Microeconomics

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

Lecture 4&5

Silvester van Koten

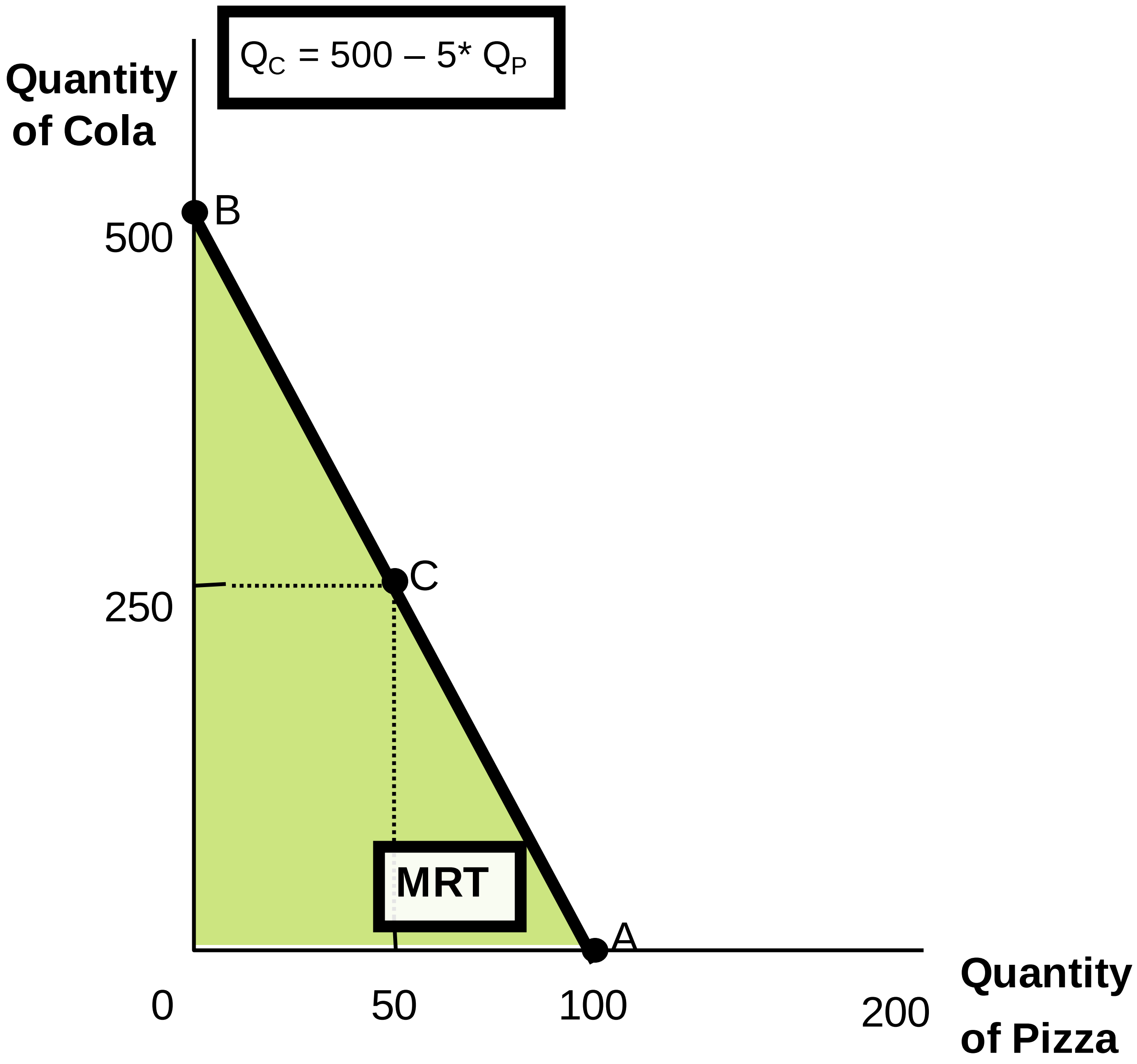
- The presentation is based on the following sources:

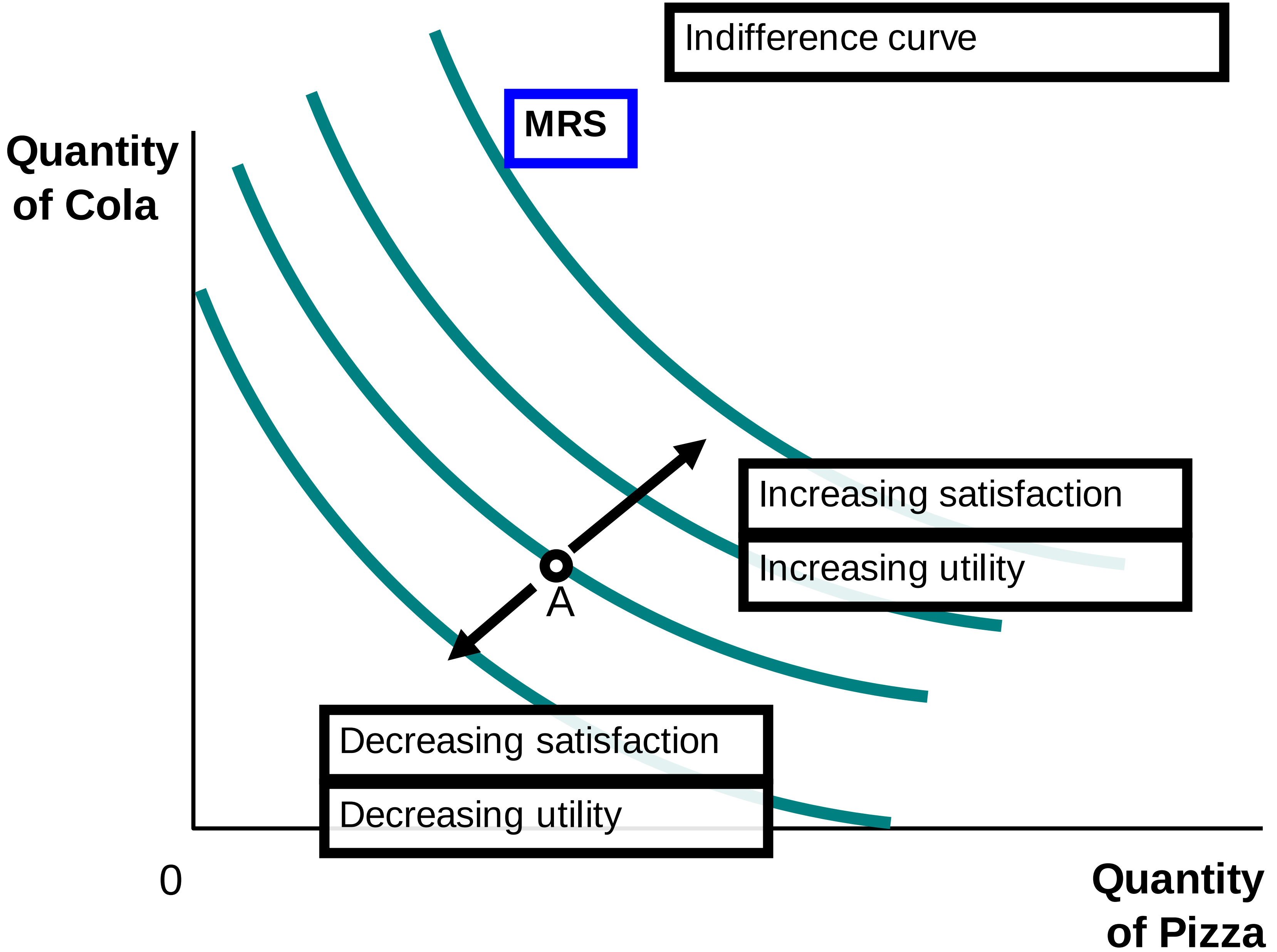
- Snyder Christopher M Walter Nicholson and Robert Stewart. 2015. Microeconomic Theory : Basic Principles and Extensions EMEA ed. Andover: Cengage Learning. +
- Varian Hal R. 2020. Intermediate Microeconomics : A Modern Approach Ninth ed. New York: W.W. Norton & Company. +
- Excerpts from Gravelle H O Hart and R Rees. 2004. Advanced Microeconomics.

