similar repulsive combinations.



## Quantity of other goods

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## Quantity

 of ElectricityEVROPSKA UNIE
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## Národohospodářská fakulta VŠE v Praze

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