Microeconomics 2

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ,
MLÁDEŽE A TĚLOVÝCHOVY

VŠE - Silvester van Koten

- The presentation is based on the following sources:

- Snyder Christopher M Walter Nicholson and Robert Stewart. 2015. Microeconomic Theory : Basic Principles and Extensions EMEA ed. Andover: Cengage Learning. +
- Varian Hal R. 2020. Intermediate Microeconomics : A Modern Approach Ninth ed. New York: W.W. Norton & Company. +
- Excerpts from Gravelle H O Hart and R Rees. 2004. Advanced Microeconomics.

Utility

A

B

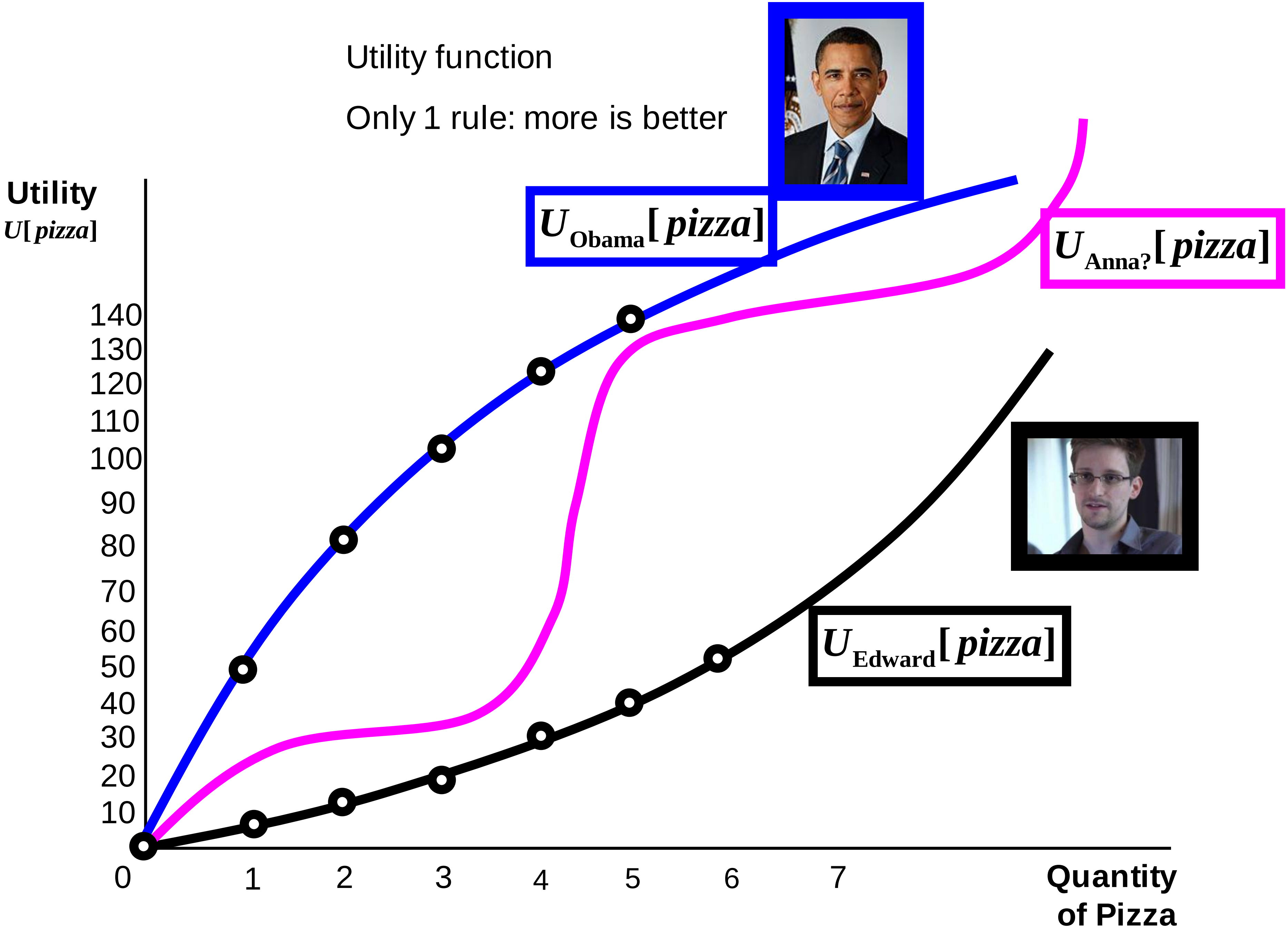
C

D

E

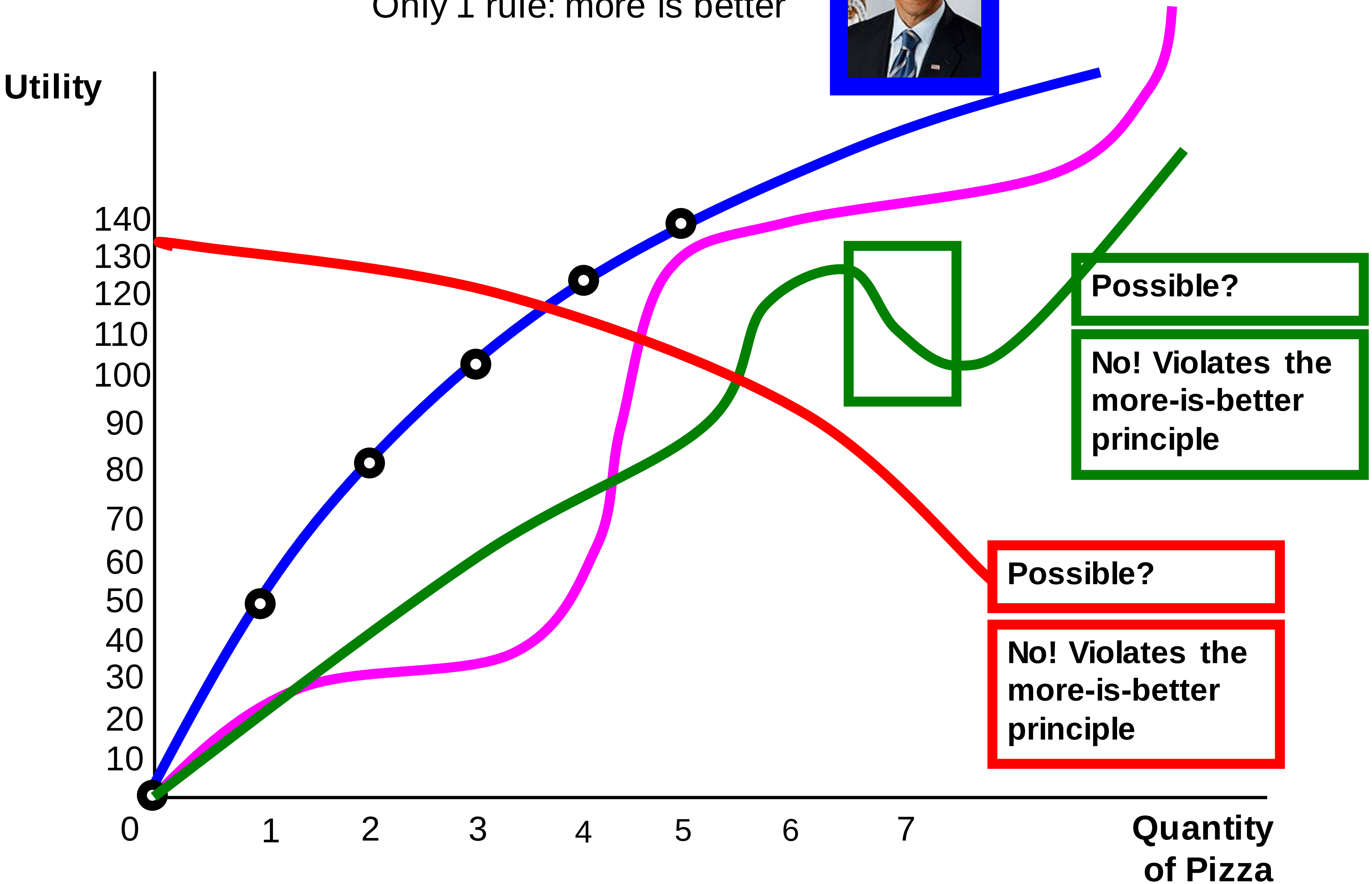
F

G


Utility function
Only 1 rule: more is better
Utility
U[pizza]
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
Quantity of Pizza
U Obama [pizza]
U Anna? [pizza]
U Edward [pizza]

Utility function

Only 1 rule: more is better



Different ways to assign utilities.

Bundle	U_1	U_2	U_3
A	3	17	-1
B	2	10	-2
C	1	.002	-3

Multiplication by 2 is an example of a **monotonic transformation**.

We typically represent a monotonic transformation by a function $f(u)$ that transforms each number u into some other number $f(u)$, in a way that preserves the order of the numbers in the sense that $u_1 > u_2$ implies $f(u_1) > f(u_2)$. A monotonic transformation and a monotonic function are essentially the same thing.

Examples of monotonic transformations are multiplication by a positive number (e.g., $f(u) = 3u$), adding any number (e.g., $f(u) = u + 17$), raising u to an odd power (e.g., $f(u) = u^3$), and so on.[1]

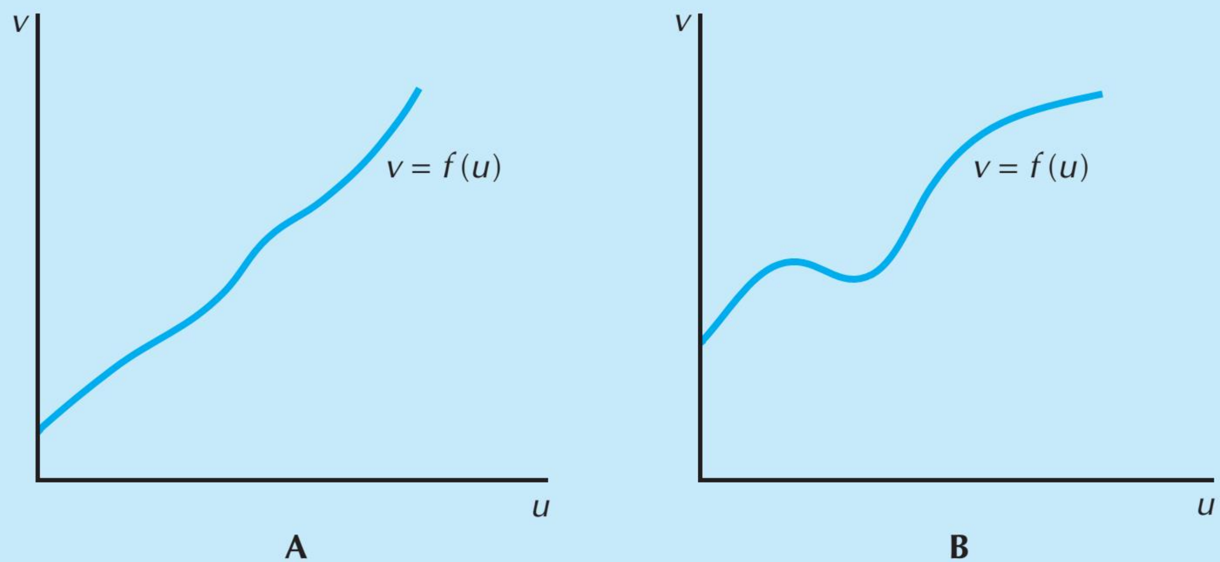
- Ordinality implies:
  - Utility function is *invariant* under any monotonous transformation
  - (But what about risk preferences?)
    - (these are affected, unless it is a monotonous linear transformation – Micro 3)

- Any monotonic

FT

The rate of change of $f(u)$ as u changes can be measured by looking at the change in f between two values of u, divided by the change in u:

$$\frac{\Delta f}{\Delta u} = \frac{f(u_2) - f(u_1)}{u_2 - u_1}.$$



A positive monotonic transformation. Panel A illustrates a monotonic function—one that is always increasing. Panel B illustrates a function that is *not* monotonic, since it sometimes increases and sometimes decreases.

If $f(u)$ is *any* monotonic transformation of a utility function that represents some particular preferences, then $f(u(x_1, x_2))$ is also a utility function that represents those same preferences.

1. To say that $u(x_1, x_2)$ represents some particular preferences means that $u(x_1, x_2) > u(y_1, y_2)$ if and only if $(x_1, x_2) \succ (y_1, y_2)$.

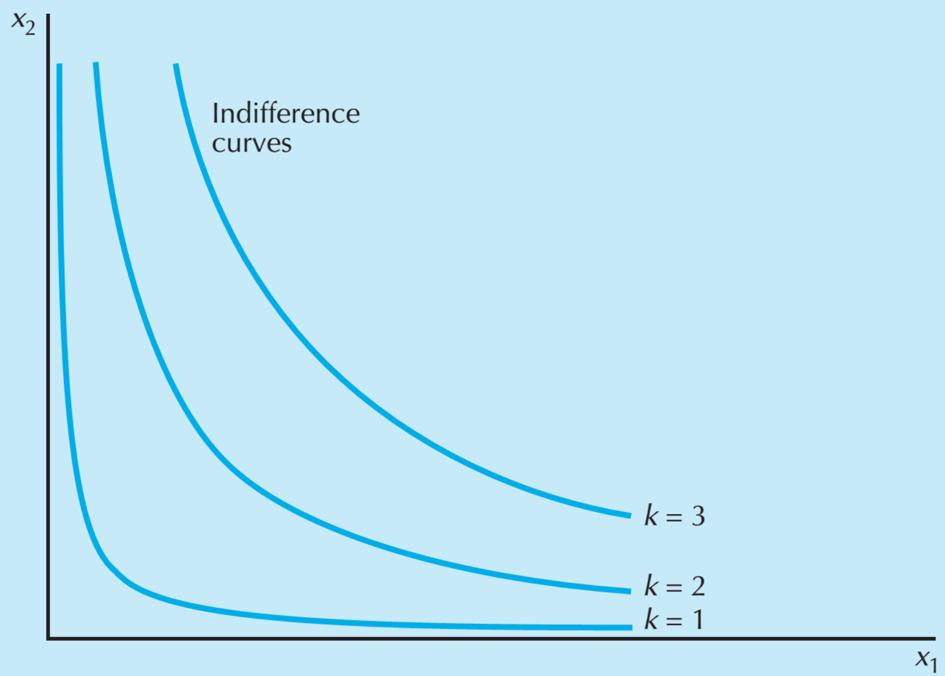
2. But if $f(u)$ is a monotonic transformation, then $u(x_1, x_2) > u(y_1, y_2)$ if and only if $f(u(x_1, x_2)) > f(u(y_1, y_2))$.

3. Therefore, $f(u(x_1, x_2)) > f(u(y_1, y_2))$ if and only if $(x_1, x_2) \succ (y_1, y_2)$, so the function $f(u)$ represents the preferences in the same way as the original utility function $u(x_1, x_2)$.

Suppose that the utility function is given by: $u(x_1, x_2) = x_1 x_2$. What do the indifference curves look like?

We know that a typical indifference curve is just the set of all x_1 and x_2 such that $k = x_1 x_2$ for some constant k. Solving for x_2 as a function of x_1, we see that a typical indifference curve has the formula:

$$x_2 = \frac{k}{x_1}.$$



Indifference curves. The indifference curves $k = x_1 x_2$ for different values of k.

Let's consider another example. Suppose that we were given a utility function $v(x_1, x_2) = x_1^2 x_2^2$. What do its indifference curves look like? By the standard rules of algebra we know that:

$$v(x_1, x_2) = x_1^2 x_2^2 = (x_1 x_2)^2 = u(x_1, x_2)^2.$$

This means that the utility function $v(x_1, x_2) = x_1^2 x_2^2$ has to have exactly the same shaped indifference curves as those depicted in Figure 4.3. The labeling of the indifference curves will be different—the labels that were $1, 2, 3, \cdots$ will now be $1, 4, 9, \cdots$—but the set of bundles that has $v(x_1, x_2) = 9$ is exactly the same as the set of bundles that has $u(x_1, x_2) = 3$. Thus $v(x_1, x_2)$ describes exactly the same preferences as $u(x_1, x_2)$ since it *orders* all of the bundles in the same way.

Like an Olympic ranking, utility is an ordinal measure (not cardinal)

What are cardinal measures?

Money, speed, distance are cardinal measures

Is 100C 2x hotter than 50C?

Is 10C 2x hotter than 5C?

How many times is 10C hotter than -5C?