Consumer choice

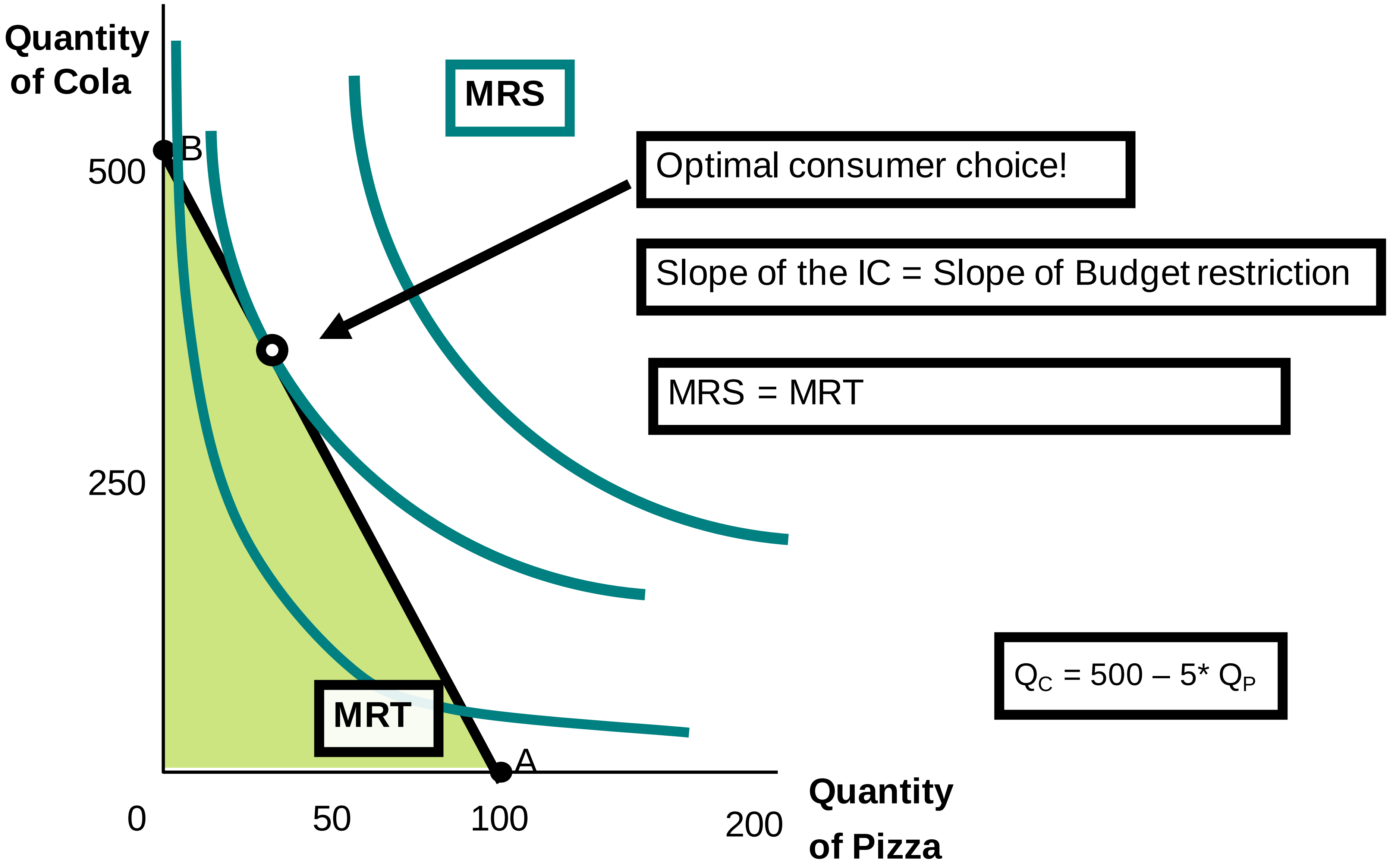
Chap. 5

Consumer Choice



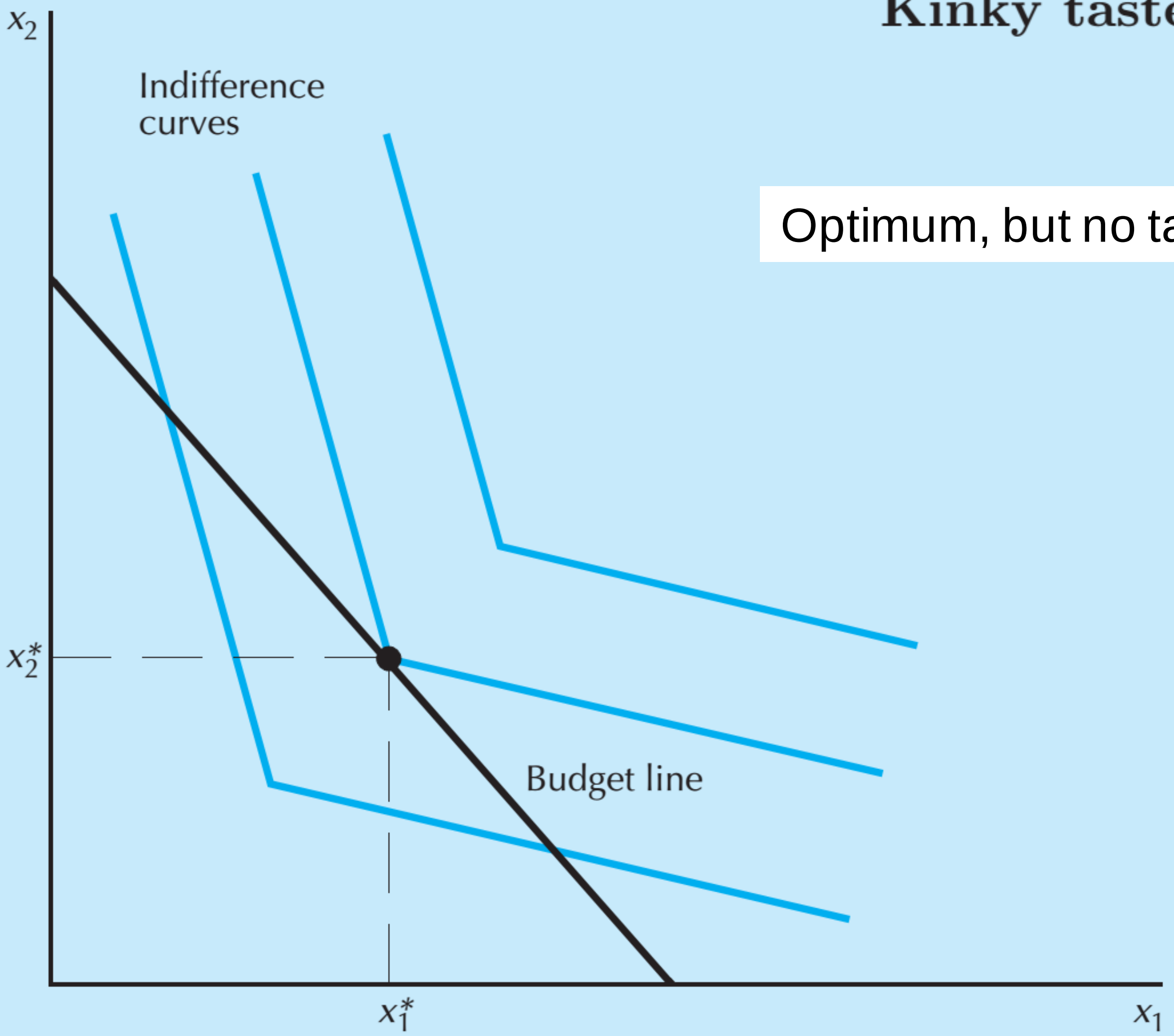

Indifference curve
MRS
Quantity
of Cola
Increasing satisfaction
Increasing utility
A
Decreasing satisfaction
Decreasing utility
0
Quantity
of Pizza

Consumer Choice



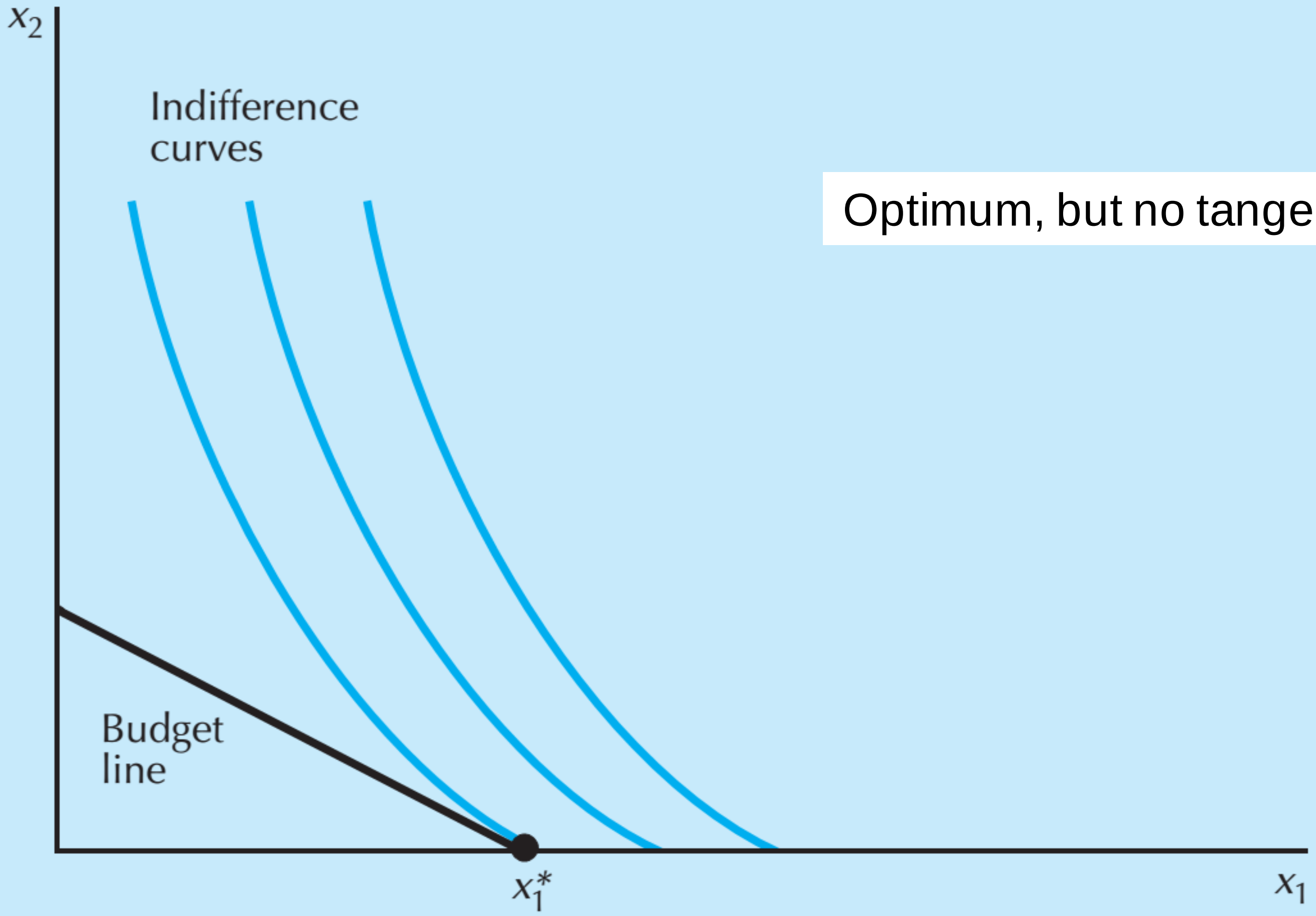
Kinky tastes.

Optimum, but no tangent



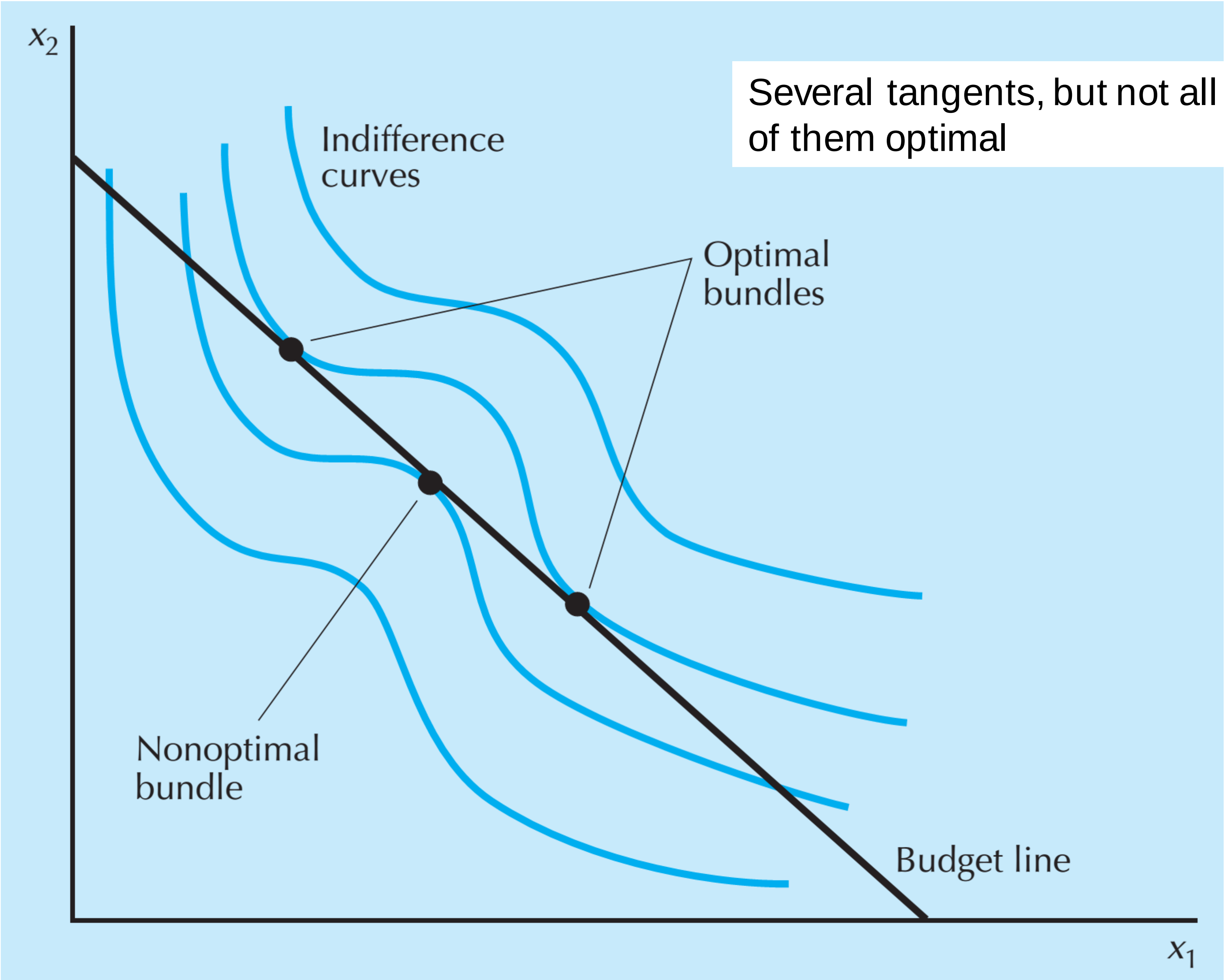
Kinky tastes. Here is an optimal consumption bundle where the indifference curve doesn't have a tangent.

Optimum, but no tangent



Boundary optimum. The optimal consumption involves consuming zero units of good 2. The indifference curve is not tangent to the budget line.

Several tangents, but not all of them optimal



For the moment:

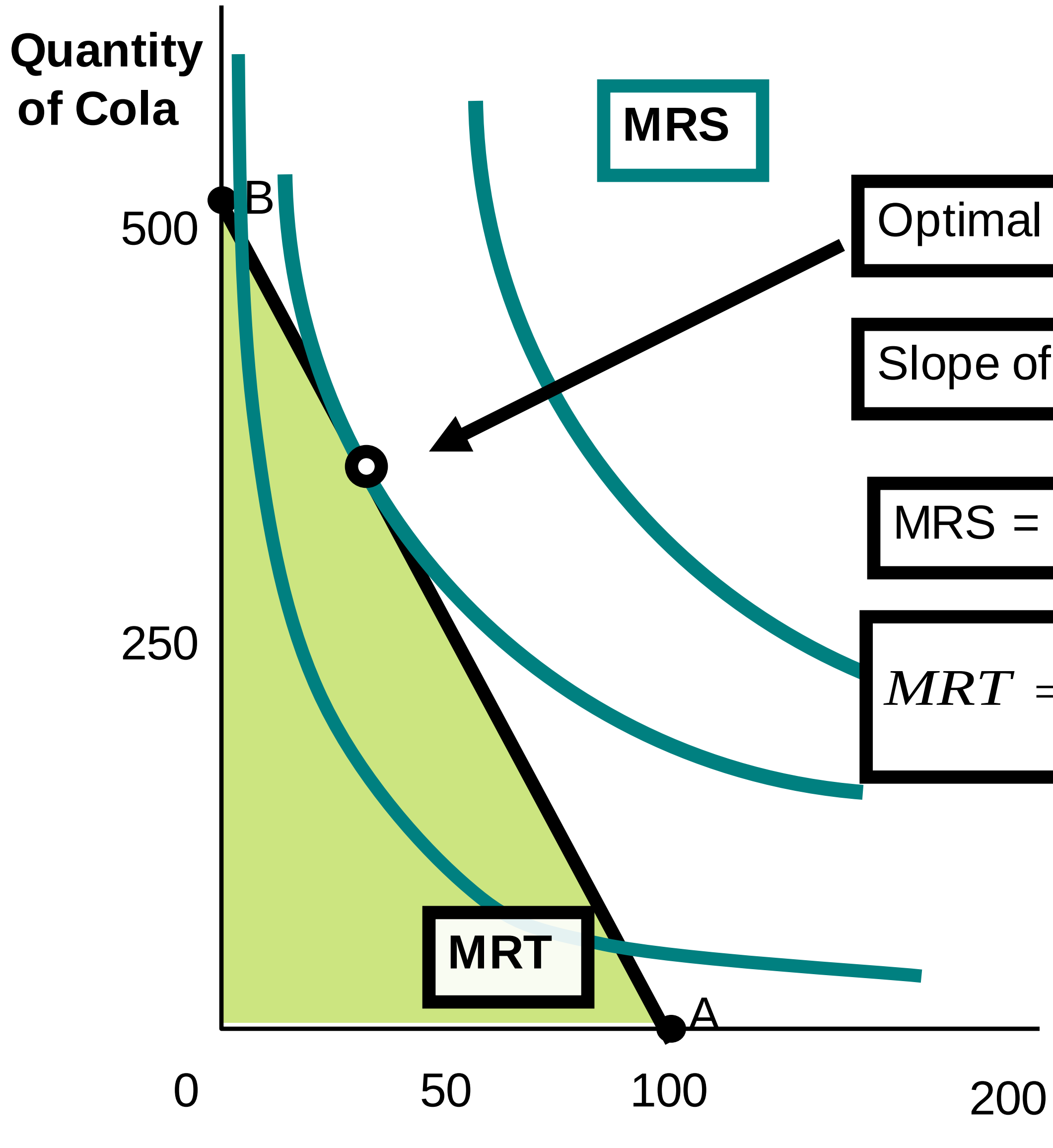
- Rule out “kinky tastes”
- Rule out corner solutions & focus on interior solutions

Generally:

- Assume convex preferences

If we are willing to rule out "kinky tastes" we can forget about the example given in Figure 5.2.[1] And if we are willing to restrict ourselves only to *interior* optima, we can rule out the other example. If we have an interior optimum with smooth indifference curves, the slope of the indifference curve and the slope of the budget line must be the same ... because if they were different the indifference curve would cross the budget line, and we couldn't be at the optimal point.

Consumer Choice



Optimal consumer choice!

Slope of the IC = Slope of Budget restriction

MRS = MRT

$$MRT = -\frac{P_P}{P_C}$$

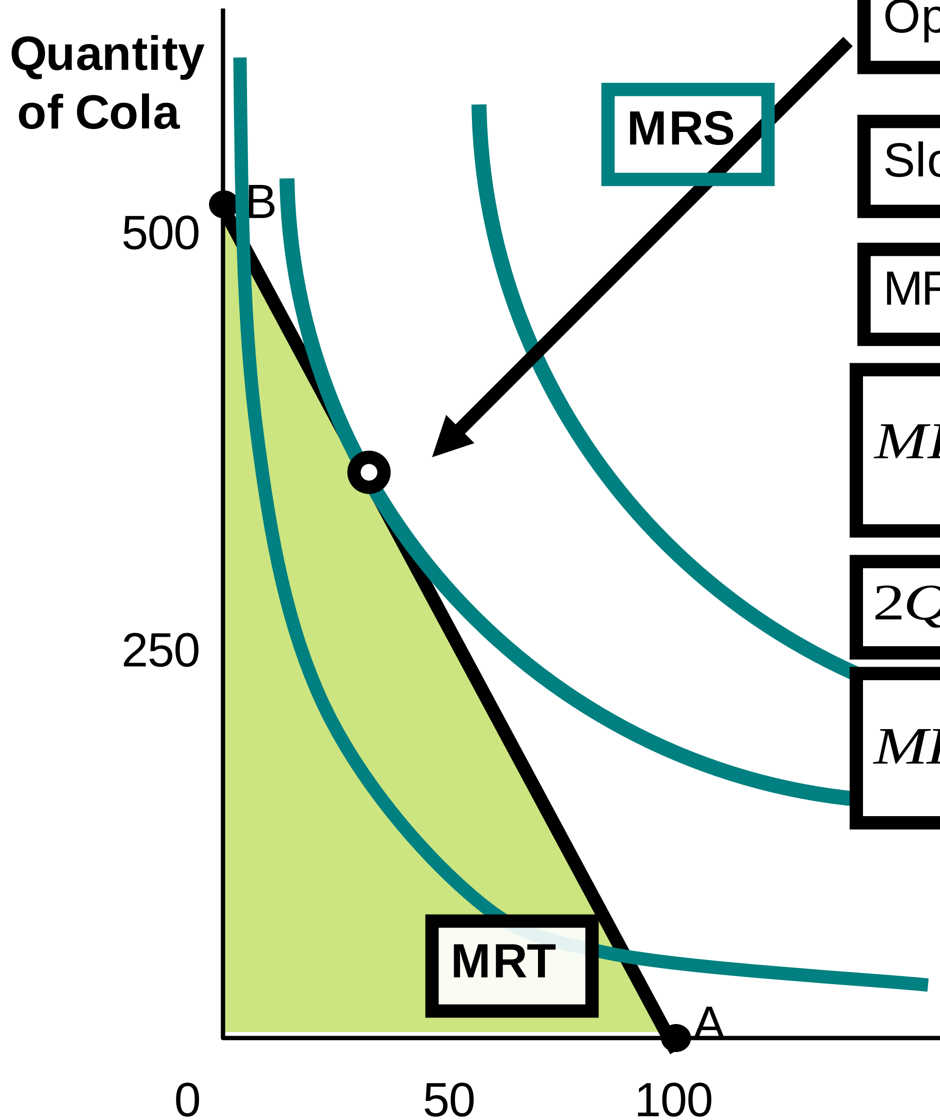
$$MRS = -\frac{MU_p}{MU_c}$$

$$\frac{MU_p}{MU_c} = \frac{P_P}{P_C}$$

- $\overbrace{Q_C * P_C + Q_P * P_P}^{\text{Expenditure}} = \overbrace{M}^{\text{Budget}}$
- $Q_C * P_C = M - Q_P * P_P$
- $Q_C = (M - Q_P * P_P) / P_C$
- $Q_C = M / P_C - (P_P / P_C) * Q_P$

$$MRT = \frac{\Delta Q_C}{\Delta Q_P} = -\frac{P_P}{P_C}$$

Consumer Choice



Optimal consumer choice!

Slope of the IC = Slope of Budget restriction

MRS = MRT

$$MRT = -\frac{P_P}{P_C}$$

$$MRS = -\frac{MU_p}{MU_c}$$

$$2Q_P + 10Q_C = 360$$

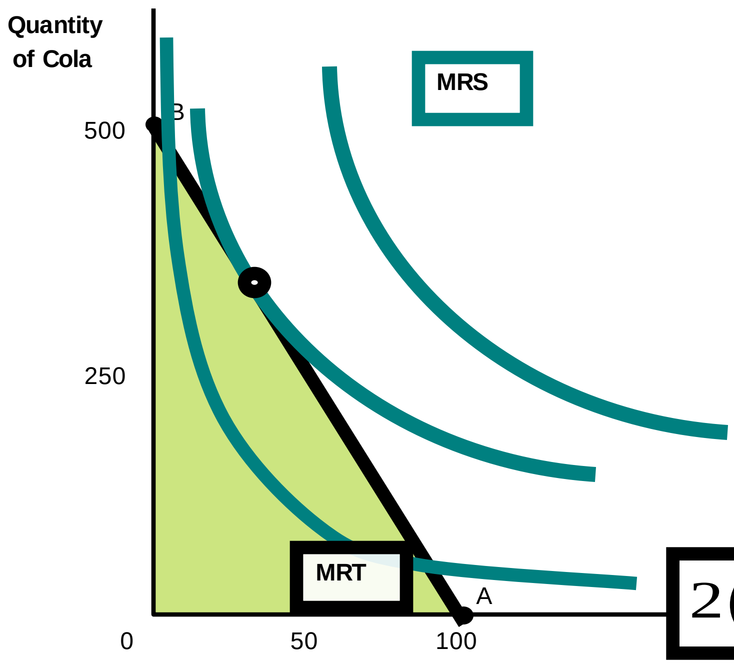
$$U = Q_P \cdot Q_C$$

$$MRT = -\frac{1}{5}$$

$$MRS = -\frac{Q_C}{Q_P}$$

$$\frac{Q_C}{Q_P} = \frac{1}{5} \Leftrightarrow Q_P = 5Q_C$$

Consumer Choice



$$2Q_P + 10Q_C = 360$$

$$U = Q_P \cdot Q_C$$

$$MRT = -\frac{1}{5}$$

$$MRS = -\frac{Q_C}{Q_P}$$

$$\frac{Q_C}{Q_P} = \frac{1}{5} \Leftrightarrow Q_P = 5Q_C$$

$$2(Q_P) + 10Q_C = 360 \Leftrightarrow 2(5Q_C) + 10Q_C = 360$$

$$\Leftrightarrow 10Q_C + 10Q_C = 360$$

$$\Leftrightarrow 20Q_C = 360$$

$$\Leftrightarrow Q_C = \tfrac{360}{20} = 18$$

$$\Leftrightarrow Q_P = 5 \cdot 18 = 90$$

$$p_1x_1 + p_2x_2 = m.$$

$$\max_{x_1,x_2} u(x_1, x_2)$$

$$\text{such that } p_1x_1 + p_2x_2 = m.$$

$$x_2(x_1) = \frac{m}{p_2} - \frac{p_1}{p_2}x_1 \longrightarrow \frac{dx_2}{dx_1} = -\frac{p_1}{p_2}$$

$$\max_{x_1} u(x_1, m/p_2 - (p_1/p_2)x_1)$$

$$\frac{\partial u(x_1, x_2(x_1))}{\partial x_1} + \frac{\partial u(x_1, x_2(x_1))}{\partial x_2}\frac{dx_2}{dx_1} = 0$$

$$\frac{\partial u(x_1, x_2(x_1))}{\partial x_1} + \frac{\partial u(x_1, x_2(x_1))}{\partial x_2}\frac{dx_2}{dx_1} = 0 \qquad \frac{dx_2}{dx_1} = -\frac{p_1}{p_2}$$

$$\frac{\partial u(x_1^*, x_2^*)/\partial x_1}{\partial u(x_1^*, x_2^*)/\partial x_2} = \frac{p_1}{p_2}$$

which just says that the marginal rate of substitution between x_1 and x_2 must equal the price ratio at the optimal choice (x_1^*, x_2^*). This is exactly the condition

$u(x_1, x_2)$ $\quad\quad$ $p_1x_1 + p_2x_2 - m = 0$

Use Lagrange

$$L = u(x_1, x_2) - \lambda(p_1x_1 + p_2x_2 - m)$$

$$\frac{\partial L}{\partial x_1} = \frac{\partial u(x_1^*, x_2^*)}{\partial x_1} - \lambda p_1 = 0$$
$$\frac{\partial L}{\partial x_2} = \frac{\partial u(x_1^*, x_2^*)}{\partial x_2} - \lambda p_2 = 0$$
$$\frac{\partial L}{\partial \lambda} = p_1x_1^* + p_2x_2^* - m = 0.$$

$$\frac{\partial u(x_1^*, x_2^*)/\partial x_1}{\partial u(x_1^*, x_2^*)/\partial x_2} = \frac{p_1}{p_2}$$

$$u(x_1, x_2) = x_1^c x_2^d.$$

$$\ln u(x_1, x_2) = c \ln x_1 + d \ln x_2$$

The maximization problem becomes:

$$\max_{x_1, x_2} c \ln x_1 + d \ln x_2$$

$$\text{such that } p_1 x_1 + p_2 x_2 = m$$

Set up the Lagrangian

Now for Lagrange's method. Set up the Lagrangian

$$L = c\ln x_1 + d\ln x_2 - \lambda(p_1x_1 + p_2x_2 - m)$$

$$\frac{\partial L}{\partial x_1} = \frac{c}{x_1} - \lambda p_1 = 0$$
$$\frac{\partial L}{\partial x_2} = \frac{d}{x_2} - \lambda p_2 = 0$$
$$\frac{\partial L}{\partial \lambda} = p_1x_1 + p_2x_2 - m = 0.$$

$$c = \lambda p_1 x_1$$
$$d = \lambda p_2 x_2.$$

$$c + d = \lambda(p_1x_1 + p_2x_2) = \lambda m$$

$$\lambda = \frac{c+d}{m}$$

$$\lambda = \frac{c+d}{m}$$

$$c = \lambda p_1 x_1$$
$$d = \lambda p_2 x_2.$$

$$x_1 = \frac{c}{c+d}\frac{m}{p_1}$$
$$x_2 = \frac{d}{c+d}\frac{m}{p_2},$$

The second way is to substitute the budget constraint into the maximization problem at the beginning. If we do this, our problem becomes

$$\max_{x_1} c \ln x_1 + d \ln(m/p_2 - x_1 p_1/p_2)$$

$$\frac{c}{x_1} - d \frac{p_2}{m - p_1 x_1} \frac{p_1}{p_2} = 0.$$

$$x_1 = \frac{c}{c+d} \frac{m}{p_1}$$

$$x_2 = m/p_2 - x_1 p_1/p_2$$

$$x_2 = \frac{d}{c+d} \frac{m}{p_2}$$

$$\boxed{\frac{MU_p}{MU_c} = \frac{P_P}{P_C}}$$

Equilibrium

What if $\dfrac{MU_p}{MU_c} > \dfrac{P_P}{P_C}$ **?**

$\downarrow$

What if $-\dfrac{MU_p}{MU_c} < -\dfrac{P_P}{P_C}$ **?**

Slope indifference curve
STEEPER than
Slope budget line

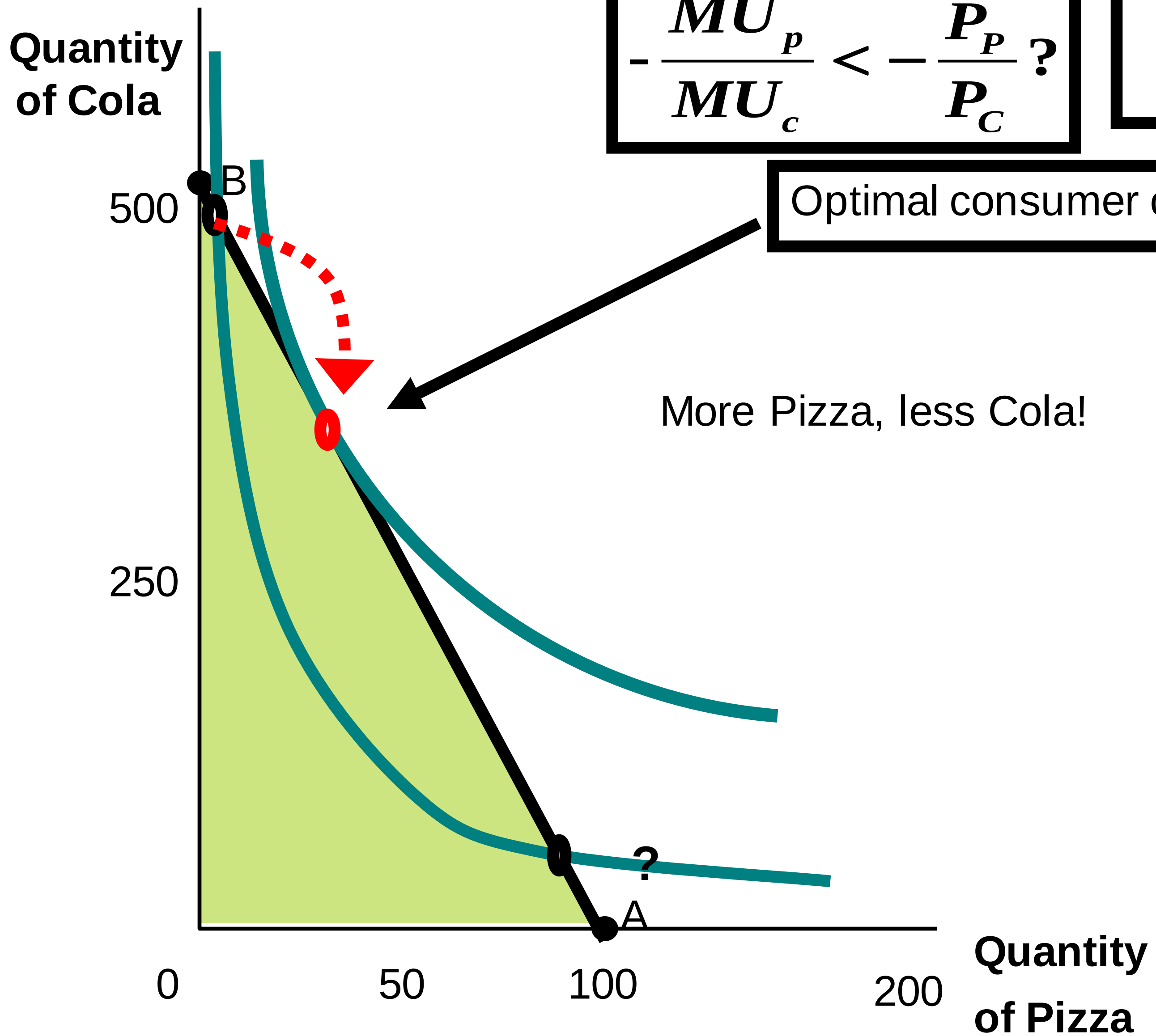
Consumer Choice

$$-\frac{MU_p}{MU_c} < -\frac{P_P}{P_C}\ ?$$

Slope indifference curve
STEEPER than
Slope budget line

Optimal consumer choice!