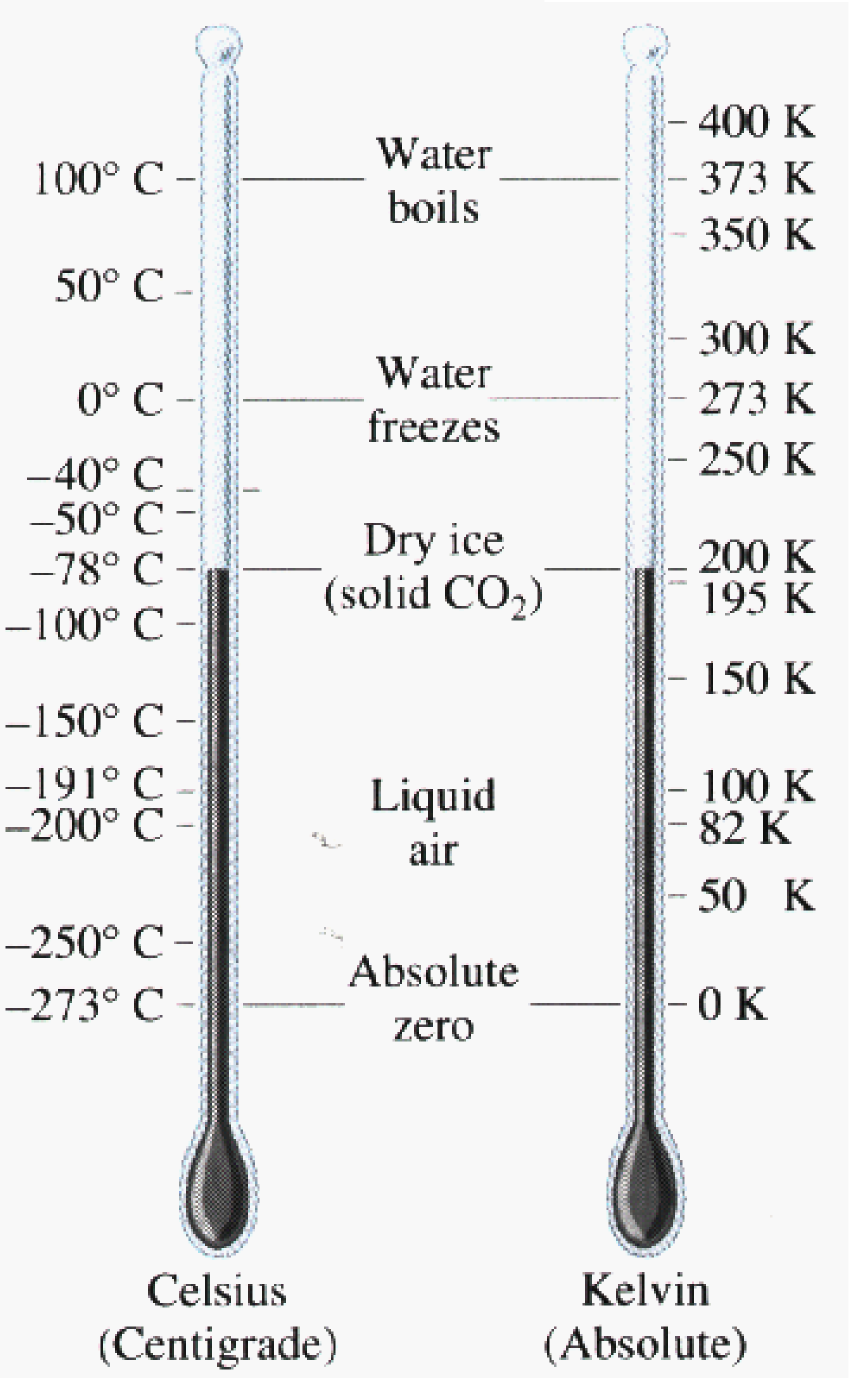
No, no no!

Degrees in Celcius is **NOT** a **CARDINAL** measure

Is 100K 2x hotter than 50K?

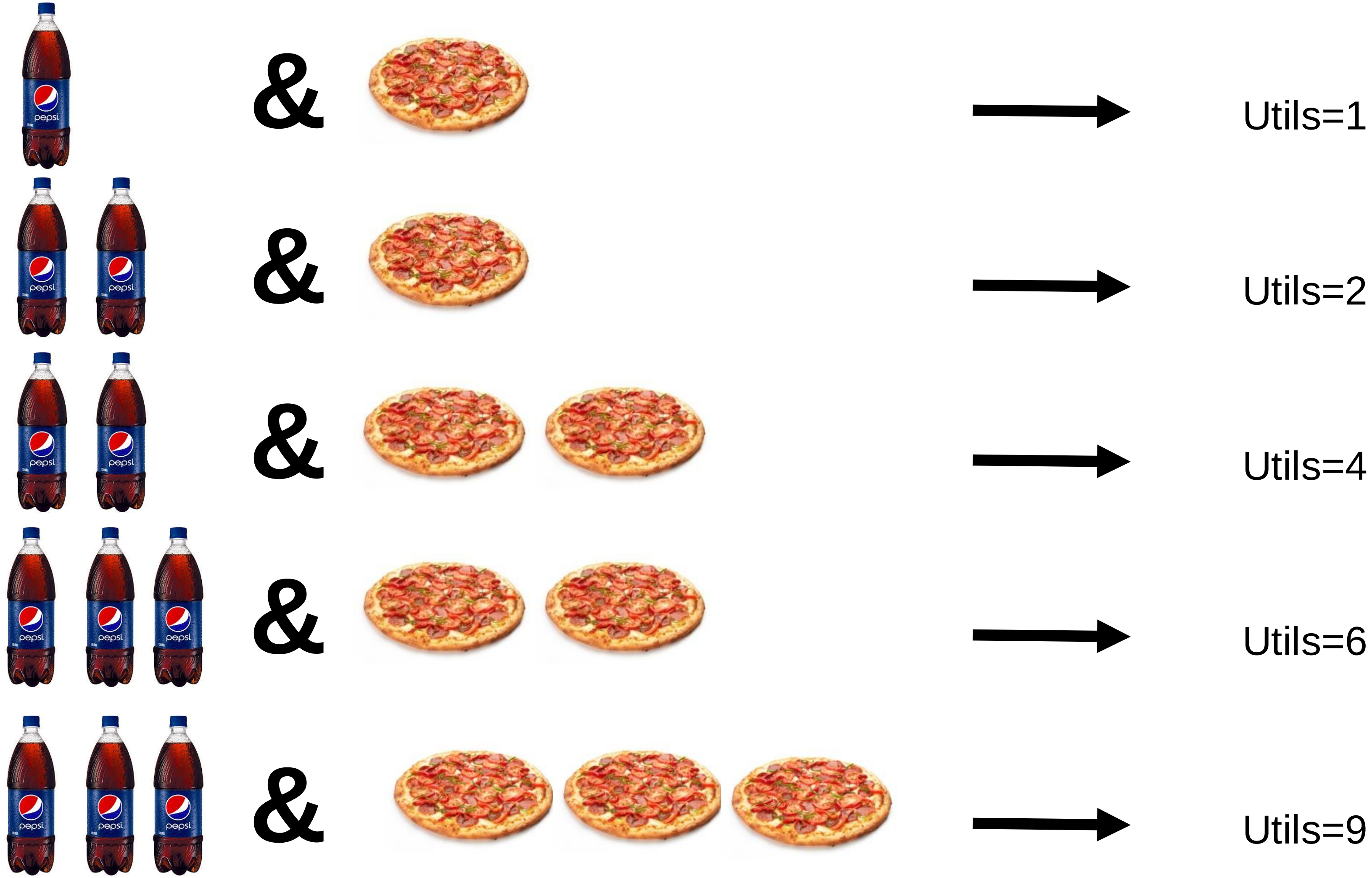
Yes!

Degrees in Kelvin is a **CARDINAL** measure



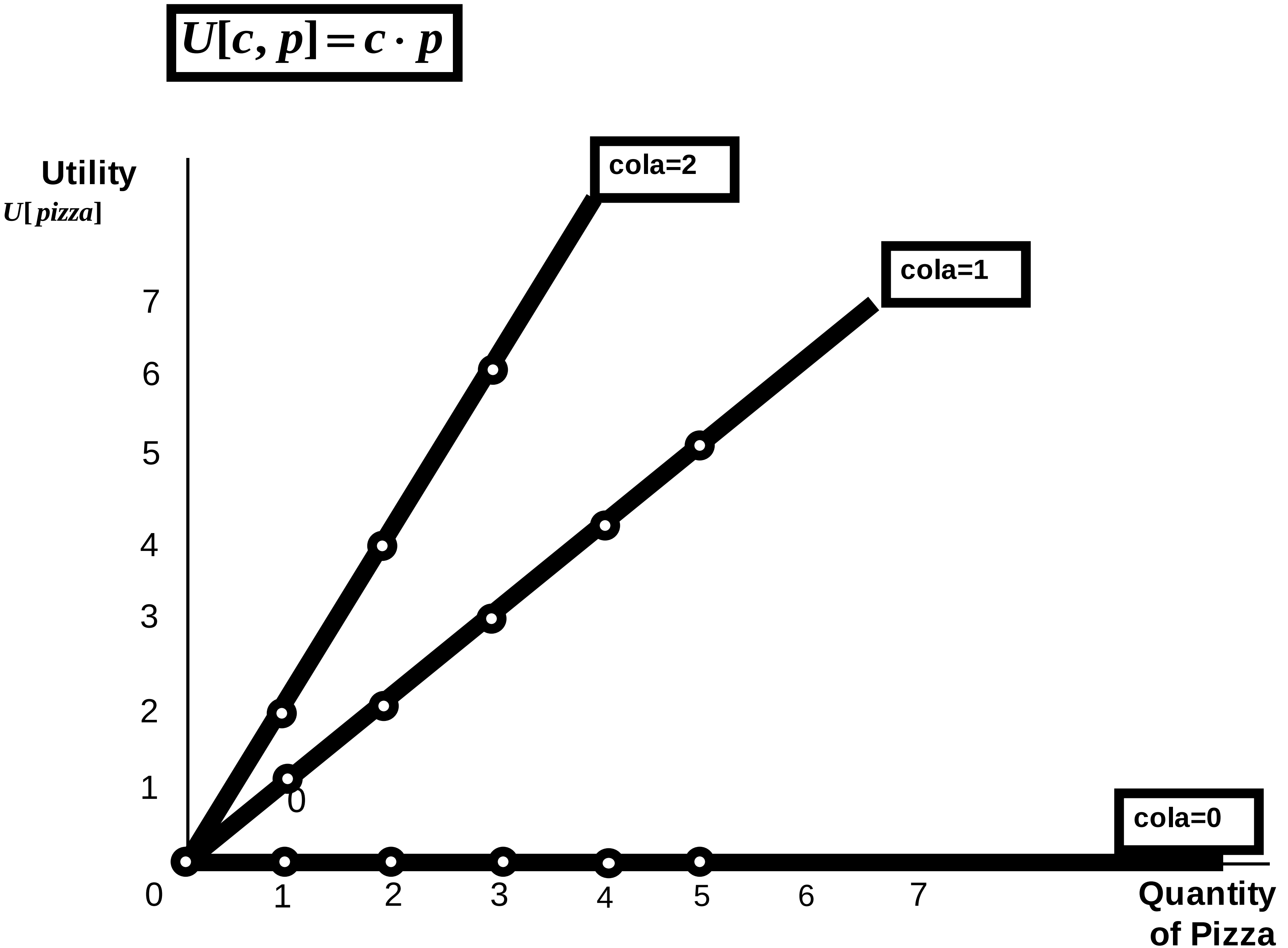
$$U[pizza]$$

$$U[cola, pizza] \equiv U[c, p]$$

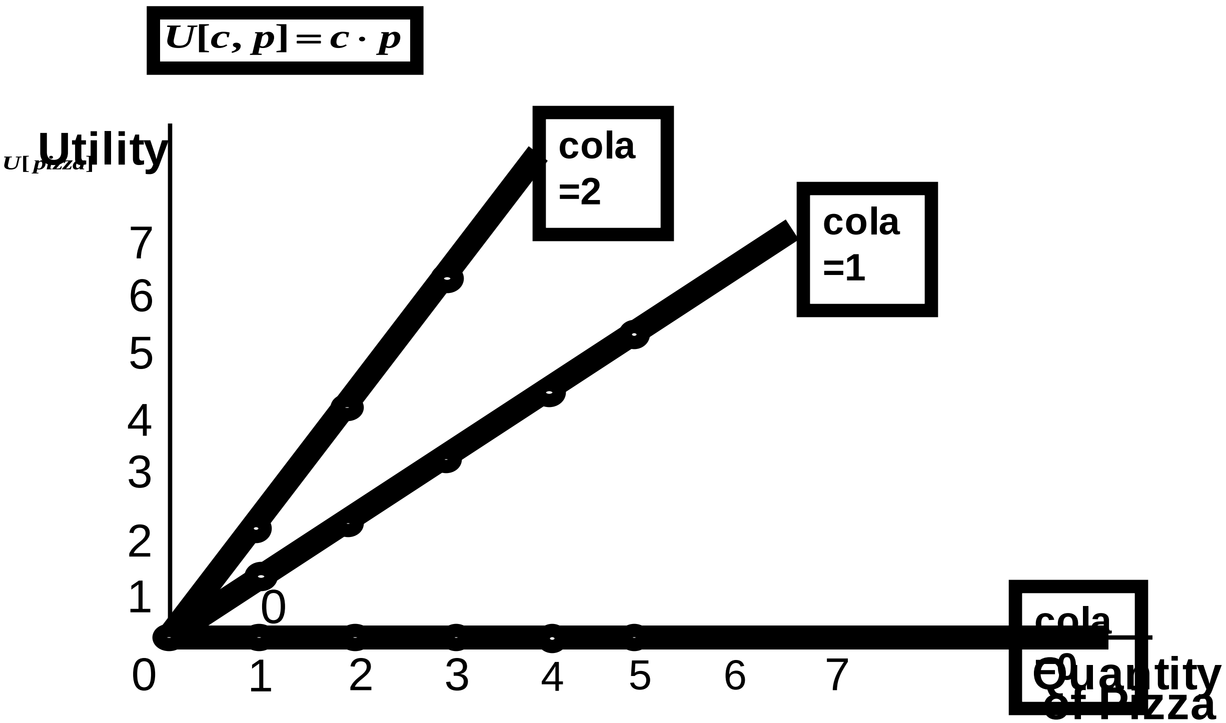


$$\boxed{U[c, p] = c \cdot p}$$

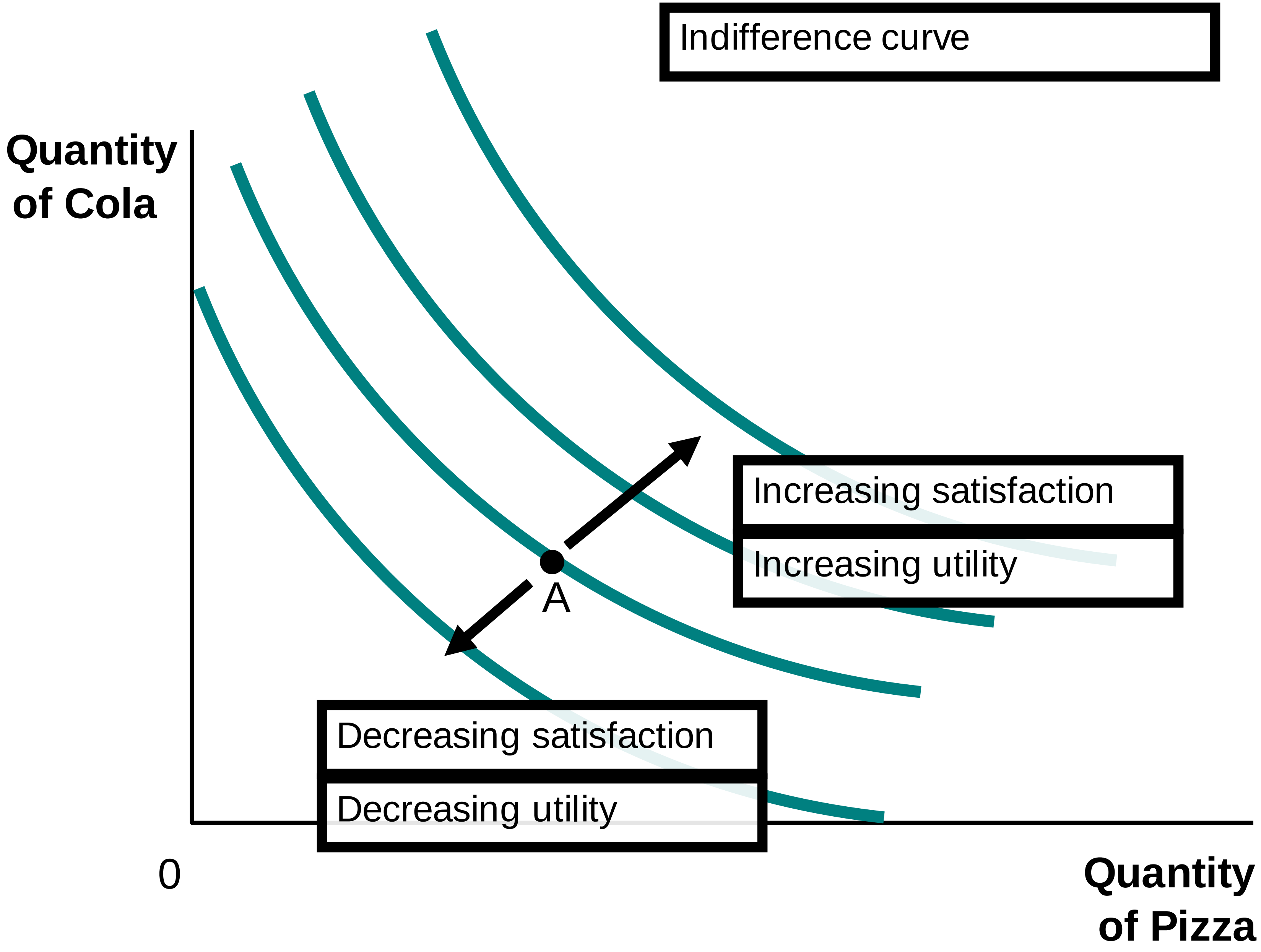
How to put this into one diagram?