$$\frac{MU_p}{MU_c} = \frac{P_P}{P_C}$$



$$\boxed{\frac{MU_p}{MU_c} = \frac{P_P}{P_C}}$$

Equilibrium

What if $\dfrac{MU_p}{MU_c} < \dfrac{P_P}{P_C}$?

$\Downarrow$

What if - $\dfrac{MU_p}{MU_c} > -\dfrac{P_P}{P_C}$?

Slope indifference curve
SHALLOWER than
Slope budget line

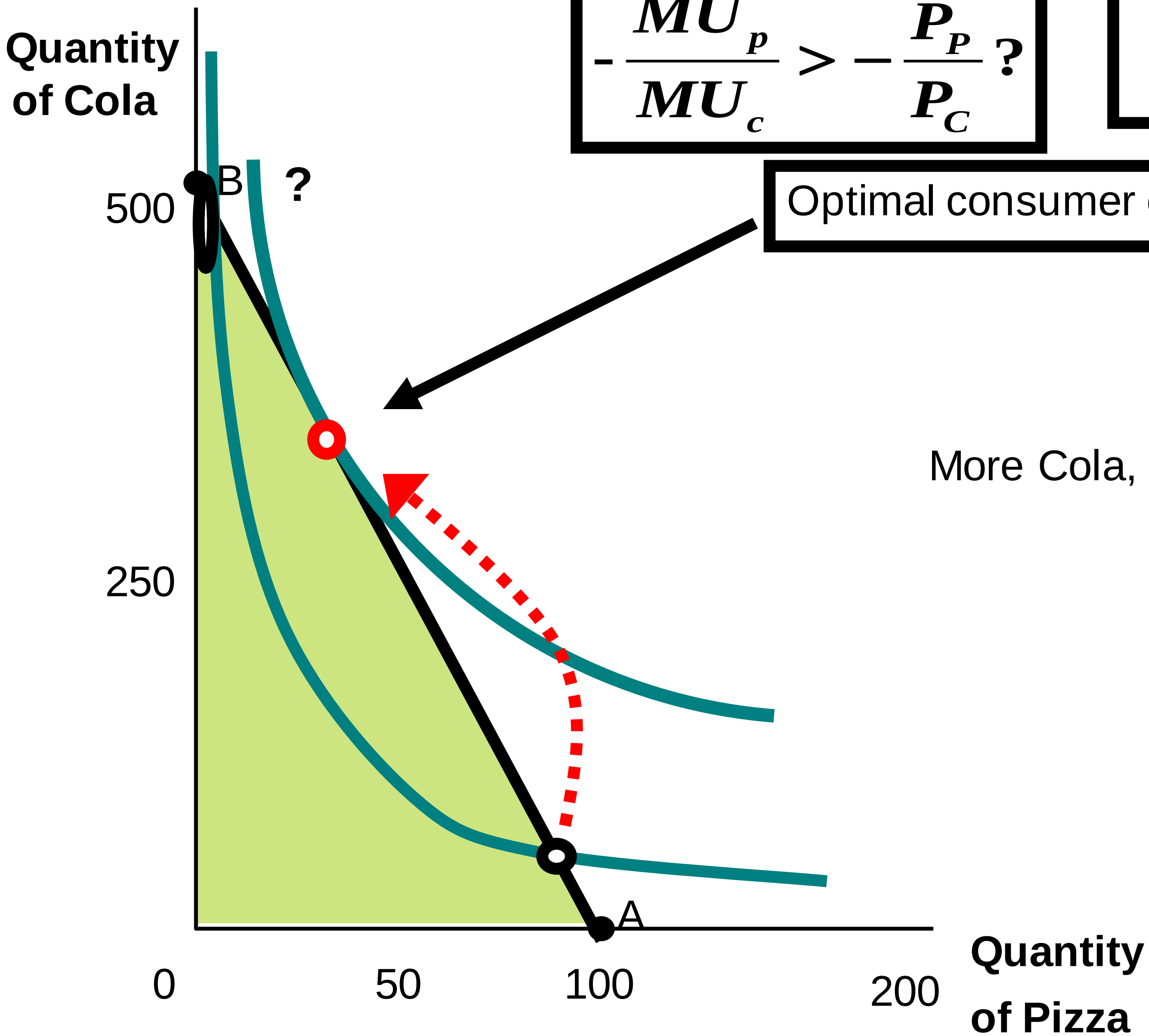
Consumer Choice

$$-\frac{MU_p}{MU_c} > -\frac{P_P}{P_C} \; ?$$

Slope indifference curve
SHALLOWER than
Slope budget line

Optimal consumer choice!

$$\frac{MU_p}{MU_c} = \frac{P_P}{P_C}$$



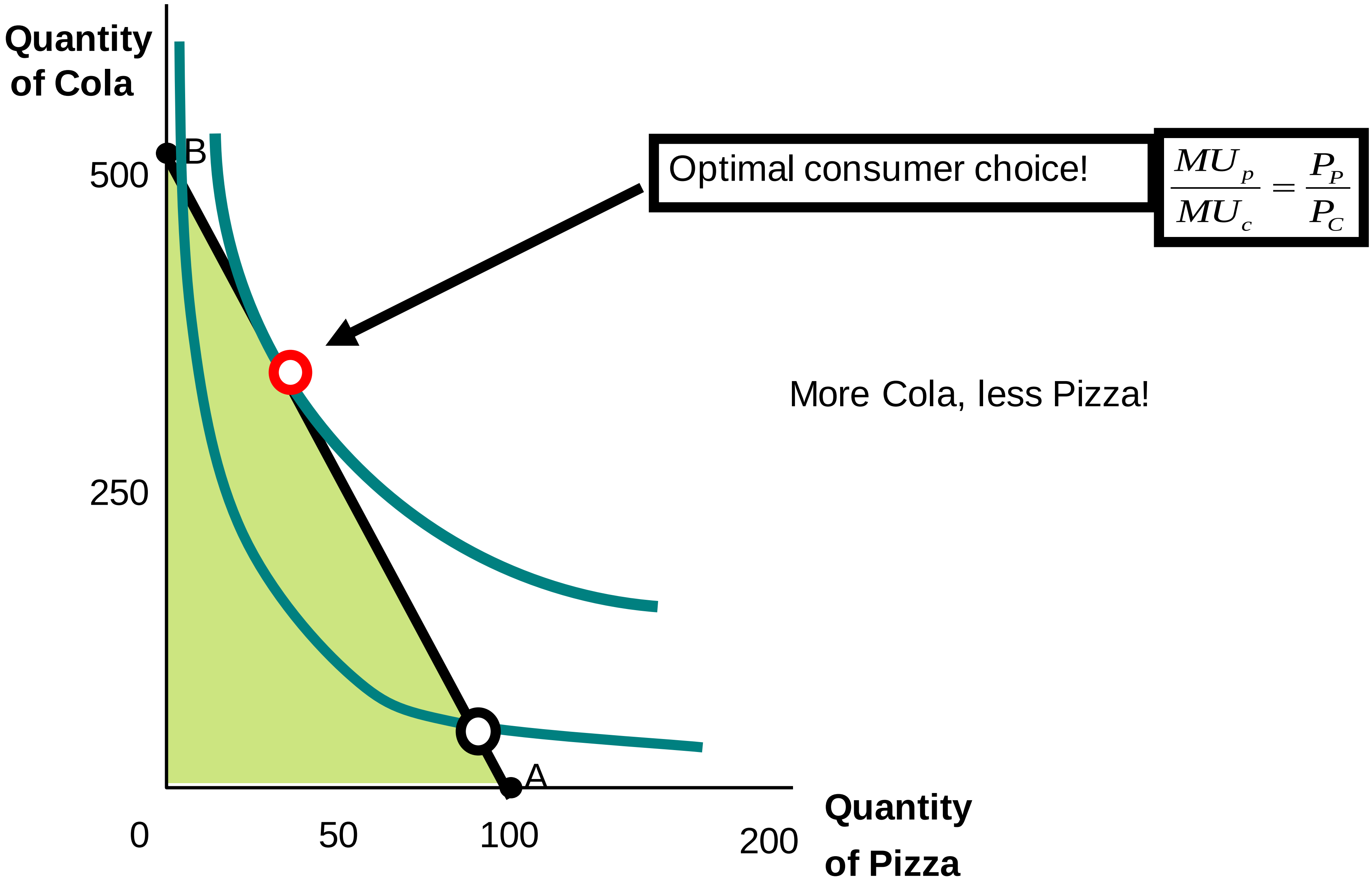
More Cola, less Pizza!

$$\boxed{\frac{MU_p}{MU_c} > \frac{P_P}{P_C}\ ?}$$ $\longrightarrow$ More Pizza, less Cola!

$$\boxed{\frac{MU_p}{MU_c} < \frac{P_P}{P_C}\ ?}$$ $\longrightarrow$ More Cola, less Pizza!

$$\boxed{\frac{MU_p}{MU_c} = \frac{P_P}{P_C}\ ?}$$ $\longrightarrow$ Good mix! Equilibrium

Consumer Choice



- Imagine that Pizza and Cola are equally expensive, so $P_P = P_C$
- Also adding one more unit:
  - of Pizza increases utility with 2
  - of Cola increases utility with 1
- You can change your consumption:
  - Exchanging Cola for Pizza: so more Pizza, less Cola
  - Exchanging Pizza for Cola: so more Cola, less Pizza
- What do you choose?
  - Exchanging Cola for Pizza: so more Pizza, less Cola!

In formulas:

$$\frac{P_P}{P_C} = \frac{1}{1} = 1 \qquad \frac{MU_p}{MU_c} = \frac{2}{1} = 2$$

$$\boxed{\frac{MU_p}{MU_c} > \frac{P_P}{P_C}}$$

Adding more pizza gives 2x more pleasure than adding more cola and adding more pizza cost the same as adding more cola

$$\boxed{\frac{MU_p}{P_P} > \frac{MU_c}{P_C}}$$

Adding more pizza gives 2x more pleasure per cost than adding more cola

Our diagram analysis: $\frac{MU_p}{MU_c} > \frac{P_P}{P_C}?$ $\rightarrow$ More Pizza, less Cola!

- Pizza cost \$4
- Cola cost \$1

- Also adding one more unit:
  - of Pizza increases utility with 1
  - of Cola increases utility with 1
- You can change your consumption:
  - Exchanging Cola for Pizza: so more Pizza, less Cola
  - Exchanging Pizza for Cola: so more Cola, less Pizza
- What do you choose?
  - Exchanging Pizza for Cola: so more Cola, less Pizza!

In formulas:

$$\frac{P_P}{P_C} = \frac{4}{1} = 4 \qquad \frac{MU_p}{MU_c} = \frac{1}{1} = 1$$

$$\boxed{\frac{MU_p}{MU_c} < \frac{P_P}{P_C}}$$

Adding more pizza gives ¼ the pleasure of adding more cola and adding more pizza cost the same as adding more cola

$$\boxed{\frac{MU_p}{P_P} < \frac{MU_c}{P_C}}$$

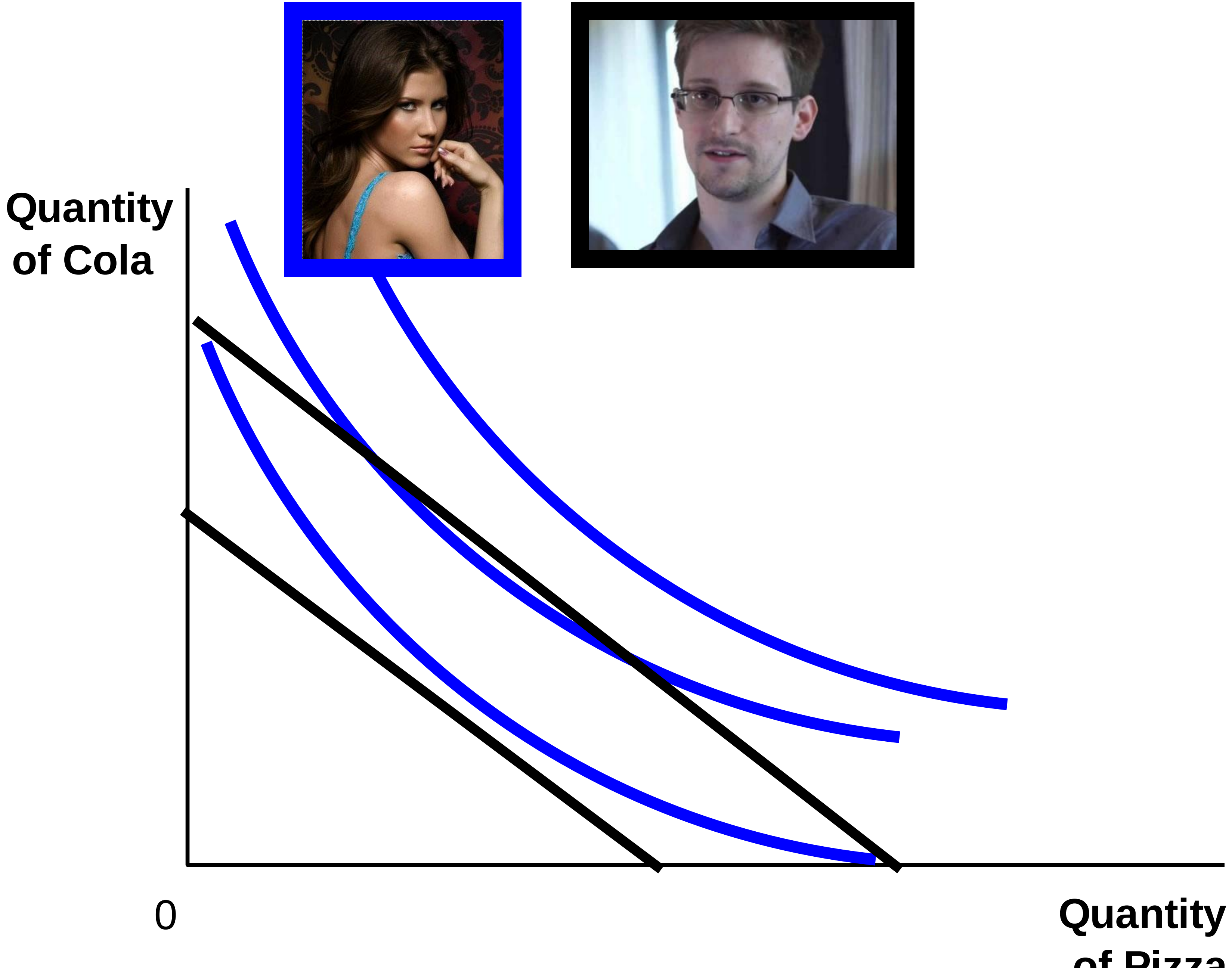
Adding more pizza gives ¼th of the pleasure per cost as adding more cola

Our diagram analysis: $\frac{MU_p}{MU_c} < \frac{P_P}{P_C}$? $\Rightarrow$ More Cola, less Pizza!

- Imagine that
  - Pizza cost $4
  - Cola cost $1
- Also:
  - adding one more unit of Pizza increases utility with 3
  - adding one more unit of Cola increases utility with 1
- You can change your consumption:
  - Exchanging Cola for Pizza: so more Pizza, less Cola
  - Exchanging Pizza for Cola: so more Cola, less Pizza
- What do you choose?
  - Exchanging Pizza for Cola: so more Cola, less Pizza!

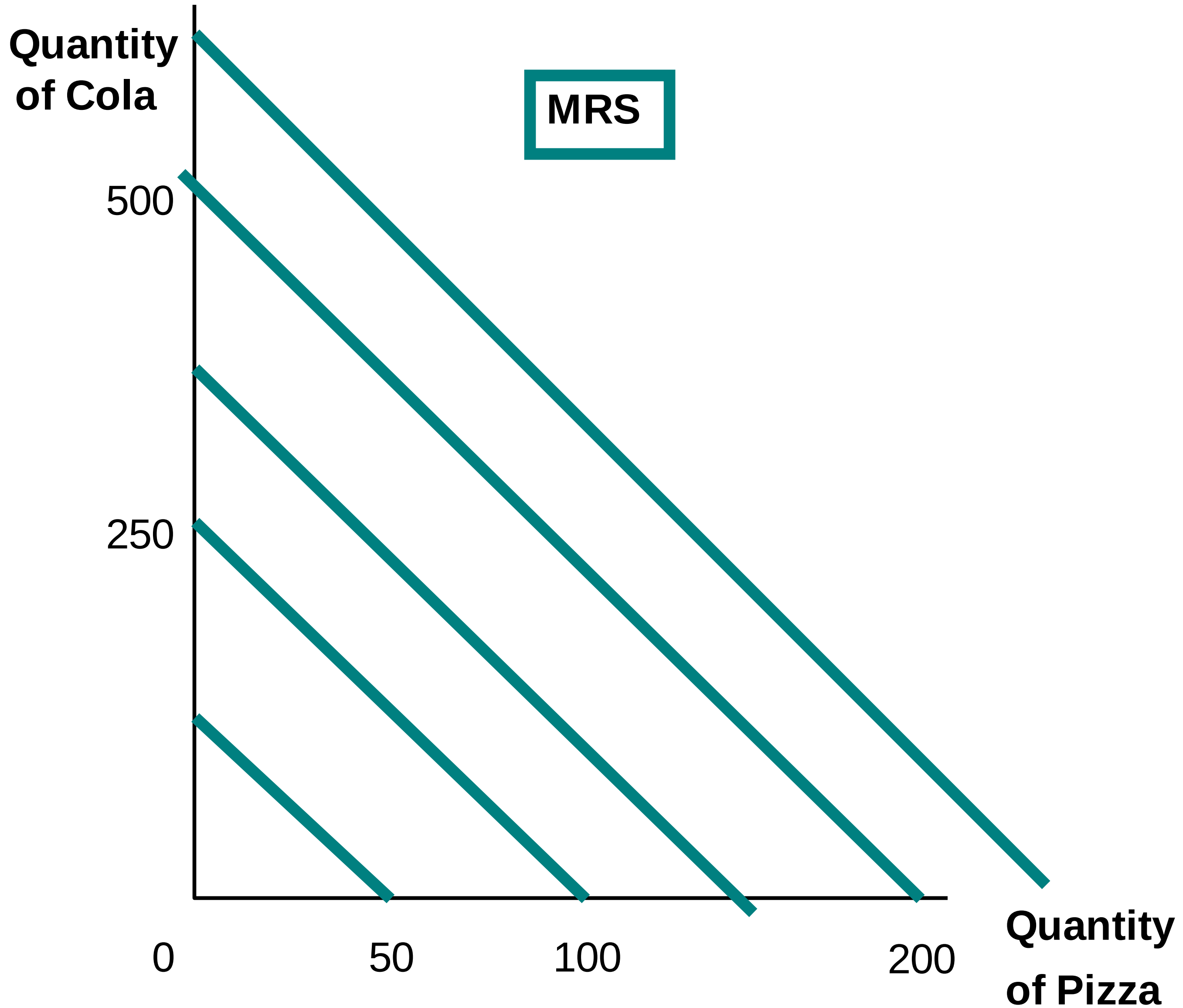
$$\frac{MU_p}{MU_c} = \frac{3}{1} = 3 \qquad \frac{P_P}{P_C} = \frac{4}{1} = 4 \qquad \frac{MU_p}{MU_c} < \frac{P_P}{P_C}$$

What if $\frac{MU_p}{MU_c} < \frac{P_P}{P_C}$ **?** $\Rightarrow$ More Cola, less Pizza!