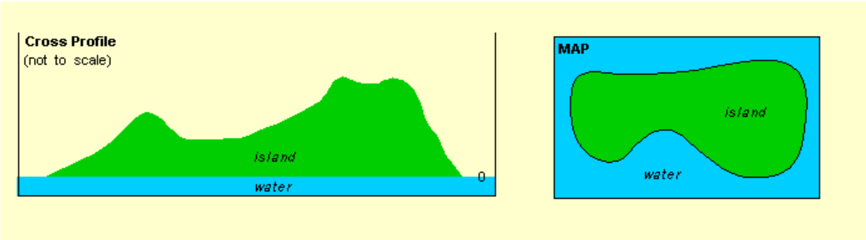


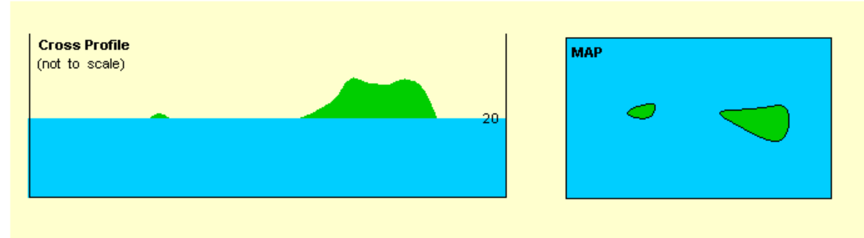
$U[c,p] = c \cdot p$
Utility
$U[pizza]$
cola=2
cola=1
cola=0
7
6
5
4
3
2
1
0
0
1
2
3
4
5
6
7
Quantity of Pizza

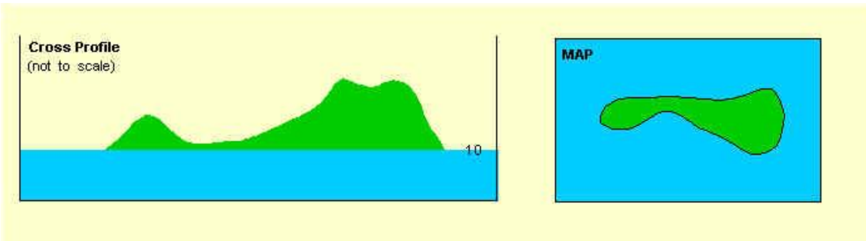


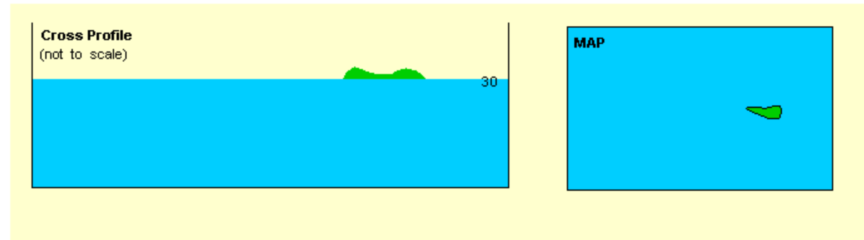
$U[c,p] = c \cdot p$
Utility
$U[pizza]$
cola =2
cola =1
cola =0
0
1
2
3
4
5
6
7
Quantity of Pizza

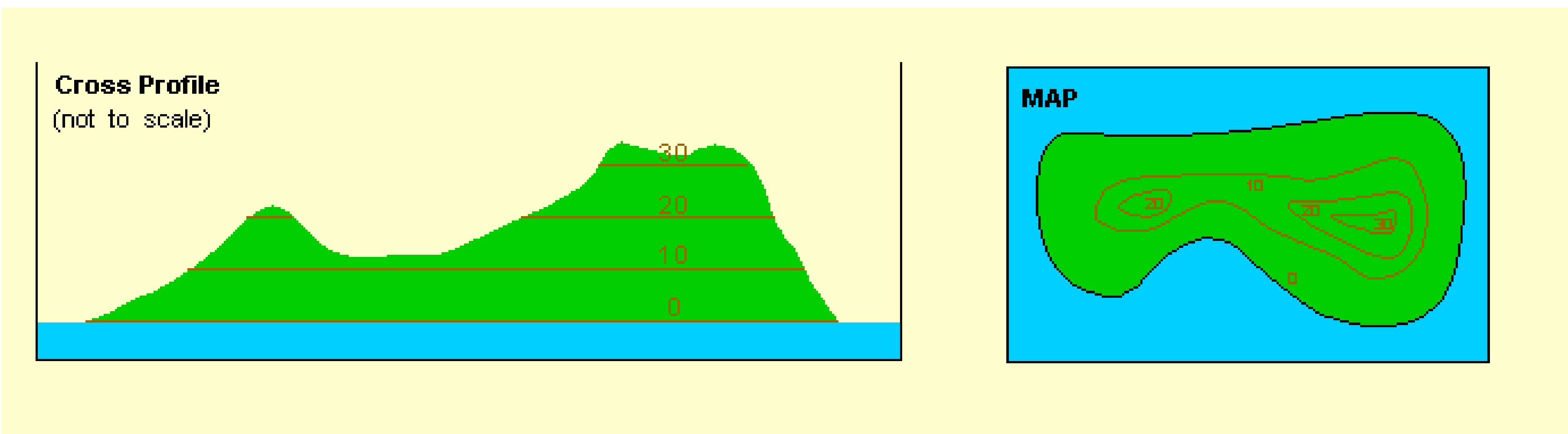

Indifference curve
Quantity
of Cola
Increasing satisfaction
Increasing utility
A
Decreasing satisfaction
Decreasing utility
0
Quantity
of Pizza