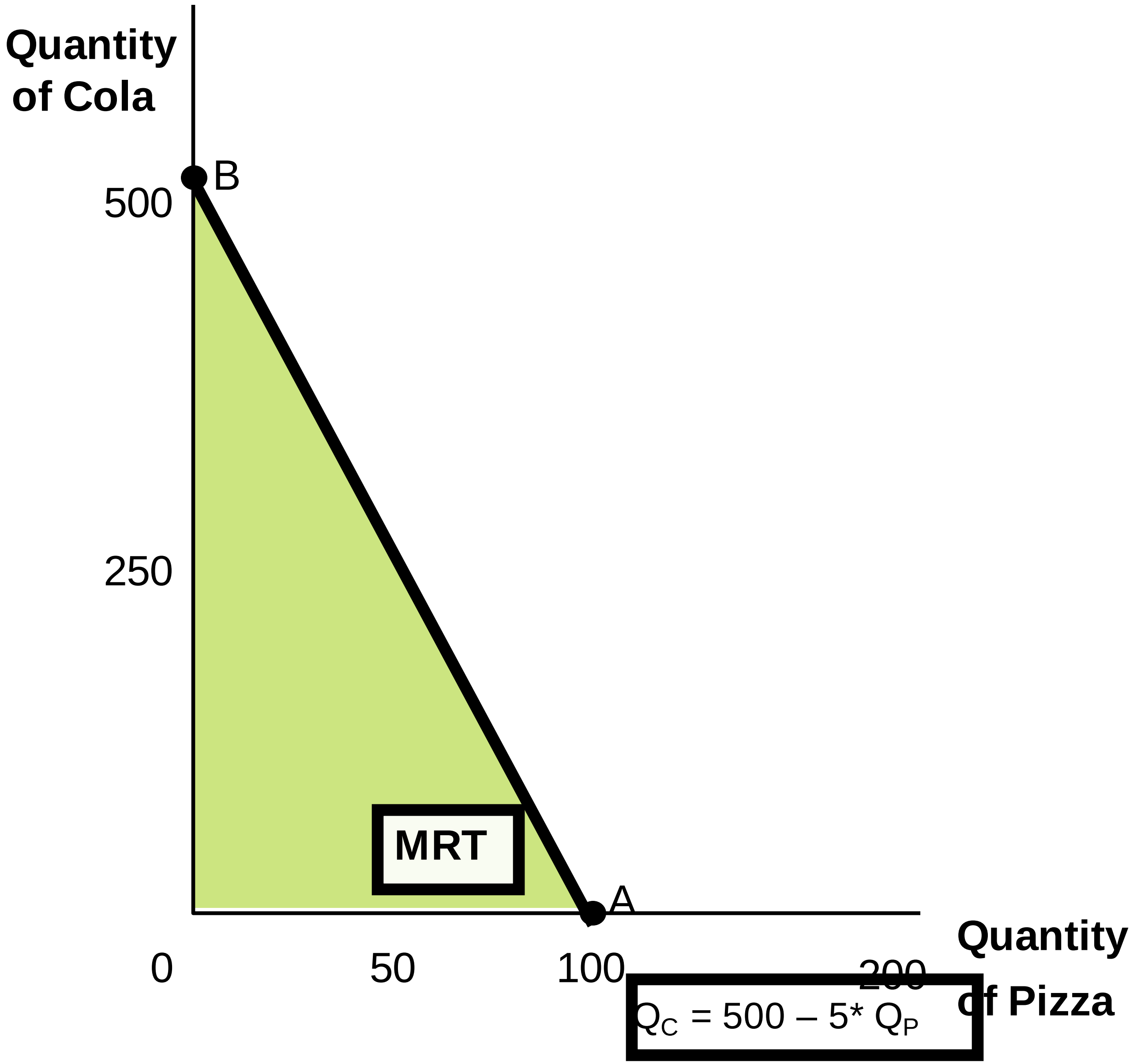

Quantity
of Cola
0
Quantity
of Pizza

- Perfect substitutes

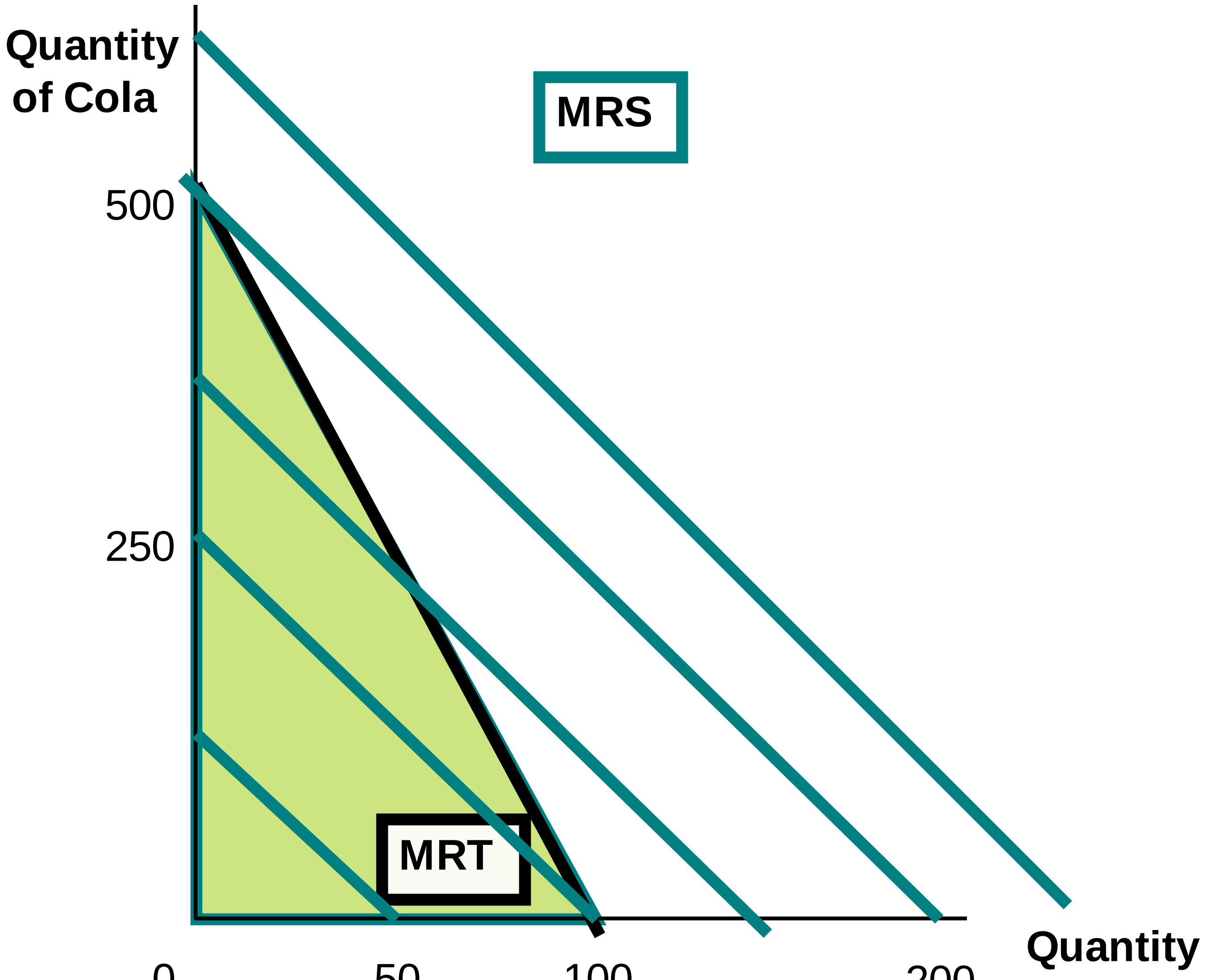
Consumer Choice



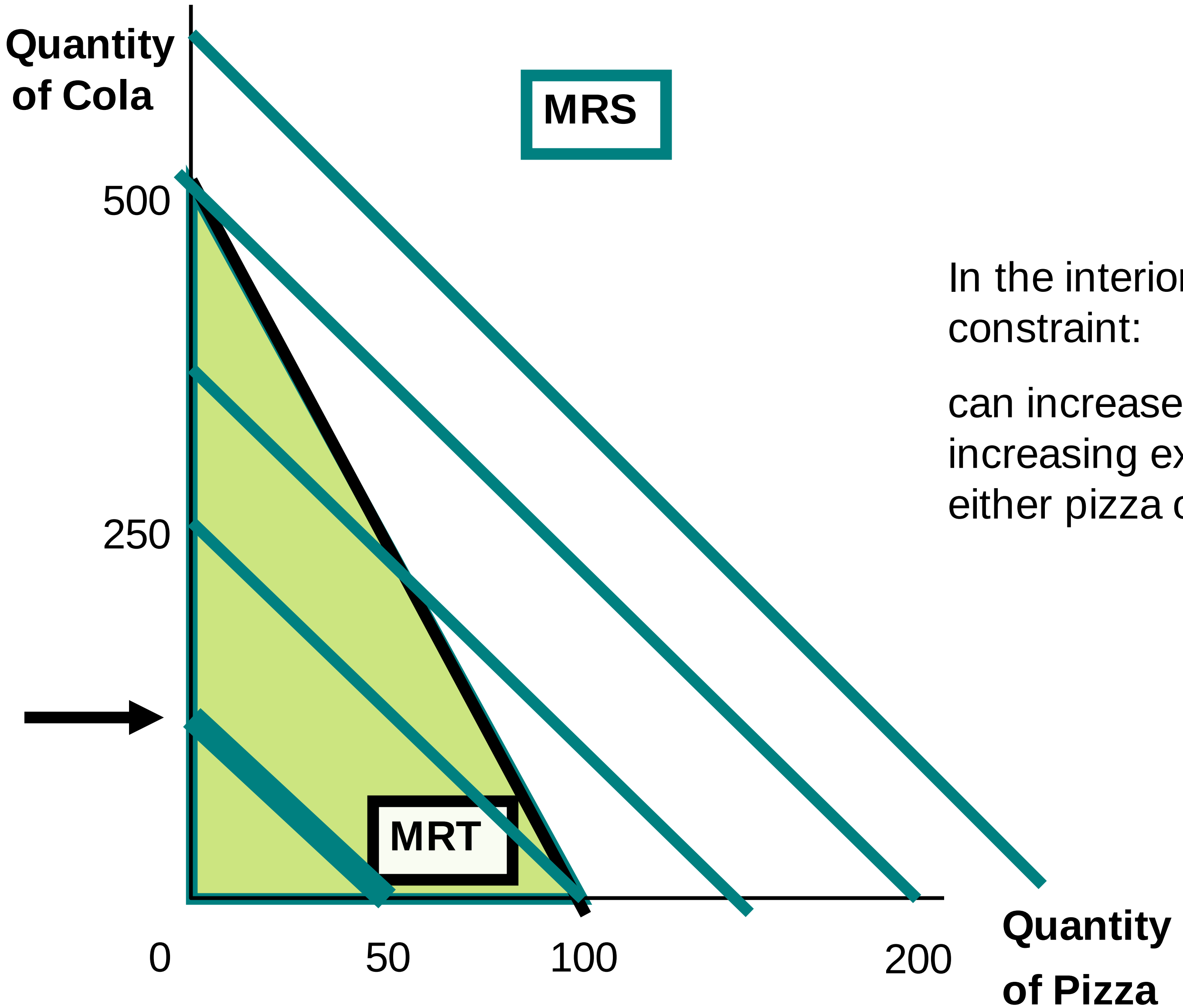
Consumer Choice



Consumer Choice



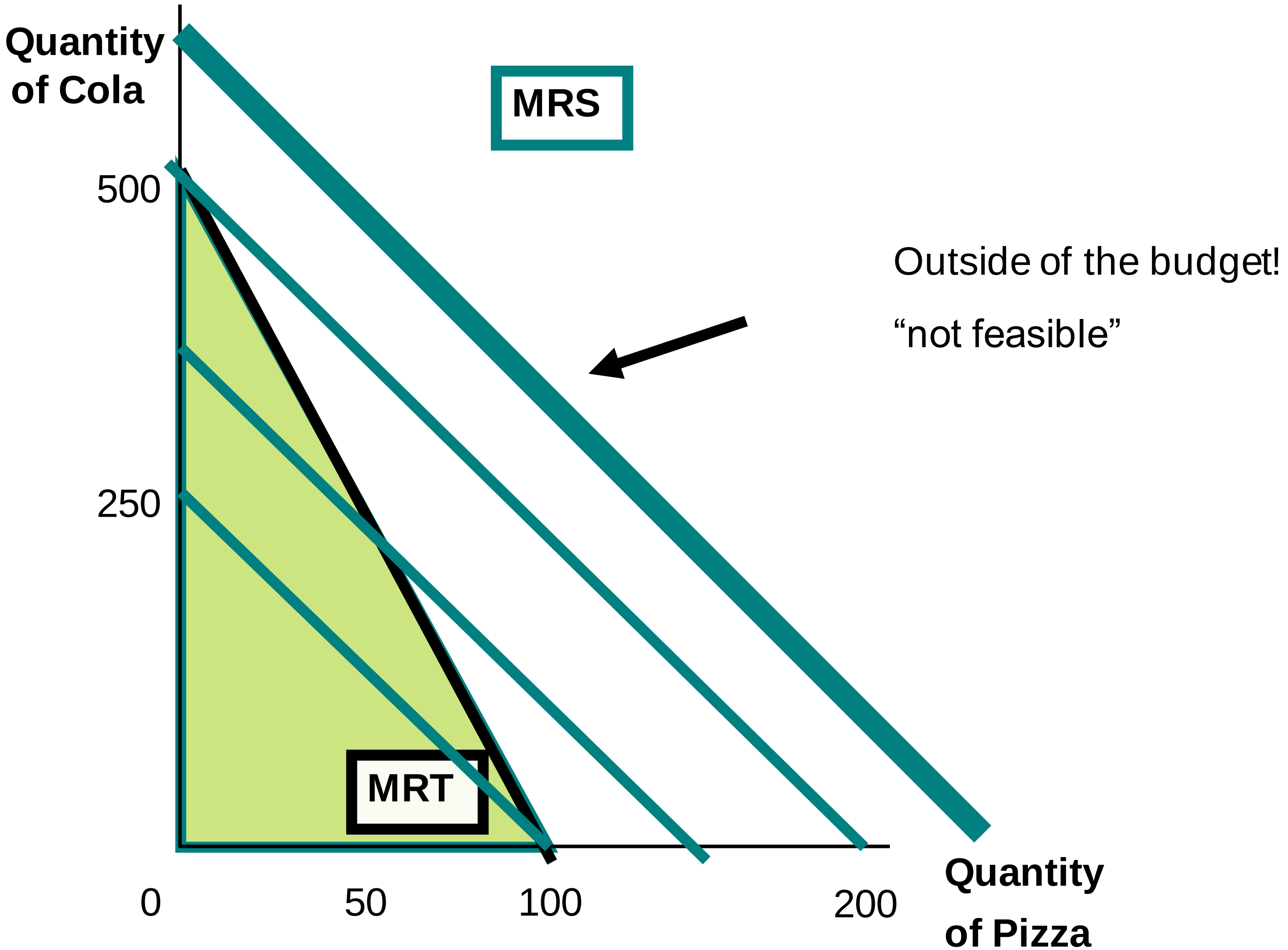
Consumer Choice



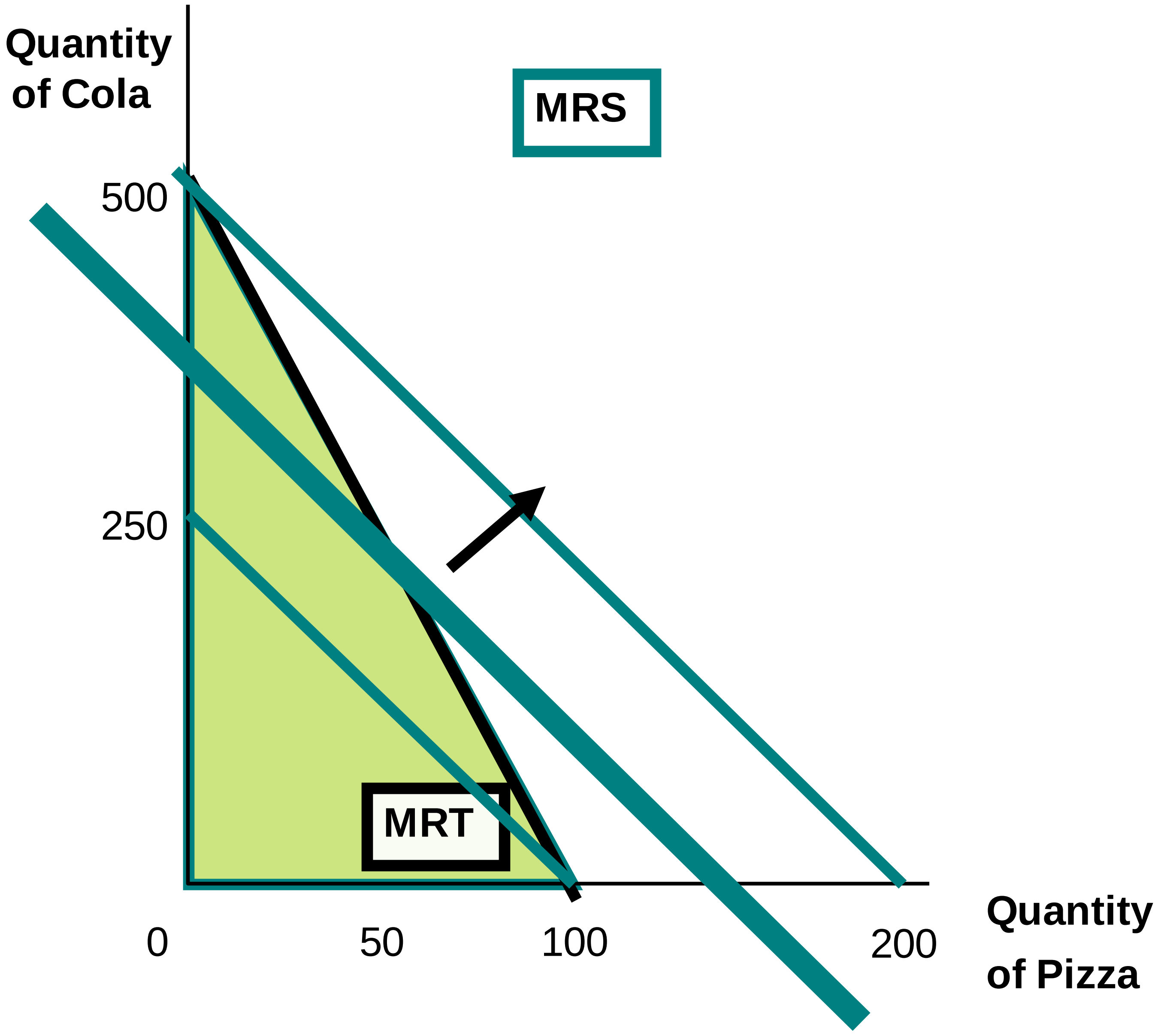
In the interior of the budget constraint:

can increase utility by increasing expenditure on either pizza or cola.