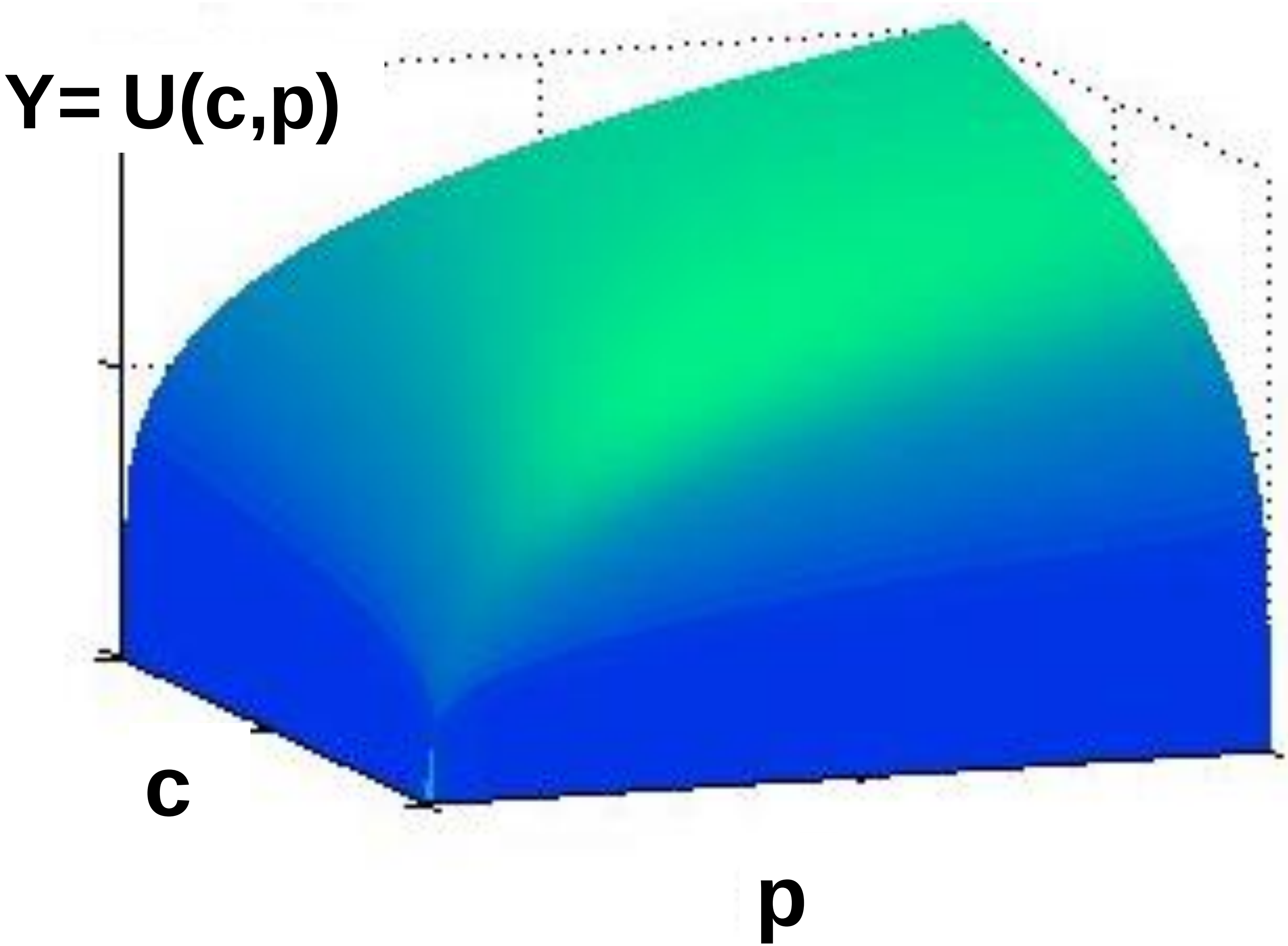

Cross Profile
(not to scale)
island
water
0
MAP
island
water

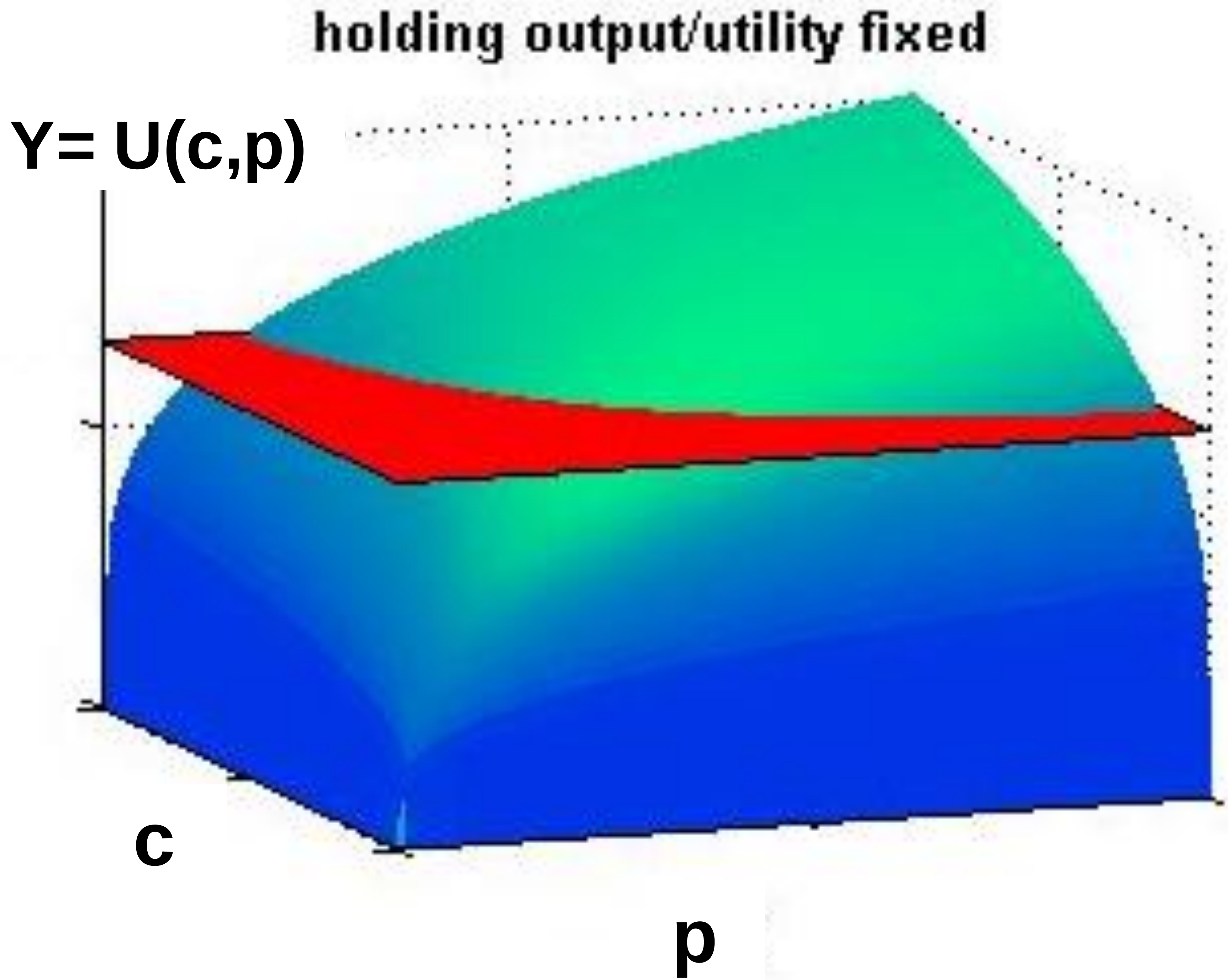

Cross Profile
(not to scale)
20
MAP


Cross Profile
(not to scale)
10
MAP

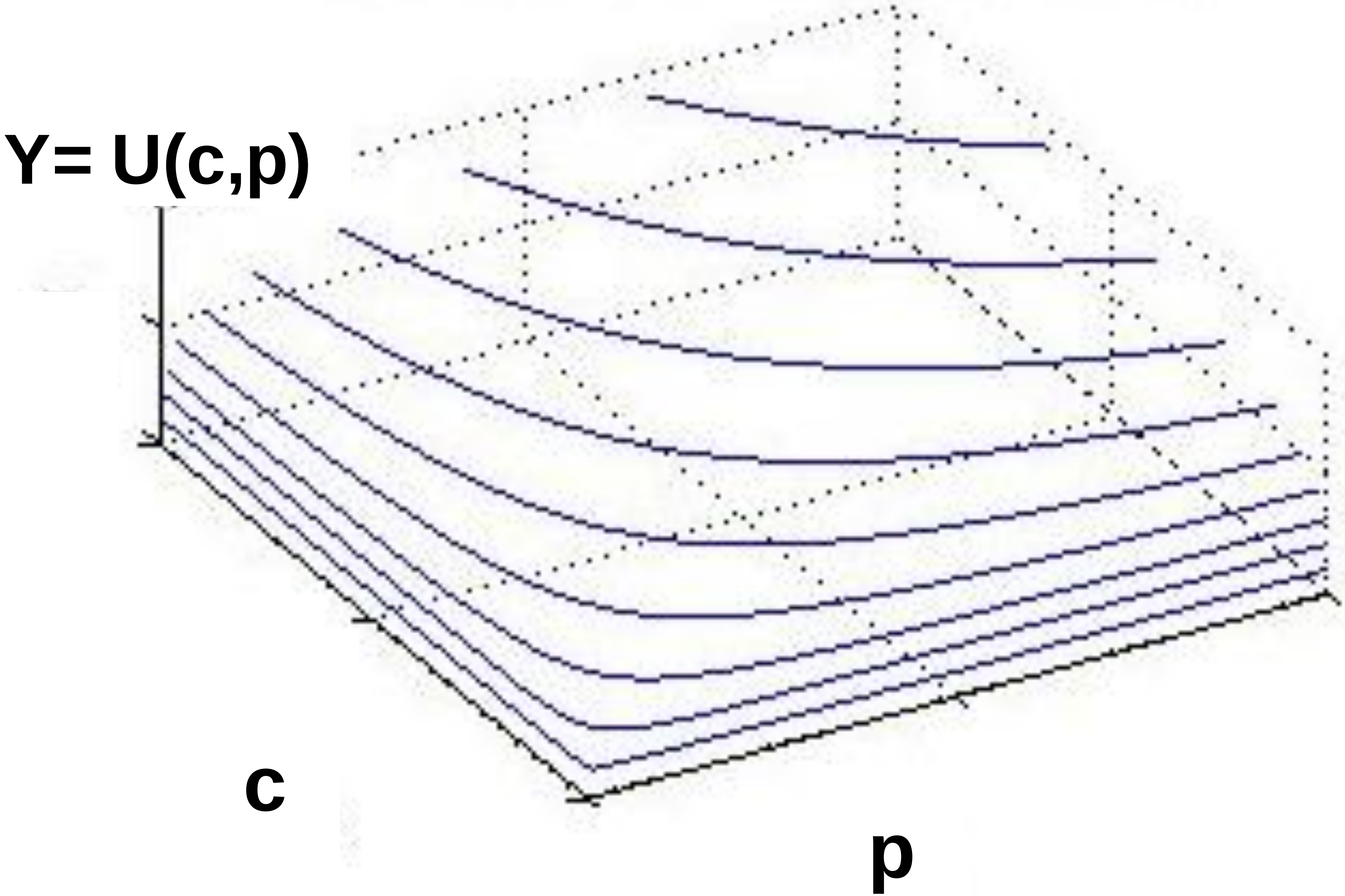

Cross Profile
(not to scale)
30
MAP

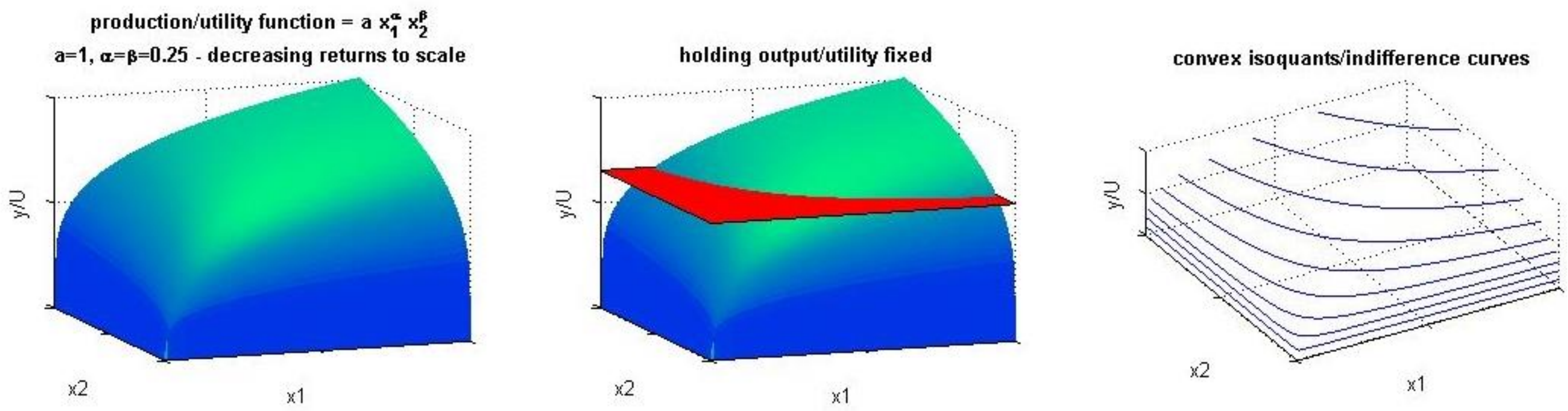

Cross Profile
(not to scale)
30
20
10
0
MAP
10
20
20
30
0

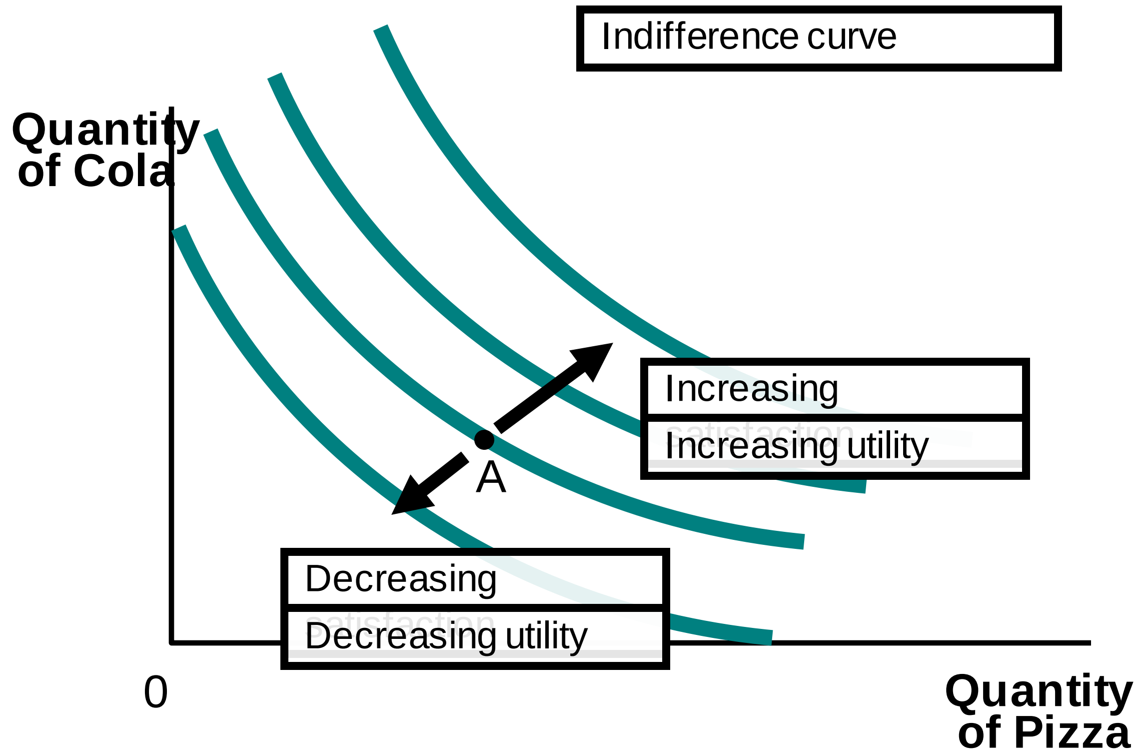

Y= U(c,p)
c
p


holding output/utility fixed
Y= U(c,p)
c
p

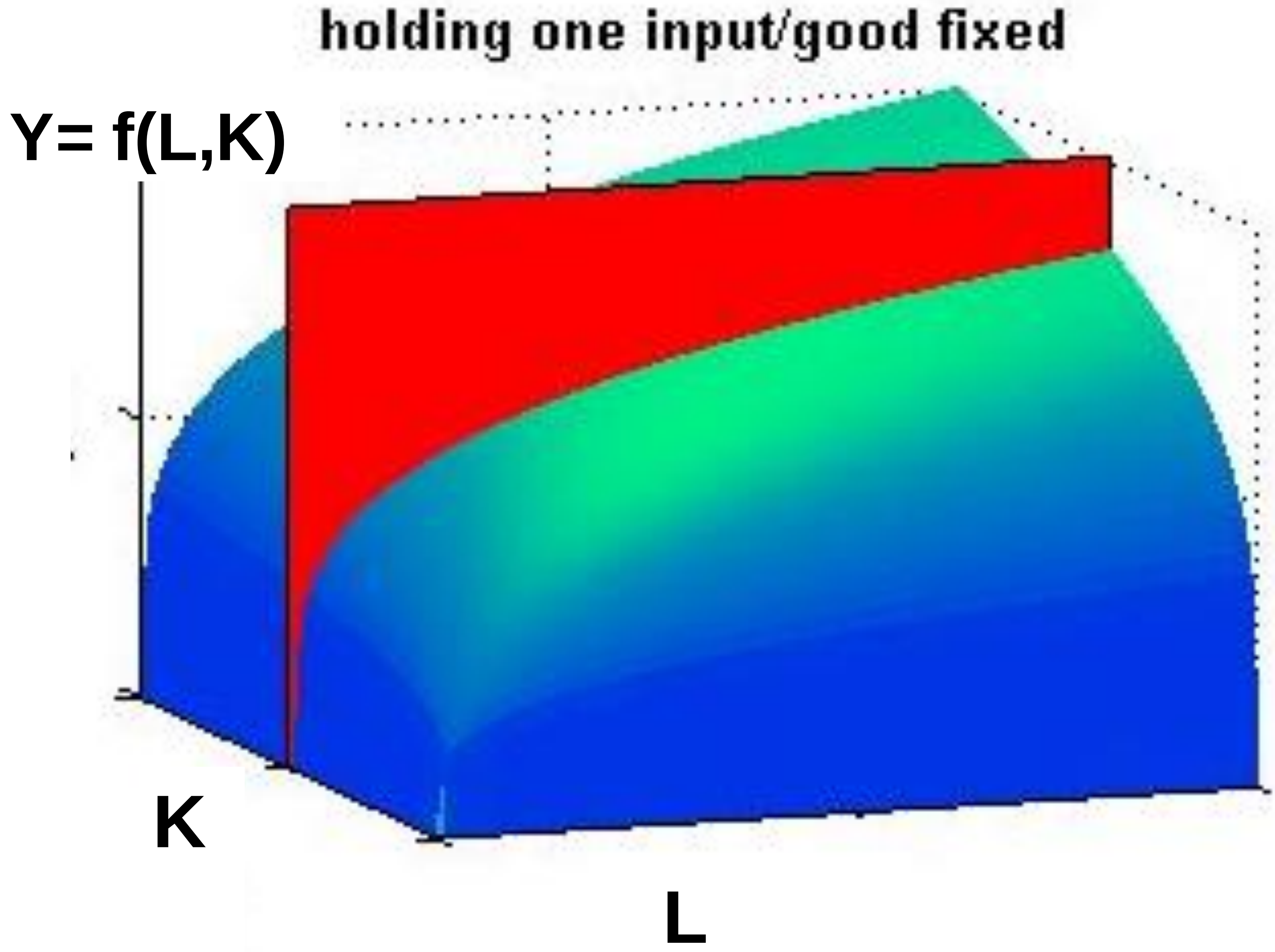
convex isoquants/indifference curves

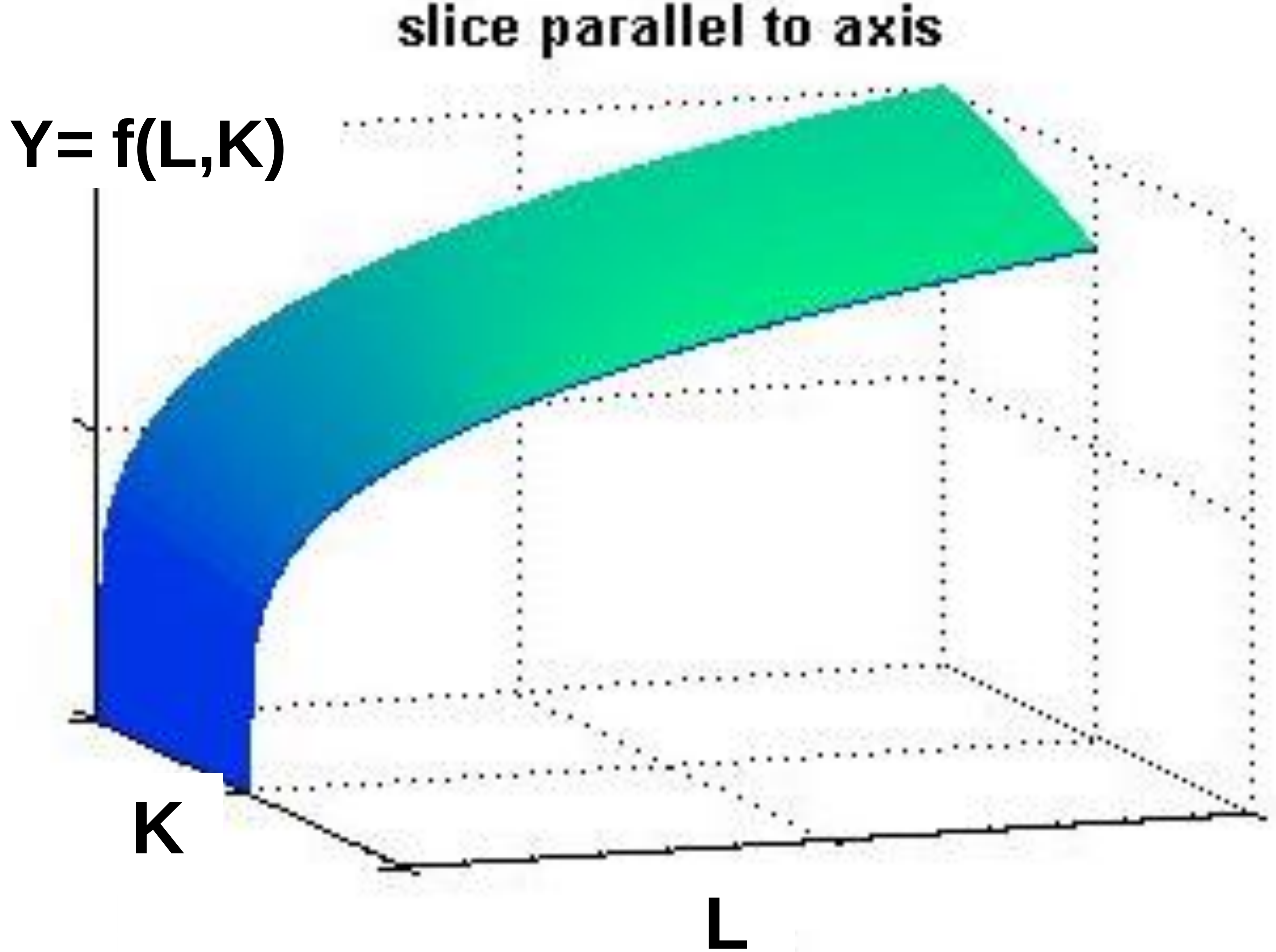




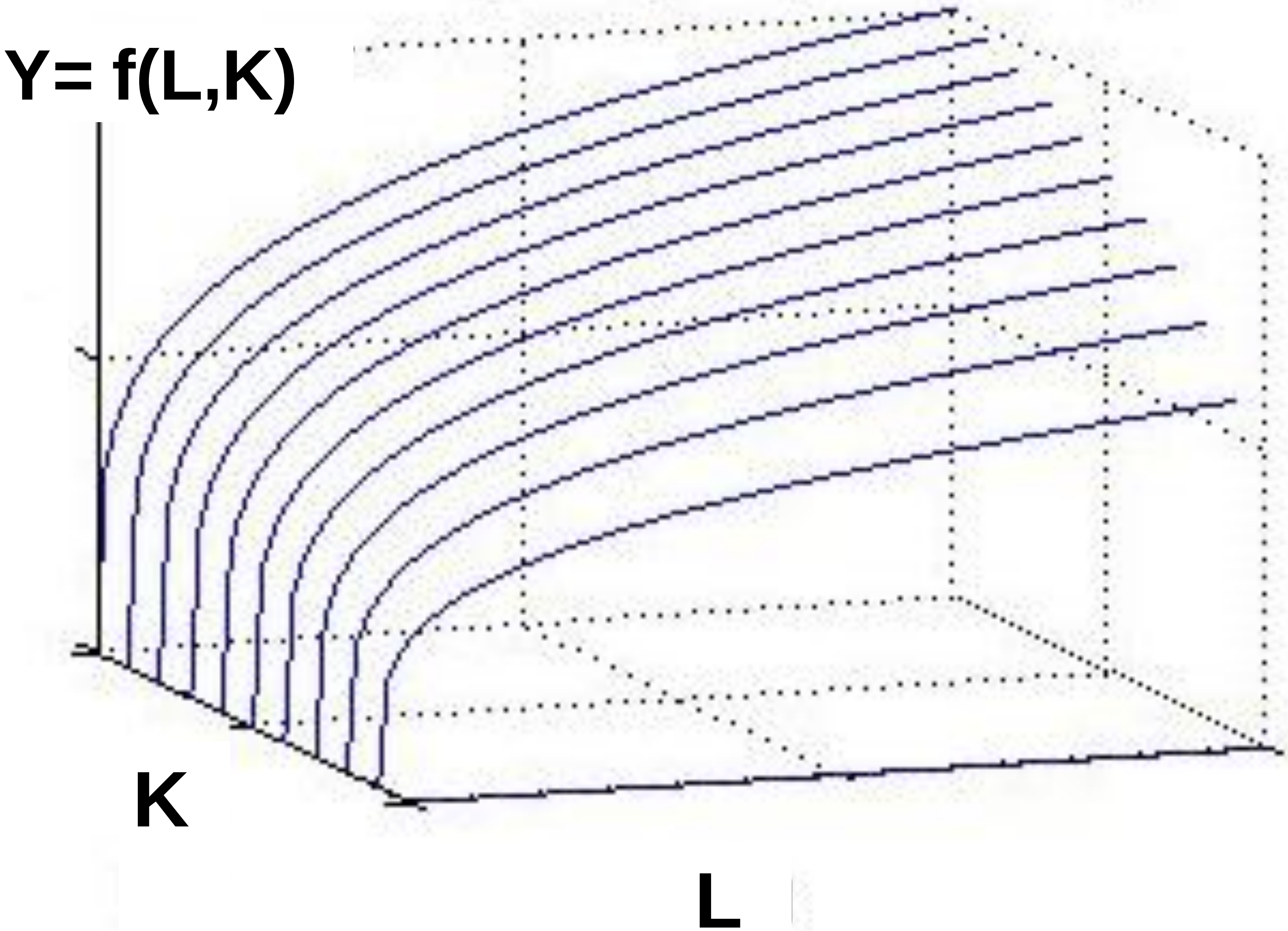


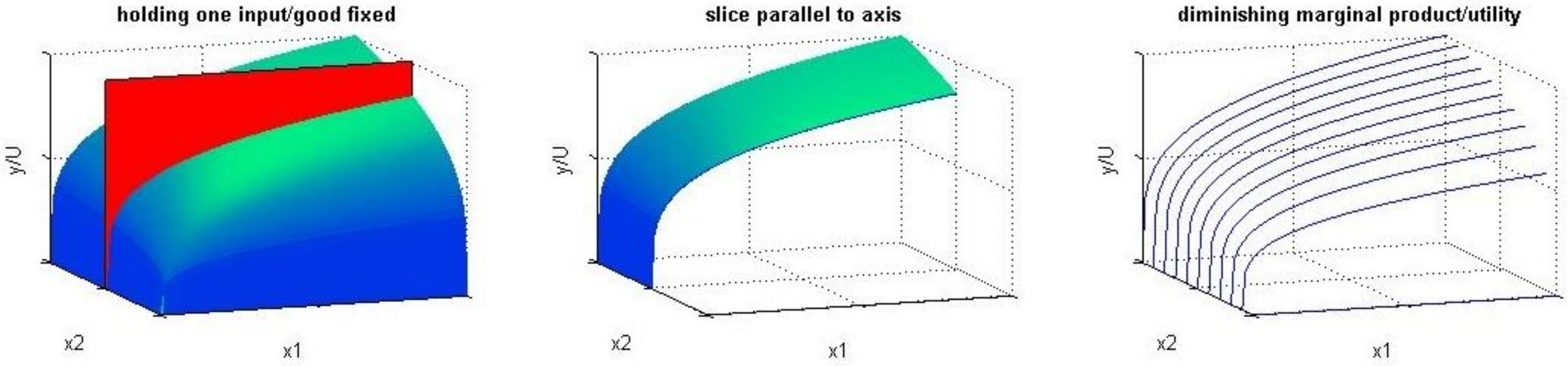
Pictures originally from
http://www2.hawaii.edu/~fuleky/anatomy/anatomy.html


holding one input/good fixed
Y= f(L,K)
K
L

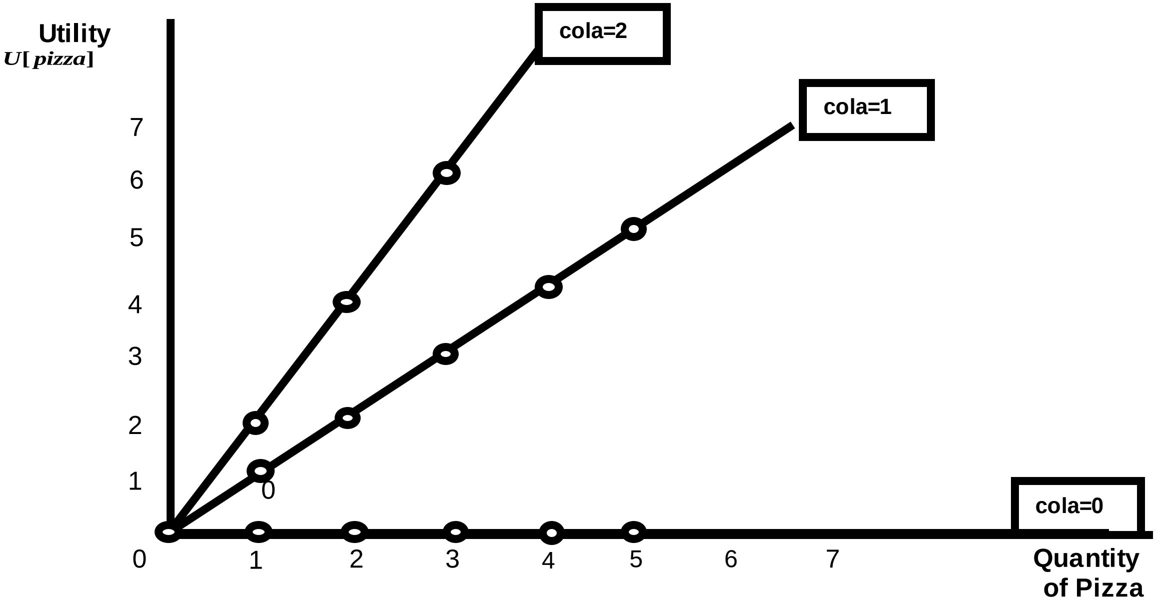

slice parallel to axis
Y= f(L,K)
K
L

diminishing marginal product/utility

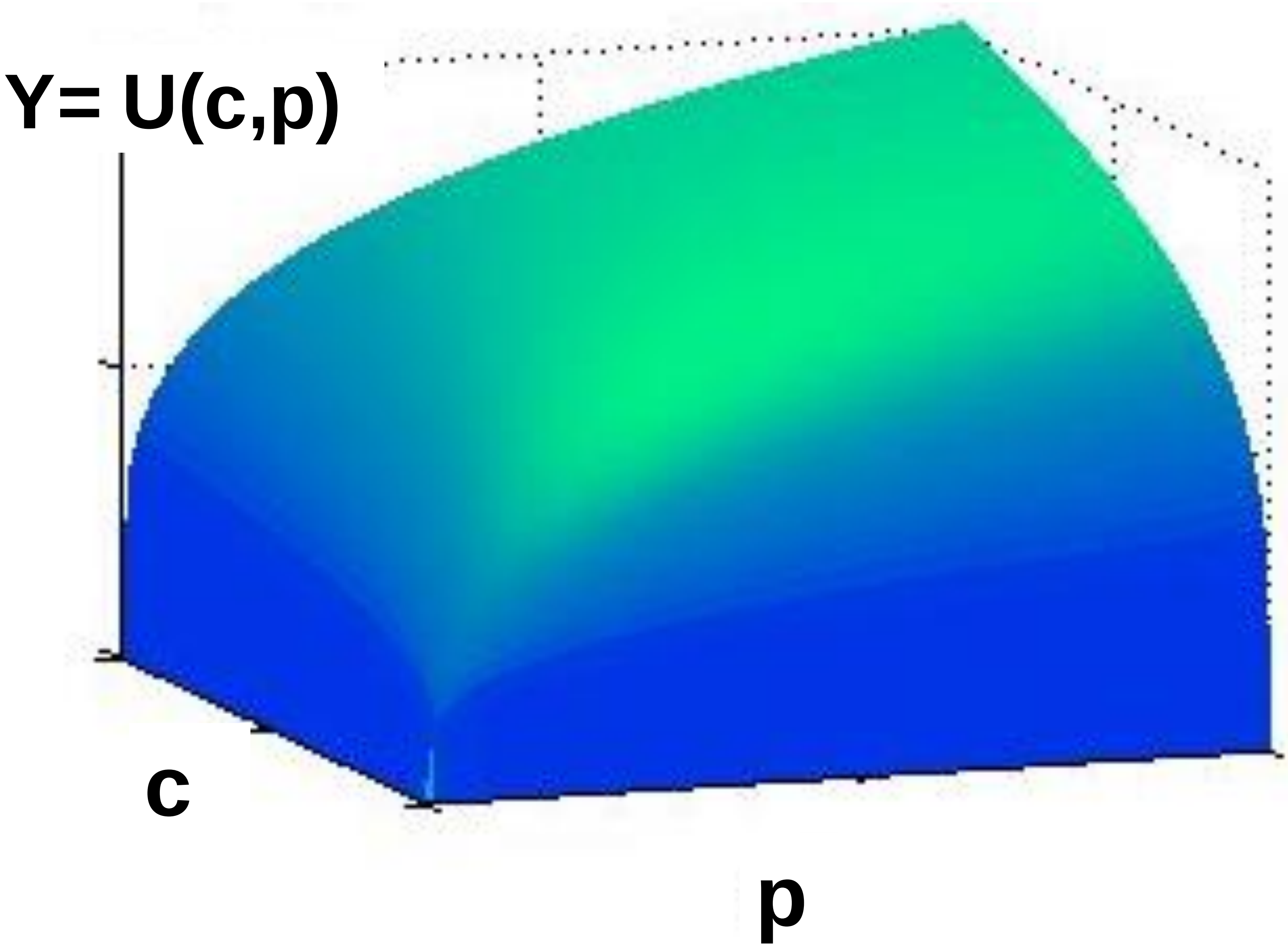


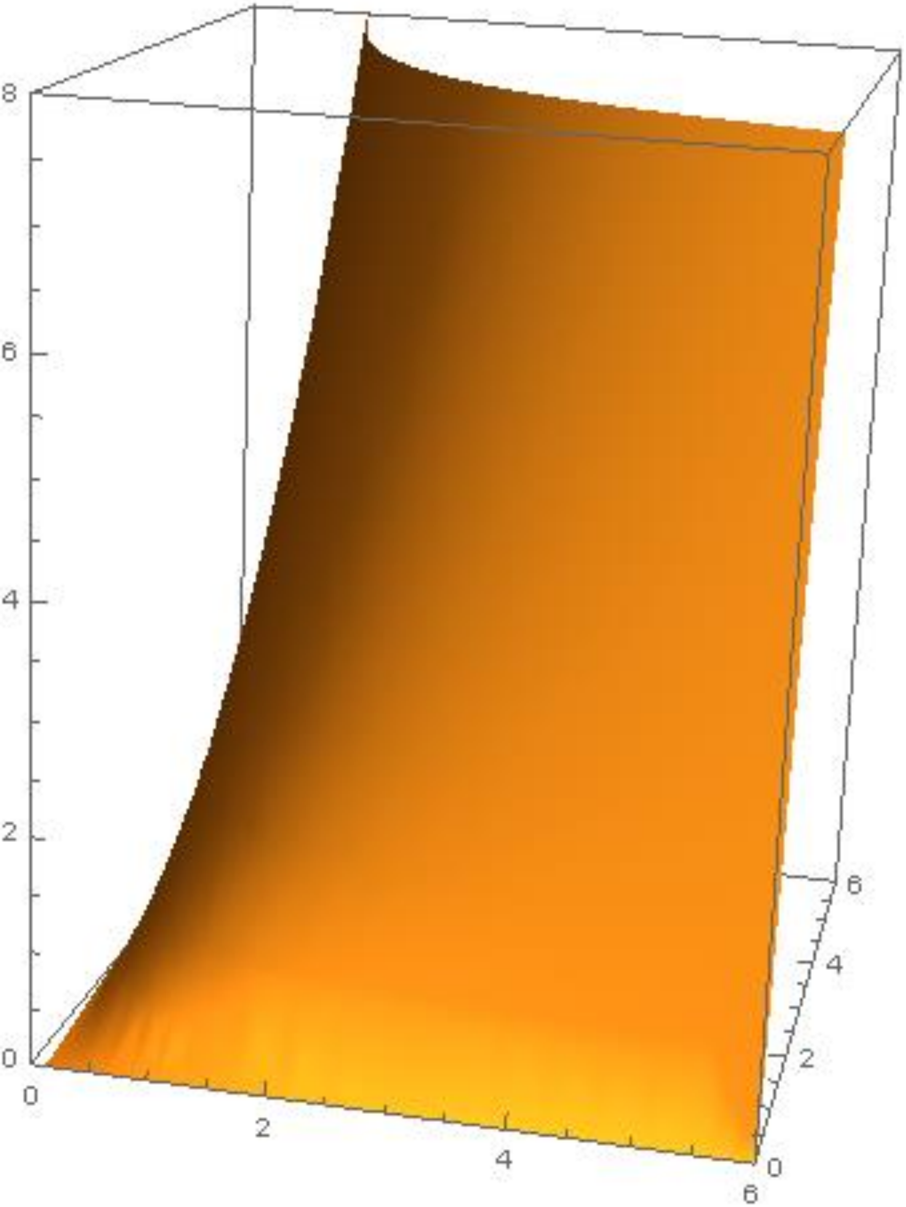


$$U[c,p] = c \cdot p$$

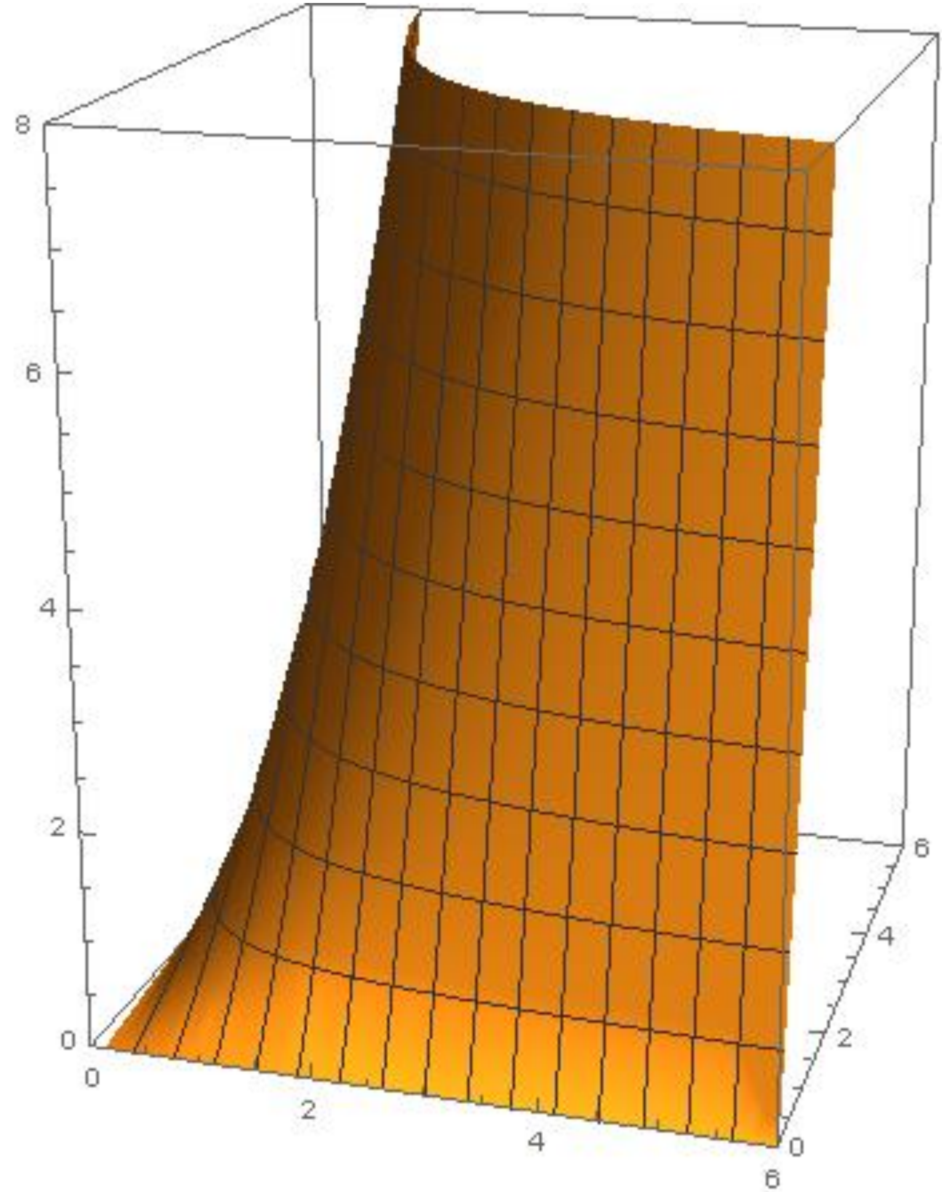


Pictures originally from http://www2.hawaii.edu/~fuleky/anatomy/anatomy.html


Y= U(c,p)
c
p



Does this utility graph represent convex preferences?



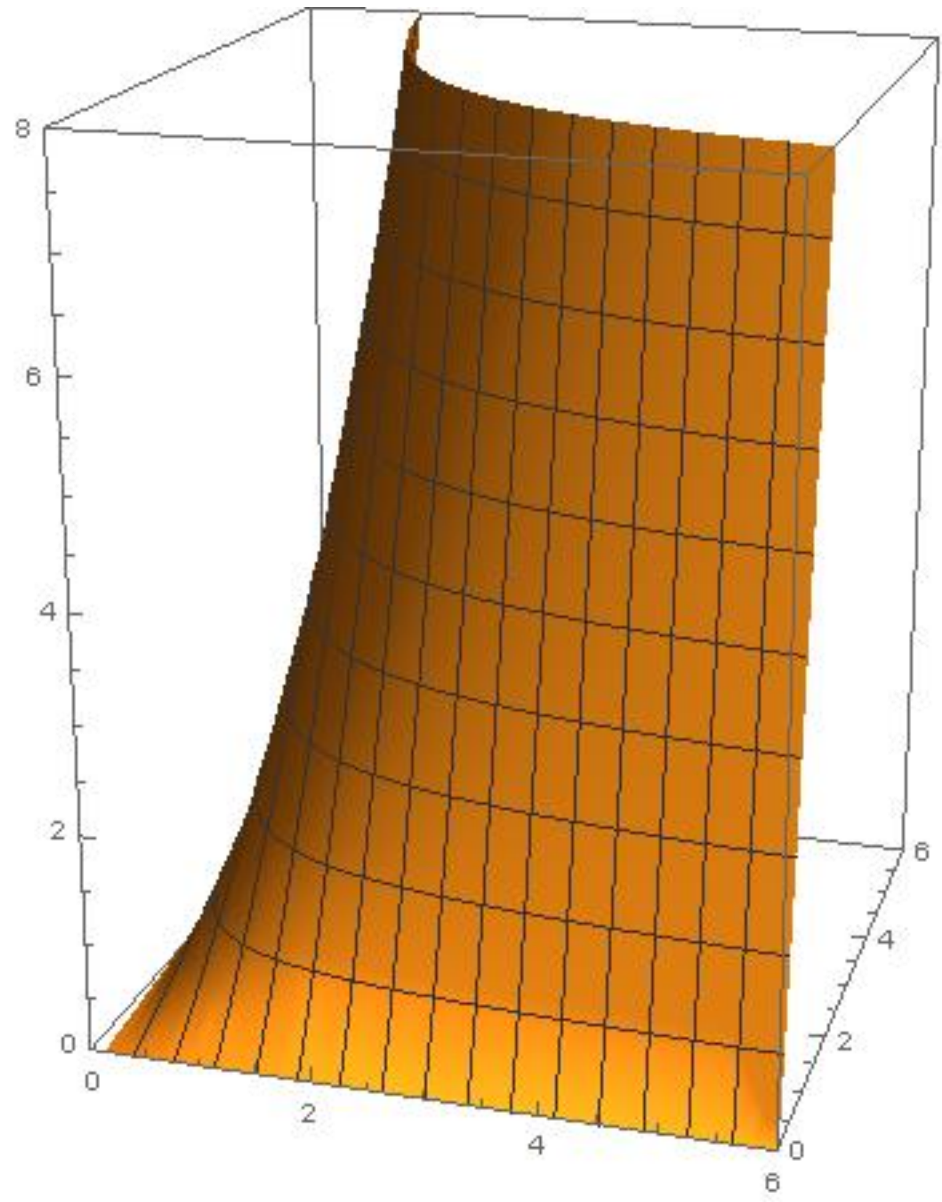
Yes!

(but not von Neumann-Morgenstern preferences – See Micro 3)

$$ParametricPlot3D[$$
$$\{x,((u\wedge.9)/x),u\},\{u,0,13\},\{x,0,6\},$$
$$PlotRange\to\{\{0,6\},\{0,6\},\{0,8\}\},$$
$$MeshStyle\to None]$$

$$y=\frac{u^{.9}}{x}=u^{.9}x^{-1}$$

$$\Leftrightarrow u^{.9}=xy\Leftrightarrow u=(xy)^{\frac{1}{.9}}\approx(xy)^{1.11}$$



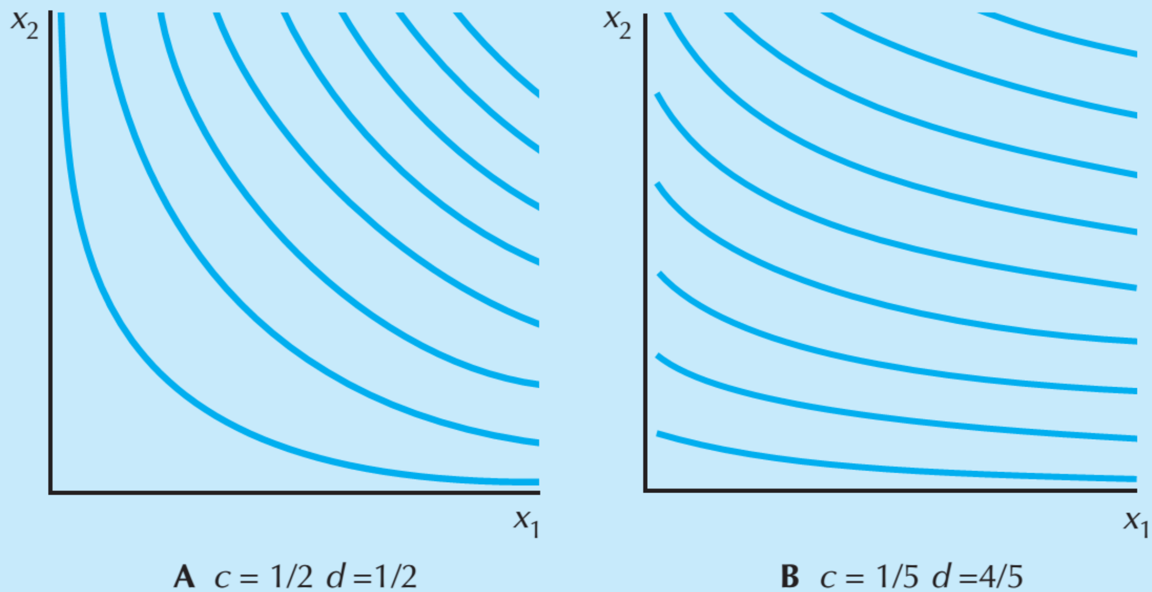
What form does the utility function have to be to create such a shape?