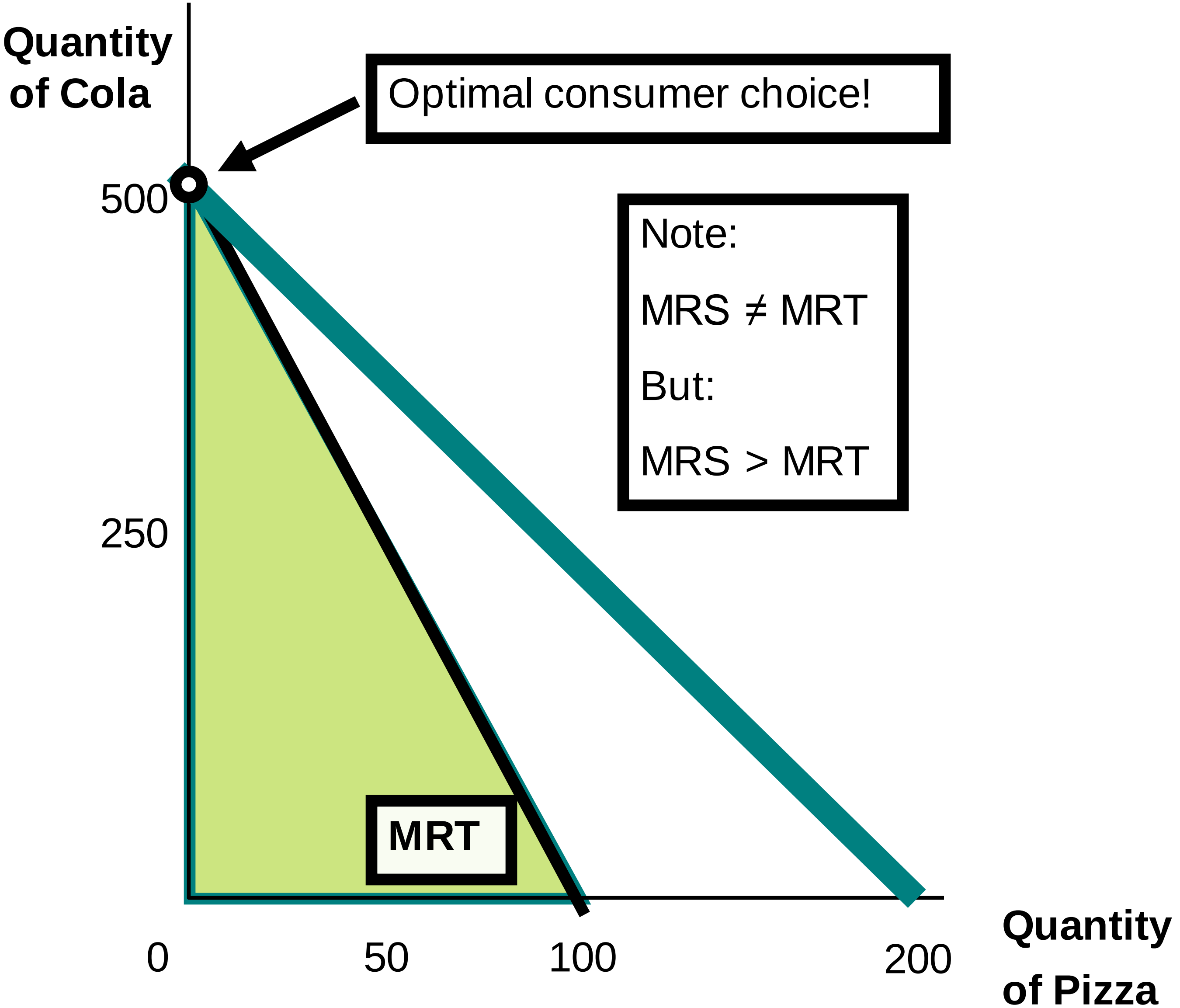
Consumer Choice

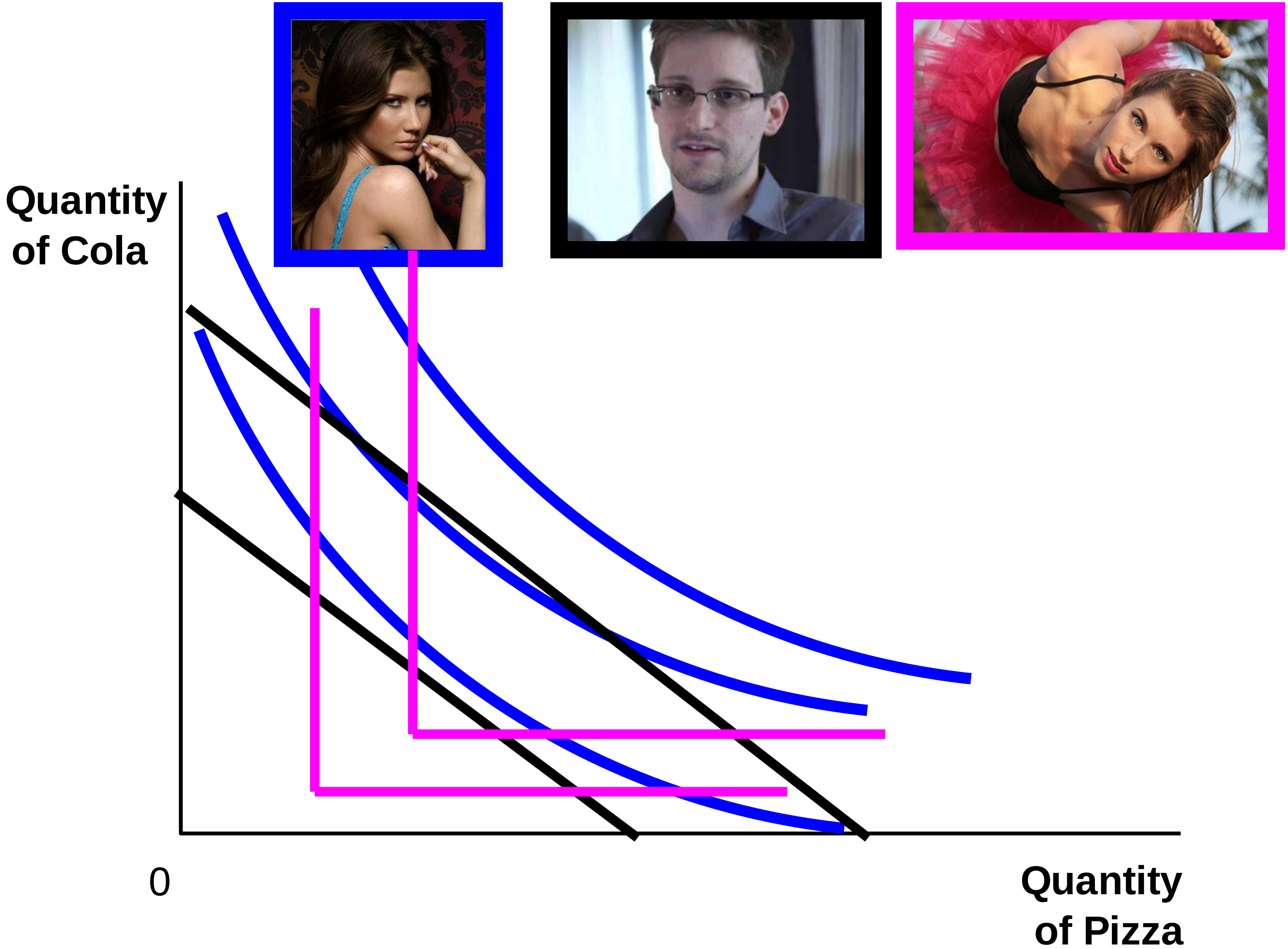


Consumer Choice



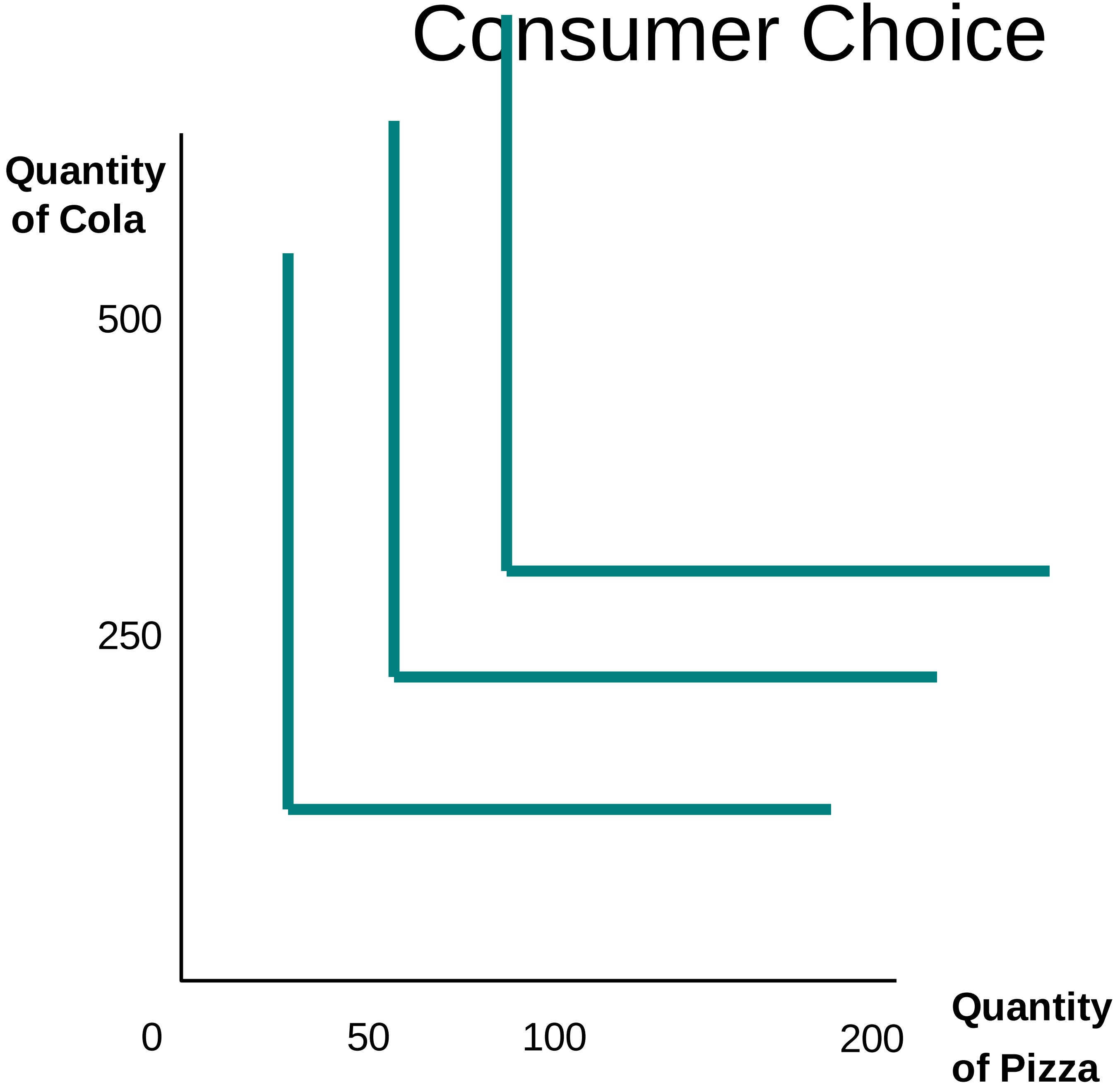
Consumer Choice



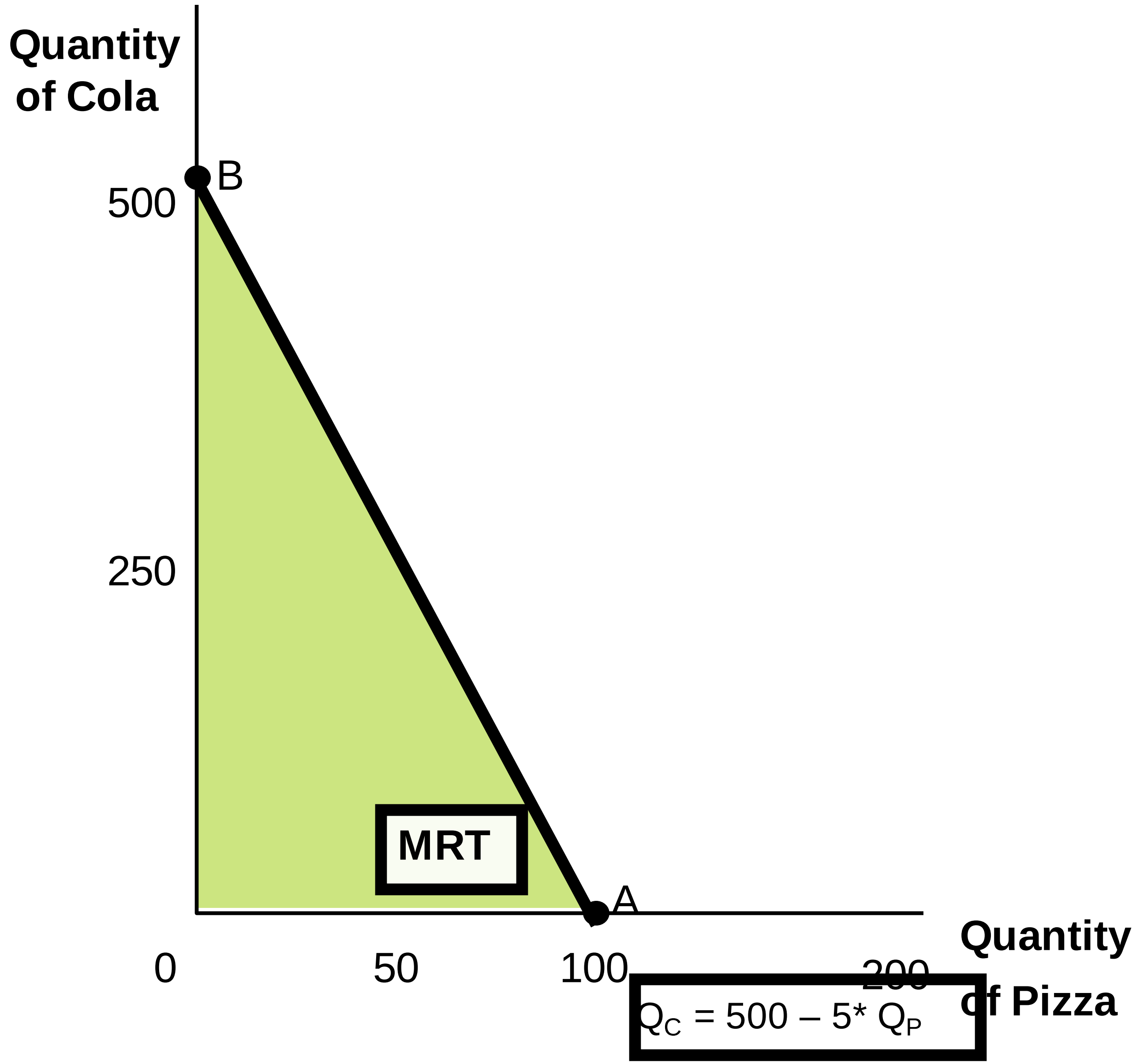

Quantity of Cola
0
Quantity of Pizza

- Perfect complements

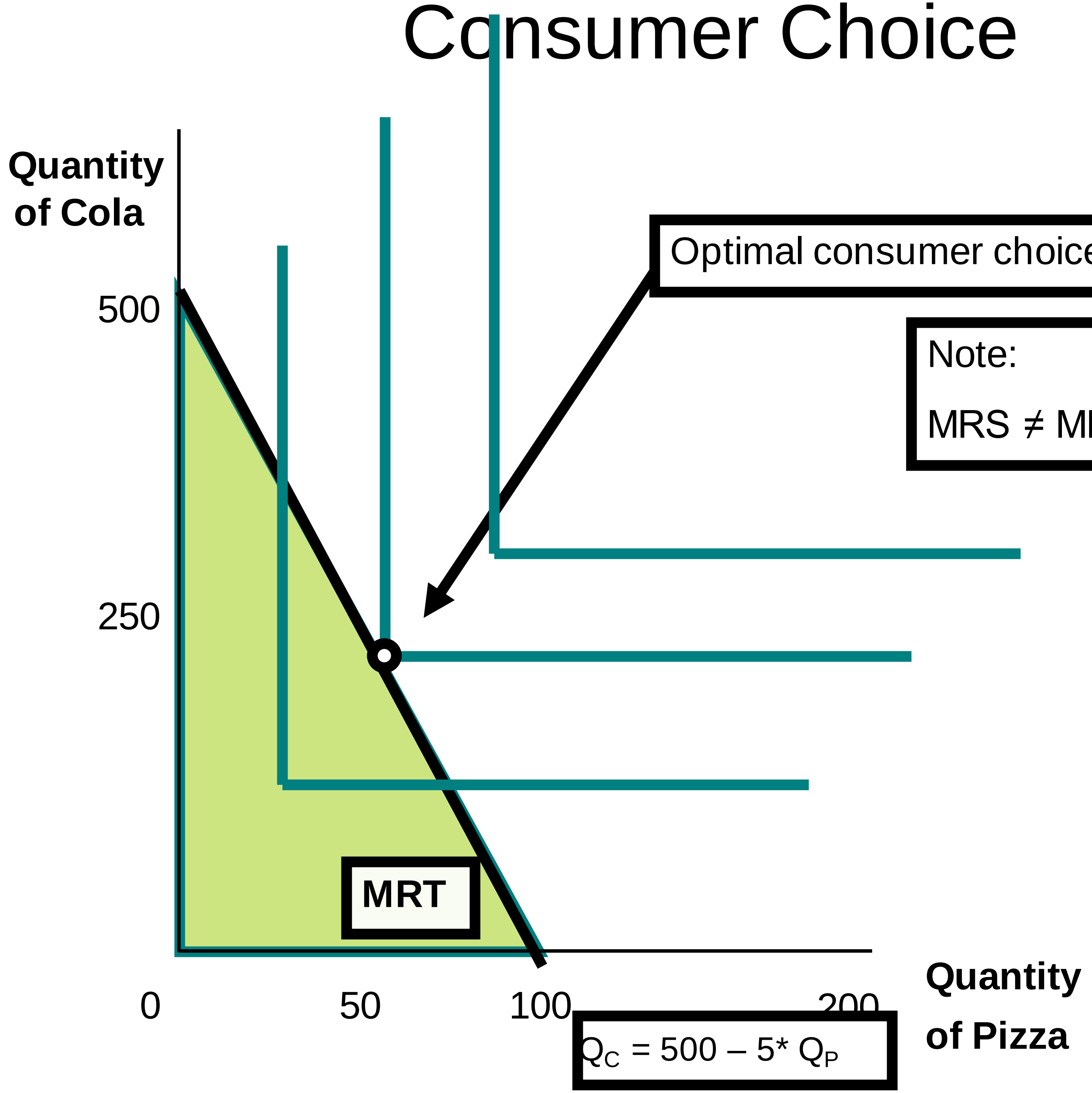
Consumer Choice



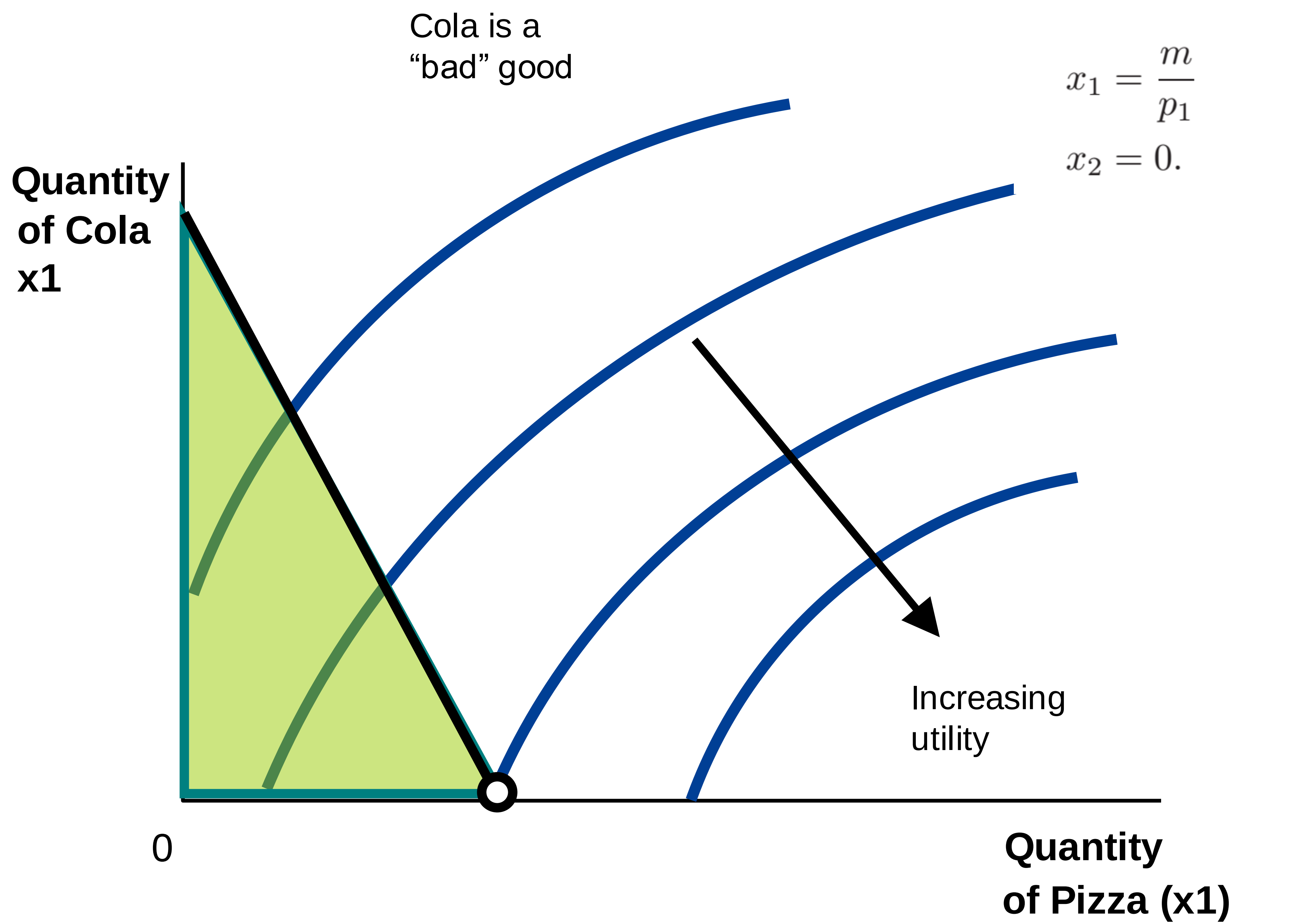
Consumer Choice

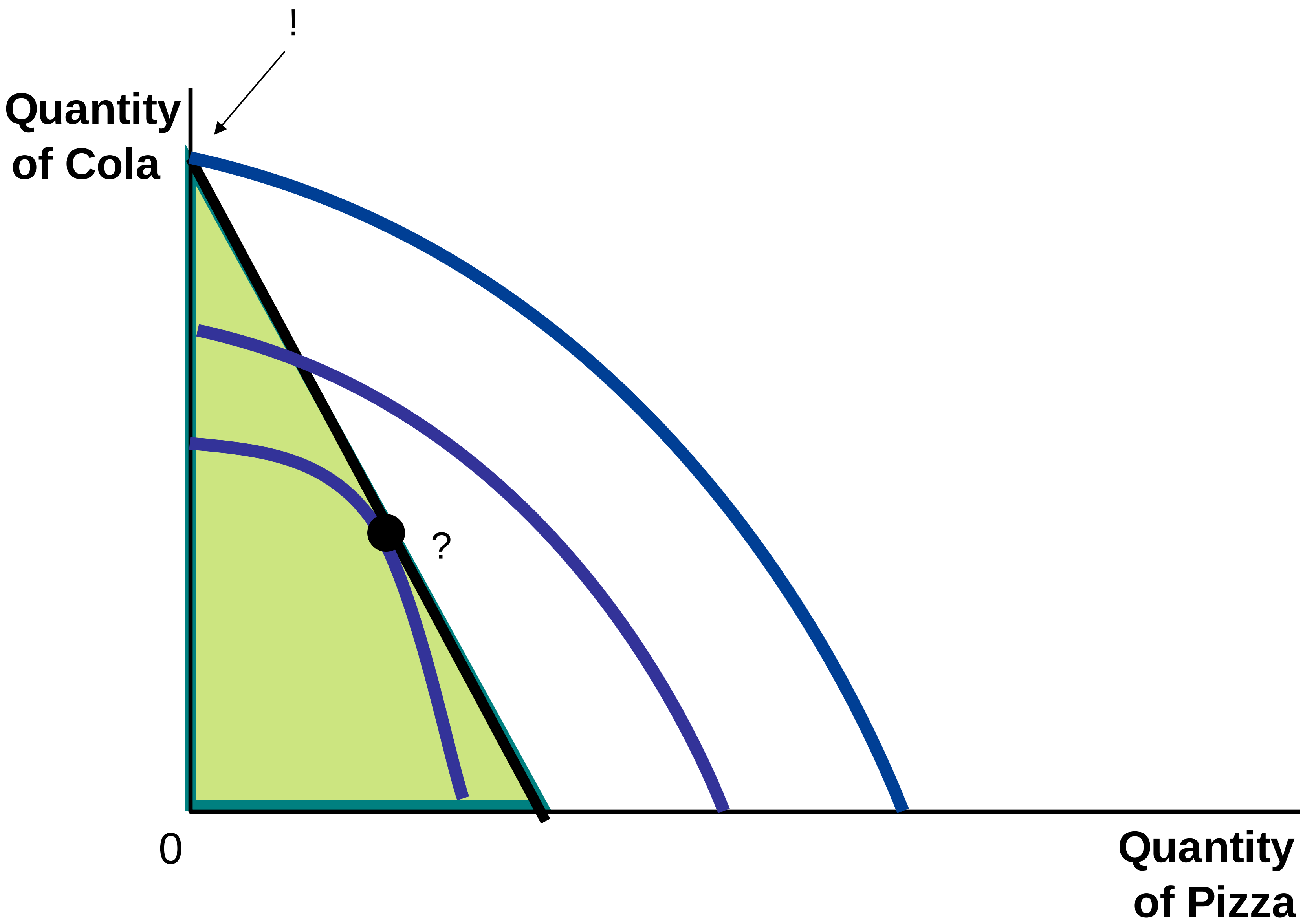


Consumer Choice



- Unusual preferences


Cola is a
“bad” good
$x_1 = \frac{m}{p_1}$
$x_2 = 0.$
Quantity
of Cola