- **Cobb-Douglas preferences**

Cobb-Douglas Preferences

Another commonly used utility function is the **Cobb-Douglas** utility function

$$u(x_1, x_2) = x_1^c x_2^d,$$



Cobb-Douglas indifference curves. Panel A shows the case where $c = 1/2$, $d = 1/2$ and panel B shows the case where $c = 1/5$, $d = 4/5$.

Of course a monotonic transformation of the Cobb-Douglas utility function will represent exactly the same preferences, and it is useful to see a couple of examples of these transformations.

$$u(x_1, x_2) = x_1^c x_2^d$$

First, if we take the natural log of utility, the product of the terms will become a sum so that we have

$$v(x_1, x_2) = \ln(x_1^c x_2^d) = c \ln x_1 + d \ln x_2.$$

For the second example, suppose that we start with the Cobb-Douglas form

$$v(x_1, x_2) = x_1^c x_2^d.$$

Then raising utility to the $1/(c + d)$ power, we have

$$x_1^{\frac{c}{c+d}} x_2^{\frac{d}{c+d}}.$$

Now define a new number

$$a = \frac{c}{c + d}.$$

We can now write our utility function as

$$v(x_1, x_2) = x_1^a x_2^{1-a}.$$

- **Linear preferences/ Perfect substitutes**

Utility functions for red and blue pencils?

$$u[x, y] = x + y \qquad ?\ \text{OK}$$

$$u[x, y] = \left(x + y\right)^2 \qquad ?\ \text{OK}$$

$$u[x, y] = x^3 + 3x^2 y + 3xy^2 + y^3 \qquad ?\text{OK}$$

Perfect substitutes

Utility functions for coins of CZK1 (*x*) and CZK2 (*y*)?

$$u[x,y] = a \cdot x + b \cdot y$$

$$u[x,y] = 2x + y$$? OK

$$u[x,y] = x + 2y$$? OK

$$u[x,y] = 2x + 4y$$? OK

$$u[x,y] = 4^x \cdot 2^y$$? OK

FT

Perfect Substitutes

Remember the red pencil and blue pencil example? All that mattered to the consumer was the total number of pencils. Thus it is natural to measure utility by the total number of pencils. Therefore we provisionally pick the utility function $u(x_1, x_2) = x_1 + x_2$. Does this work? Just ask two things: is this utility function constant along the indifference curves? Does it assign a higher label to more-preferred bundles? The answer to both questions is yes, so we have a utility function.

Of course, this isn't the only utility function that we could use. We could also use the *square* of the number of pencils. Thus the utility function $v(x_1, x_2) = (x_1 + x_2)^2 = x_1^2 + 2x_1x_2 + x_2^2$ will also represent the perfect-substitutes preferences, as would any other monotonic transformation of $u(x_1, x_2)$.

FT

What if the consumer is willing to substitute good 1 for good 2 at a rate that is different from one-to-one? Suppose, for example, that the consumer would require *two* units of good 2 to compensate him for giving up one unit of good 1. This means that good 1 is *twice* as valuable to the consumer as good 2. The utility function therefore takes the form $u(x_1, x_2) = 2x_1 + x_2$. Note that this utility yields indifference curves with a slope of -2.

In general, preferences for perfect substitutes can be represented by a utility function of the form

$$u(x_1, x_2) = ax_1 + bx_2.$$

- **Leontiev preferences/ Perfect complements**

Utility functions for left (*L*) and right (*R*) shoes?

$$u[L,R] = L \cdot R \quad ? \text{ NO}$$

$$u[L,R] = \min[L,R] \quad ? \text{ OK}$$

$$u[L,R] = LN\left[4 \cdot \min[L^2, R^2]\right] \quad ?\text{OK}$$

Marsmen have 1 left and 2 right feet. What is their U-function for shoes?

$$u[L,R] = \min[L,2R] \quad ? \text{ NO}$$

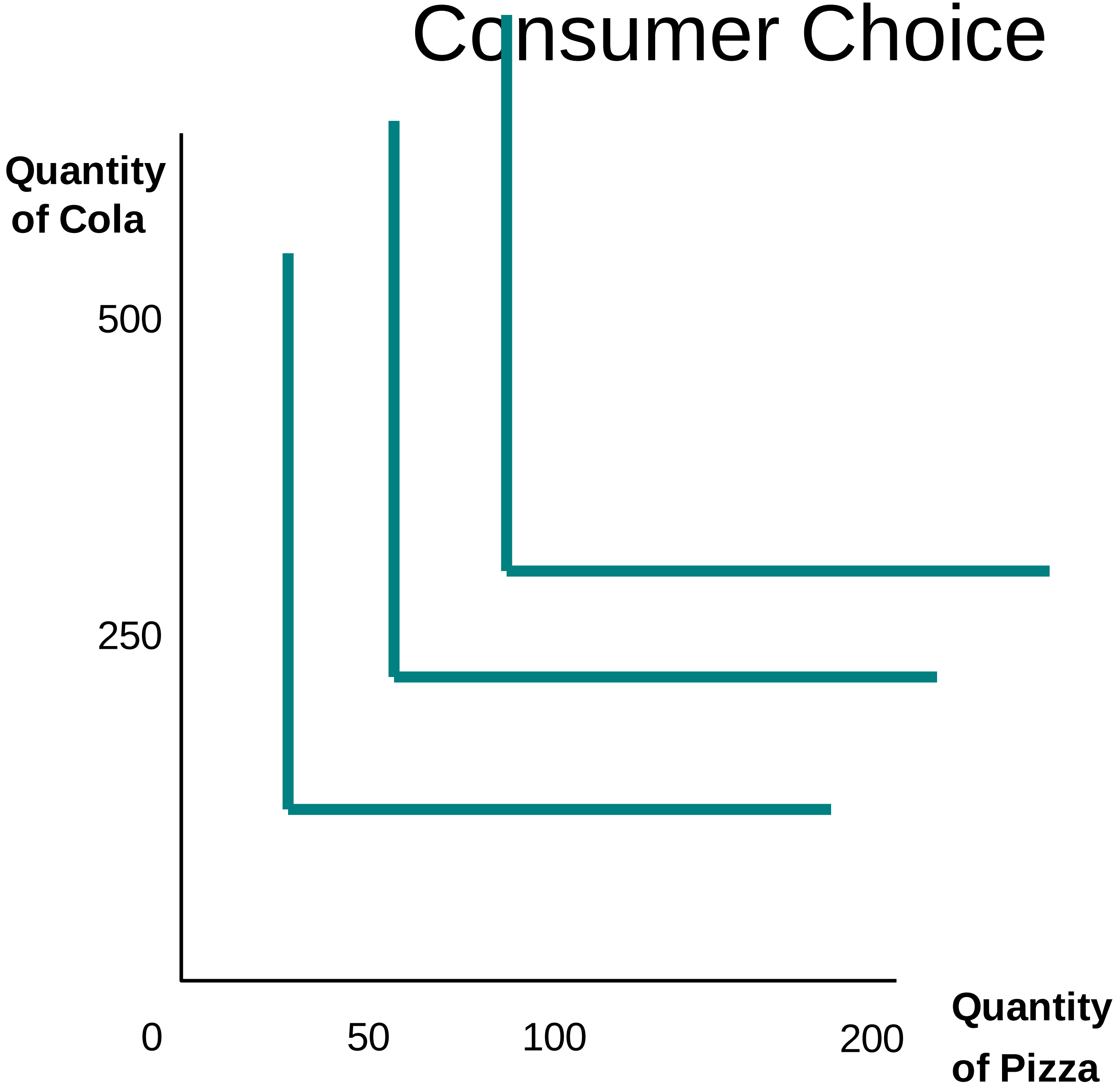
$$u[L,R] = \min[2L,R] \quad ? \text{ OK}$$

FT

Perfect Complements

This is the left shoe–right shoe case. In these preferences the consumer only cares about the number of *pairs* of shoes he has, so it is natural to choose the number of pairs of shoes as the utility function. The number of complete pairs of shoes that you have is the *minimum* of the number of right shoes you have, x_1, and the number of left shoes you have, x_2. Thus the utility function for perfect complements takes the form $u(x_1, x_2) = \min\{x_1, x_2\}$.

Consumer Choice



Quasilinear Preferences

What does such a u-function look like?

$$x_2 = u - v[x_1]$$

$$u[x_1, x_2] = u = x_2 + v[x_1]$$

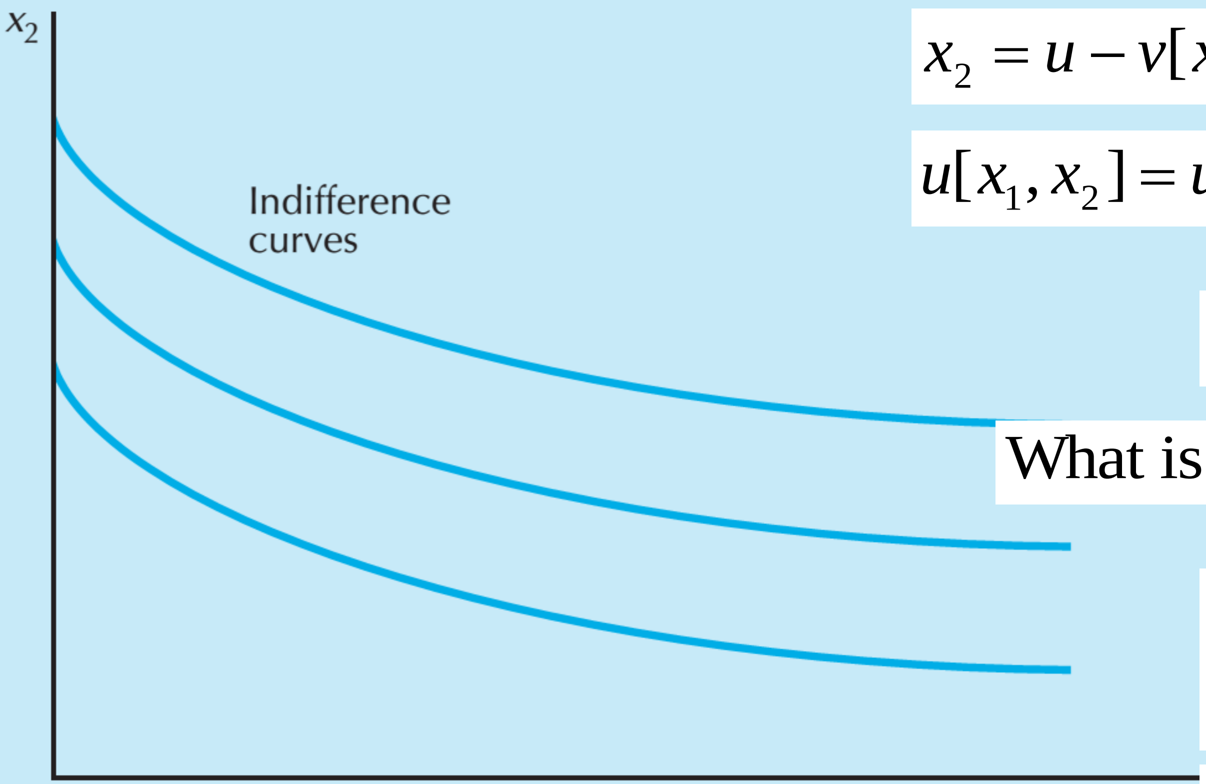
$$U[x_1, x_2] = v[x_1] + x_2$$

What is the slope of the ICs?

$$MRS = -\frac{U_1}{U_2}$$

$$= -\frac{v'[x_1]}{1}$$

$$= -v'[x_1]$$



Quasilinear preferences. Each indifference curve is a vertically shifted version of a single indifference curve.

linear" utility. Specific examples of quasilinear utility would be $u(x_1, x_2) = \sqrt{x_1} + x_2$, or $u(x_1, x_2) = \ln x_1 + x_2$. Quasilinear utility functions are not particularly realistic, but they are very easy to work with, as we'll see in several examples later on in the book.

FT

4.4 Marginal Utility

$$MU_1 = \frac{\Delta U}{\Delta x_1} = \frac{u(x_1 + \Delta x_1, x_2) - u(x_1, x_2)}{\Delta x_1},$$

that measures the rate of change in utility (ΔU) associated with a small change in the amount of good 1 (Δx_1). Note that the amount of good 2 is held fixed in this calculation.[3]

This definition implies that to calculate the change in utility associated with a small change in consumption of good 1, we can just multiply the change in consumption by the marginal utility of the good:

$$\Delta U = MU_1 \Delta x_1.$$

The marginal utility with respect to good 2 is defined in a similar manner:

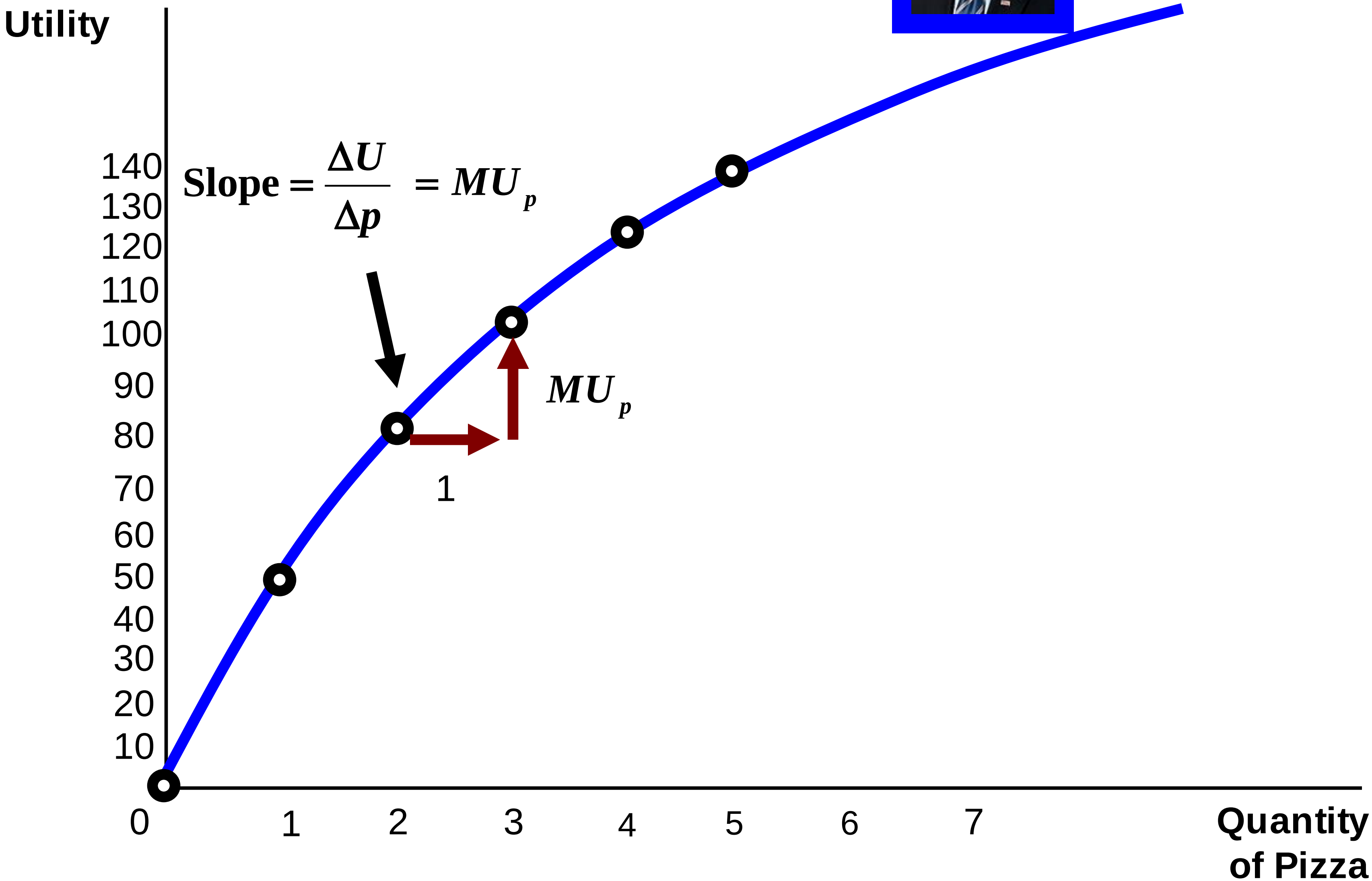
$$MU_2 = \frac{\Delta U}{\Delta x_2} = \frac{u(x_1, x_2 + \Delta x_2) - u(x_1, x_2)}{\Delta x_2}.$$

Note that when we compute the marginal utility with respect to good 2 we keep the amount of good 1 constant. We can calculate the change in utility associated with a change in the consumption of good 2 by the formula

$$\Delta U = MU_2 \Delta x_2.$$

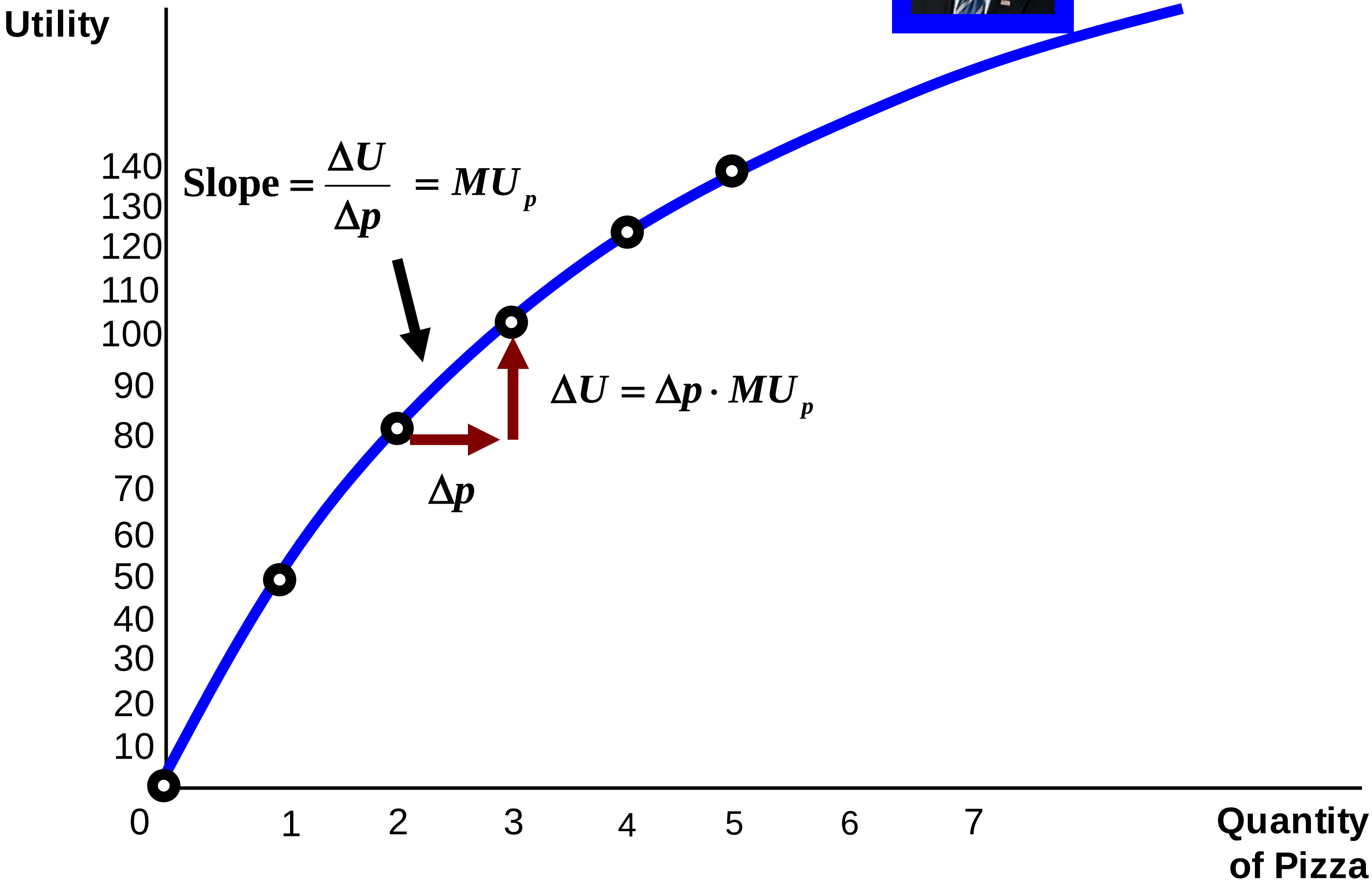
Utility function

Only 1 rule: more is better



Utility function

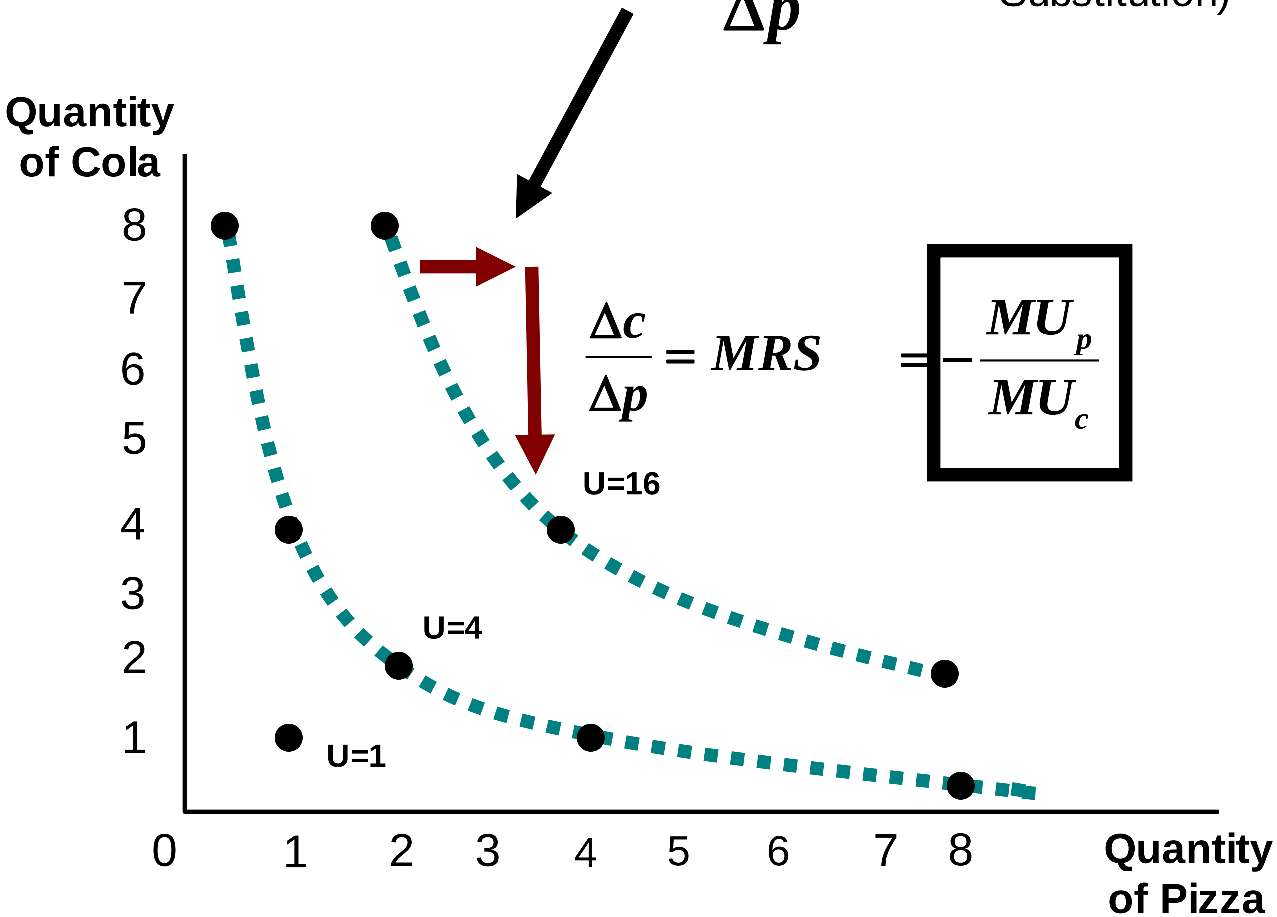
Only 1 rule: more is better



Utility function

$$U[c, p] = c \cdot p$$

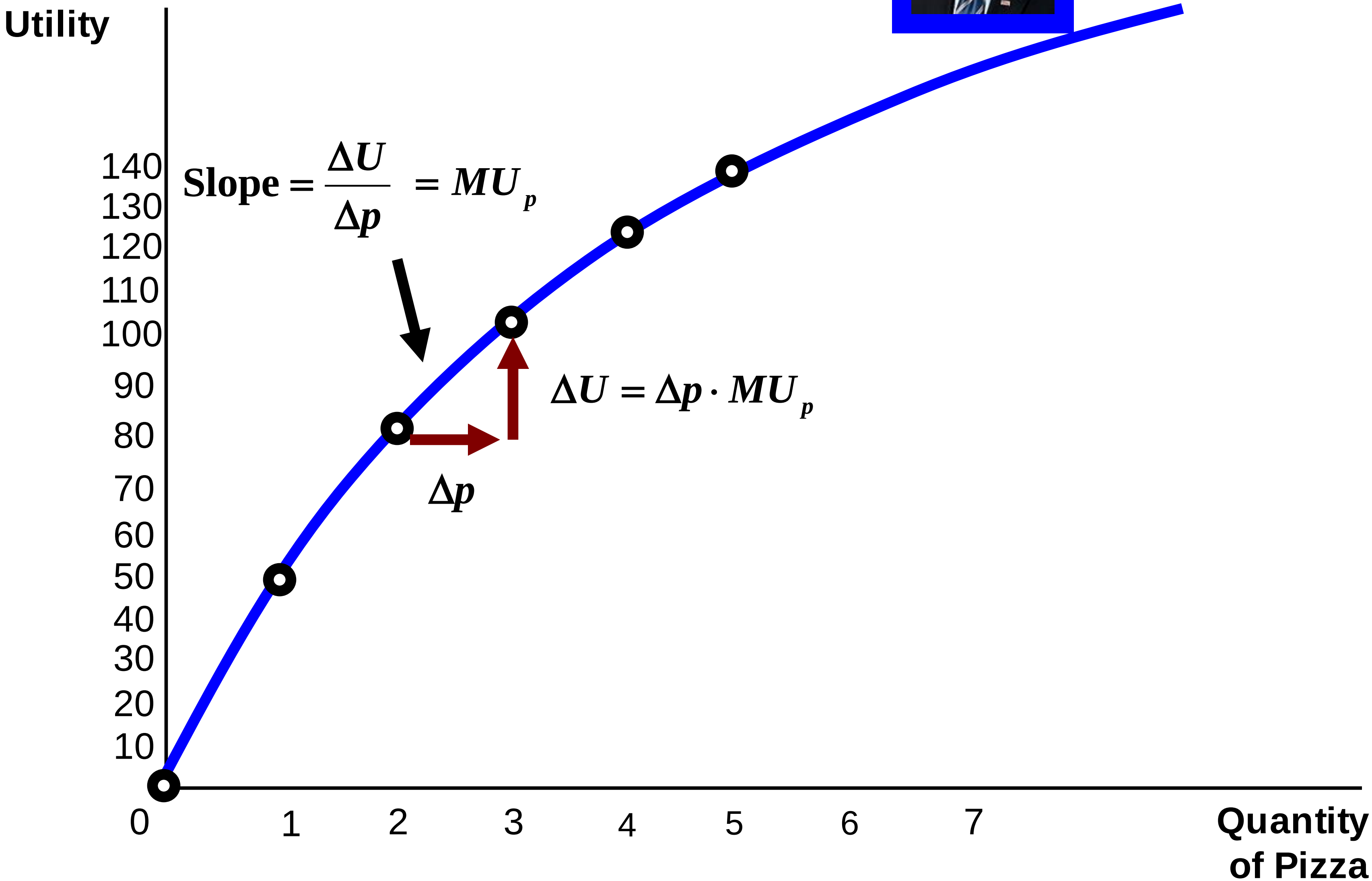
Slope? $\dfrac{\Delta c}{\Delta p}$ =MRS (Marginal Rate of Substitution)



$$\frac{\Delta c}{\Delta p} = MRS = -\frac{MU_p}{MU_c}$$

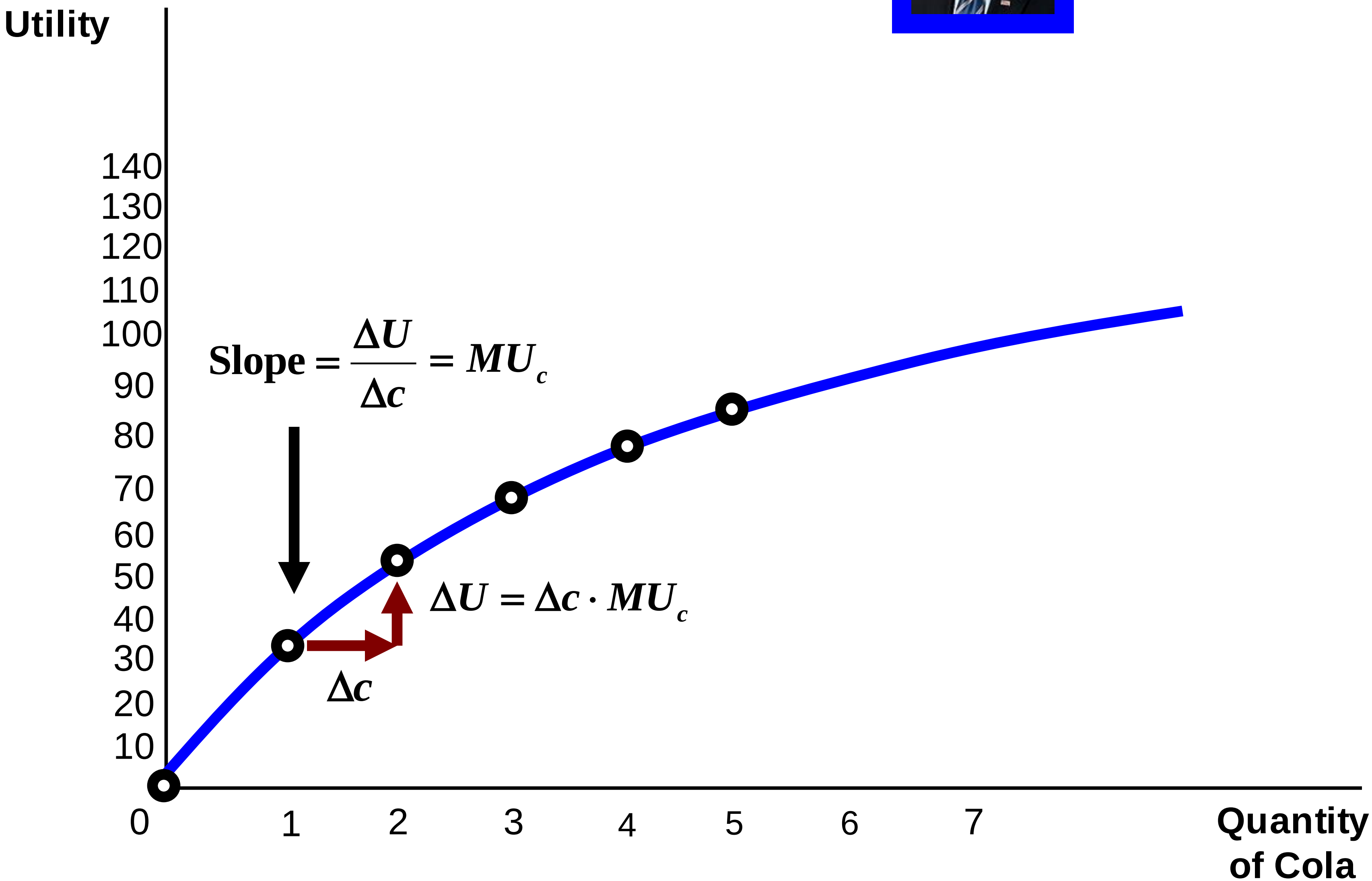
Utility function

Only 1 rule: more is better



Utility function

Only 1 rule: more is better



$$\frac{\Delta U}{\Delta p} = MU_p$$

$$\Delta U = \Delta p \cdot MU_p$$ Change in utils through more pizza

$$\frac{\Delta U}{\Delta c} = MU_c$$

$$\Delta U = \Delta c \cdot MU_c$$ Change in utils through more cola

$$\Delta U = \Delta c \cdot MU_c + \Delta p \cdot MU_p$$ Change in utils through more pizza and cola

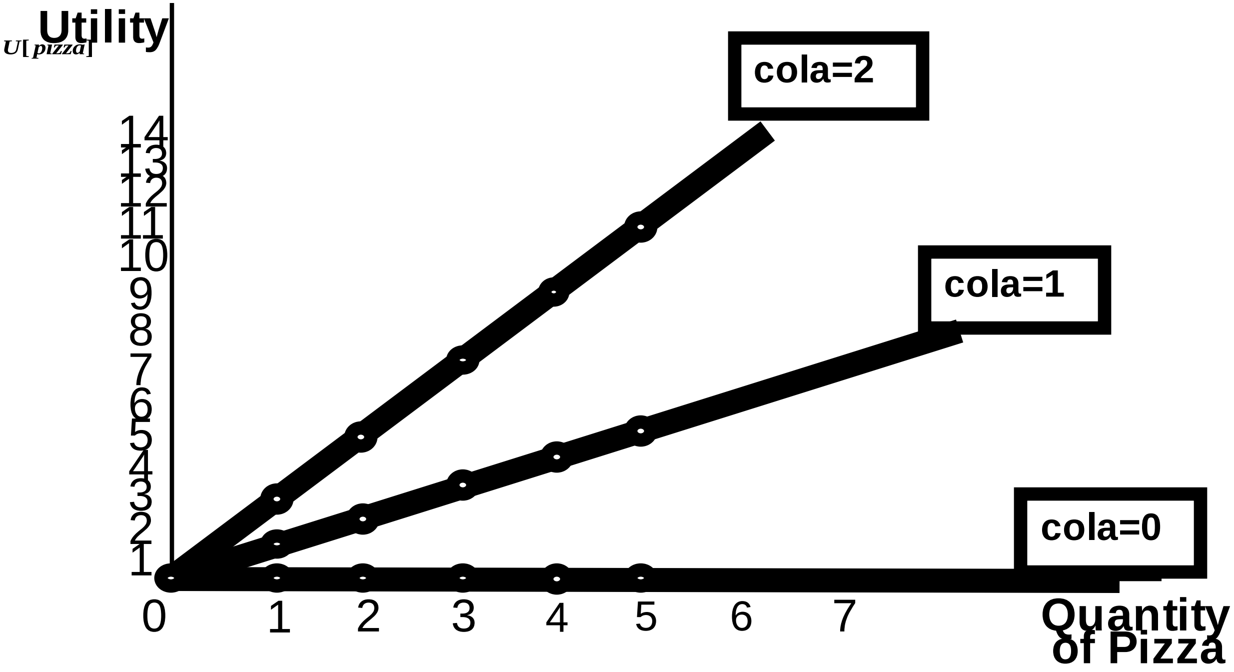
$$0 = \Delta c \cdot MU_c + \Delta p \cdot MU_p$$ On the IC

$$\Delta c \cdot MU_c = -\Delta p \cdot MU_p$$

$$\frac{\Delta c}{\Delta p} \cdot MU_c = - \cdot MU_p$$

$$\frac{\Delta c}{\Delta p} = -\frac{MU_p}{MU_c}$$

$$U[c, p] = c \cdot p$$



What is MU_p?

$$MU_p = \frac{\Delta U[c, p]}{\Delta p}$$

$$MU_p = c$$

What is MU_c?

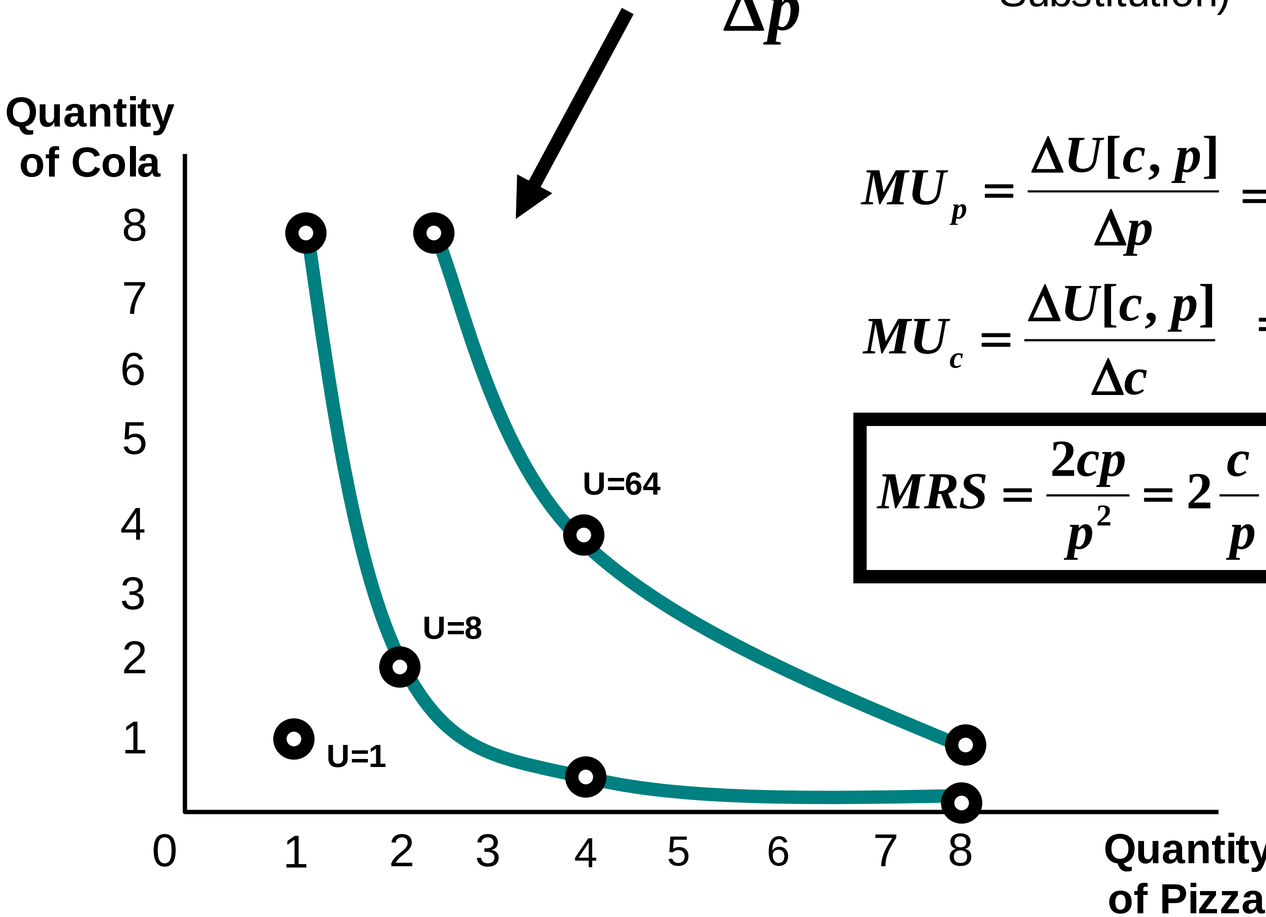
$$MU_c = \frac{\Delta U[c, p]}{\Delta c}$$

$$MU_p = p$$

Utility function

$$U[c, p] = c \cdot p^2$$

Slope? $\frac{\Delta c}{\Delta p}$ =MRS (Marginal Rate of Substitution) $= -\frac{MU_p}{MU_c}$

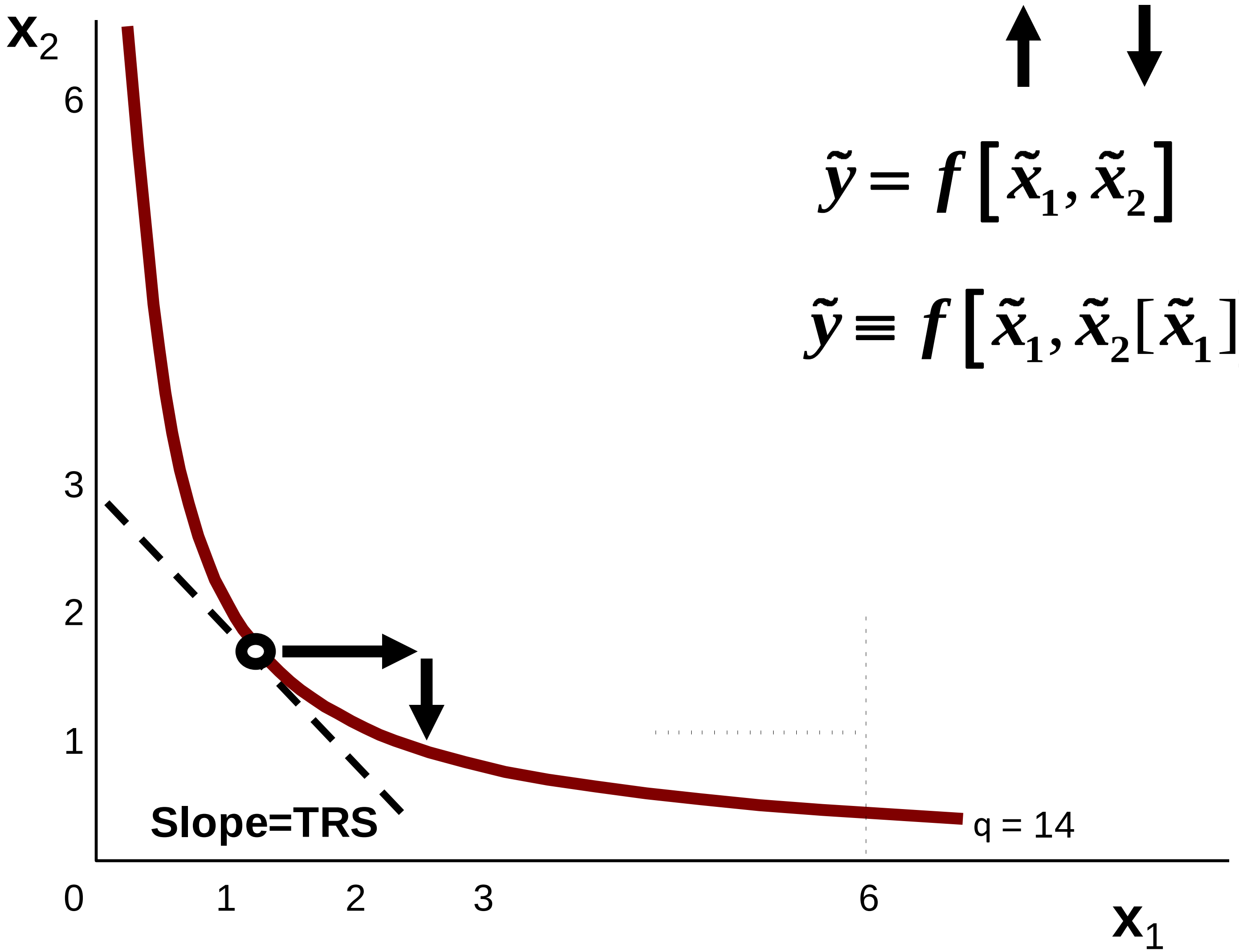


$$MU_p = \frac{\Delta U[c, p]}{\Delta p} = c \cdot 2 \cdot p^1 = 2 \cdot c \cdot p$$

$$MU_c = \frac{\Delta U[c, p]}{\Delta c} = p^2$$

$$MRS = \frac{2cp}{p^2} = 2\frac{c}{p}$$

Isoquant



$$\tilde{y} = f\left[\tilde{x}_1, \tilde{x}_2\right]$$

$$\tilde{y} \equiv f\left[\tilde{x}_1, \tilde{x}_2[\tilde{x}_1]\right]$$

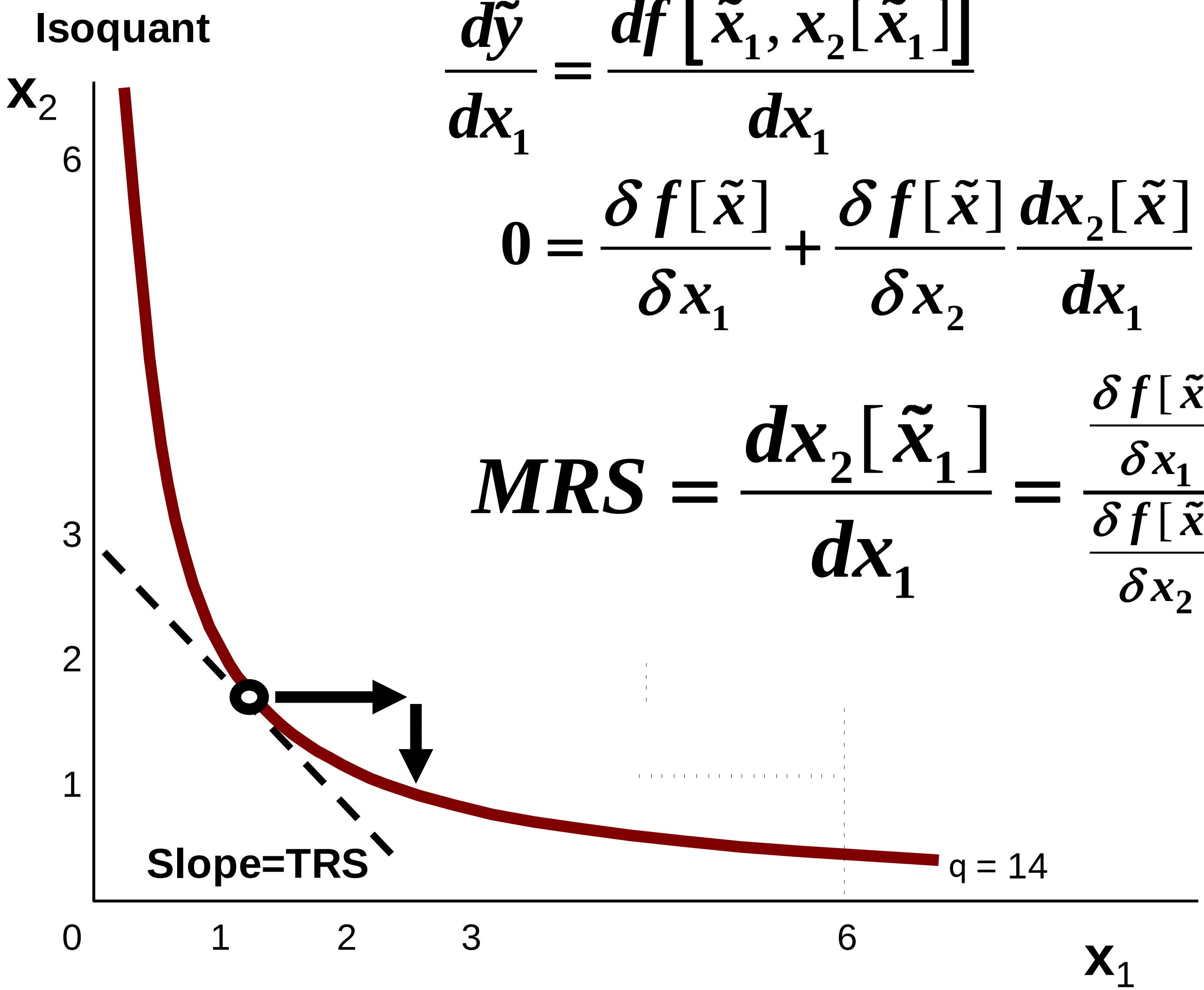
Derivation method 1: implicit function derivation

$$\tilde{y} \equiv f\left[\tilde{x}_1, x_2[\tilde{x}_1]\right]$$

$$\frac{d\tilde{y}}{dx_1} = \frac{df\left[\tilde{x}_1, x_2[\tilde{x}_1]\right]}{dx_1}$$

$$0 = \frac{\delta f[\tilde{x}]}{\delta x_1} + \frac{\delta f[\tilde{x}]}{\delta x_2}\frac{dx_2[\tilde{x}]}{dx_1}$$

$$MRS = \frac{dx_2[\tilde{x}_1]}{dx_1} = \frac{\frac{\delta f[\tilde{x}]}{\delta x_1}}{\frac{\delta f[\tilde{x}]}{\delta x_2}} \equiv \frac{MU_1}{MU_2}$$



- This implies:
  - Utility function is *invariant* under any monotonous transformation
  - (We knew this already)
  - Why?
    - Because utility is an ordinal variable

$$f(x, y)$$

$$MRS = \frac{MU_1}{MU_2} = \frac{f(x, y)_1}{f(x, y)_2}$$

$$g[f(x, y)]$$

$$\mathrm{MRS} = \frac{MU_1}{MU_2} = \frac{g[f(x_1, x_2)]_1}{g[f(x_1, x_2)]_2}$$

$$= \frac{\frac{dg[f(x_1,x_2)]}{df} f(x_1, x_2)_1}{\frac{dg[f(x_1,x_2)]}{df} f(x_1, x_2)_2} = \frac{f(x_1, x_2)_1}{f(x_1, x_2)_2}$$

FT The utility function, and therefore the marginal utility function, is not uniquely determined. Any monotonic transformation of a utility function leaves you with another equally valid utility function. Thus, if we multiply utility by 2, for example, the marginal utility is multiplied by 2. Thus the

But the *ratio* of marginal utilities gives us an observable magnitude—namely the marginal rate of substitution. The ratio of marginal utilities is independent of the particular transformation of the utility function you choose to use. Look at what happens if you multiply utility by 2. The MRS becomes

$$\text{MRS} = -\frac{2MU_1}{2MU_2}.$$

The same sort of thing occurs when we take any monotonic transformation of a utility function. Taking a monotonic transformation is just rela-

Suppose that we take a monotonic transformation of a utility function, say, $v(x_1, x_2) = f(u(x_1, x_2))$. Let's calculate the MRS for this utility function. Using the chain rule

$$\begin{aligned}\text{MRS} = -\frac{\partial v/\partial x_1}{\partial v/\partial x_2} &= -\frac{\partial f/\partial u}{\partial f/\partial u}\frac{\partial u/\partial x_1}{\partial u/\partial x_2} \\ &= -\frac{\partial u/\partial x_1}{\partial u/\partial x_2}\end{aligned}$$

The MRS for Cobb-Douglas preferences is easy to calculate by using the formula derived above.

If we choose the log representation where

$$u(x_1, x_2) = c \ln x_1 + d \ln x_2,$$

then we have

1. The text said that raising a number to an odd power was a monotonic transformation. What about raising a number to an even power? Is this a monotonic transformation? (Hint: consider the case $f(u) = u^2$.)

4.1. The function $f(u) = u^2$ is a monotonic transformation for positive u, but not for negative u.

2. Which of the following are monotonic transformations? (1) $u = 2v - 13$; (2) $u = -1/v^2$; (3) $u = 1/v^2$; (4) $u = \ln v$; (5) $u = -e^{-v}$; (6) $u = v^2$; (7) $u = v^2$ for $v > 0$; (8) $u = v^2$ for $v < 0$.

(1) Yes.

(2) No (works for v positive).

(3) No (works for v negative).

(4) Yes (only defined for v positive)

(5) Yes

(6) No

(7) Yes

(8) No

3. We claimed in the text that if preferences were monotonic, then a diagonal line through the origin would intersect each indifference curve exactly once. Can you prove this rigorously? (Hint: what would happen if it intersected some indifference curve twice?)

4.3. Suppose that the diagonal intersected a given indifference curve at two points, say (x, x) and (y, y). Then either $x > y$ or $y > x$, which means that one of the bundles has more of *both* goods. But if preferences are monotonic, then one of the bundles would have to be preferred to the other.

4. What kind of preferences are represented by a utility function of the form $u(x_1, x_2) = \sqrt{x_1 + x_2}$? What about the utility function $v(x_1, x_2) = 13x_1 + 13x_2$?

4.4. Both represent perfect substitutes.

5. What kind of preferences are represented by a utility function of the form $u(x_1, x_2) = x_1 + \sqrt{x_2}$? Is the utility function $v(x_1, x_2) = x_1^2 + 2x_1\sqrt{x_2} + x_2$ a monotonic transformation of $u(x_1, x_2)$?

4.5. Quasilinear preferences

Yes.

6. Consider the utility function $u(x_1, x_2) = \sqrt{x_1 x_2}$. What kind of preferences does it represent? Is the function $v(x_1, x_2) = x_1^2 x_2$ a monotonic transformation of $u(x_1, x_2)$? Is the function $w(x_1, x_2) = x_1^2 x_2^2$ a monotonic transformation of $u(x_1, x_2)$?

4.6. The utility function represents Cobb-Douglas preferences.

No.

Yes.

7. Can you explain why taking a monotonic transformation of a utility function doesn't change the marginal rate of substitution?

4.7. Because the MRS is measured *along* an indifference curve, and utility remains constant along an indifference curve.

$$MRS_U = -\frac{U_x[x,y]}{U_y[x,y]}$$

$$V[x,y] = g\big[U[x,y]\big], \text{ where } g'(\cdot) > 0$$

$$V_x[x,y] = g'\big[U[x,y]\big] \cdot U_x[x,y]$$

$$V_y[x,y] = g'\big[U[x,y]\big] \cdot U_y[x,y]$$

$$MRS_V = -\frac{V_x[x,y]}{V_y[x,y]}$$

$$= -\frac{g'\big[U[x,y]\big] \cdot U_x[x,y]}{g'\big[U[x,y]\big] \cdot U_y[x,y]}$$

$$= -\frac{U_x[x,y]}{U_y[x,y]} = MRS_U$$

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

Národohospodářská fakulta VŠE v Praze