x1
Increasing
utility
0
Quantity
of Pizza (x1)

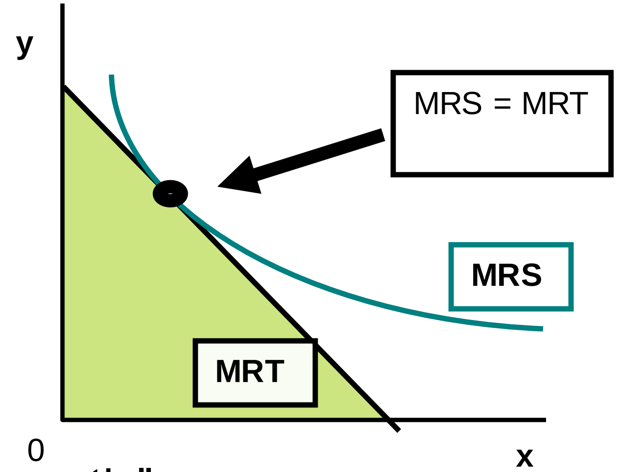

!
Quantity
of Cola
?
0
Quantity
of Pizza

Exercises

Wanda's utility function is $U(x, y) = x + 63y - 3y^2$.

- Her income is 184,
- the price of x is 1
- the price of y is 33.

How many units of good y will Wanda demand?



MRT three methods: "intuitive", "algebraic", and "mathematic"

1. **Intuitive:**
   - One more x costs \$1 (P_x)
   - By how much should Wanda **reduce** her consumption of y to save \$1?
   - 1/33 ($1/P_y$)
   - Thus MRT = -1/33 (P_x/P_y)

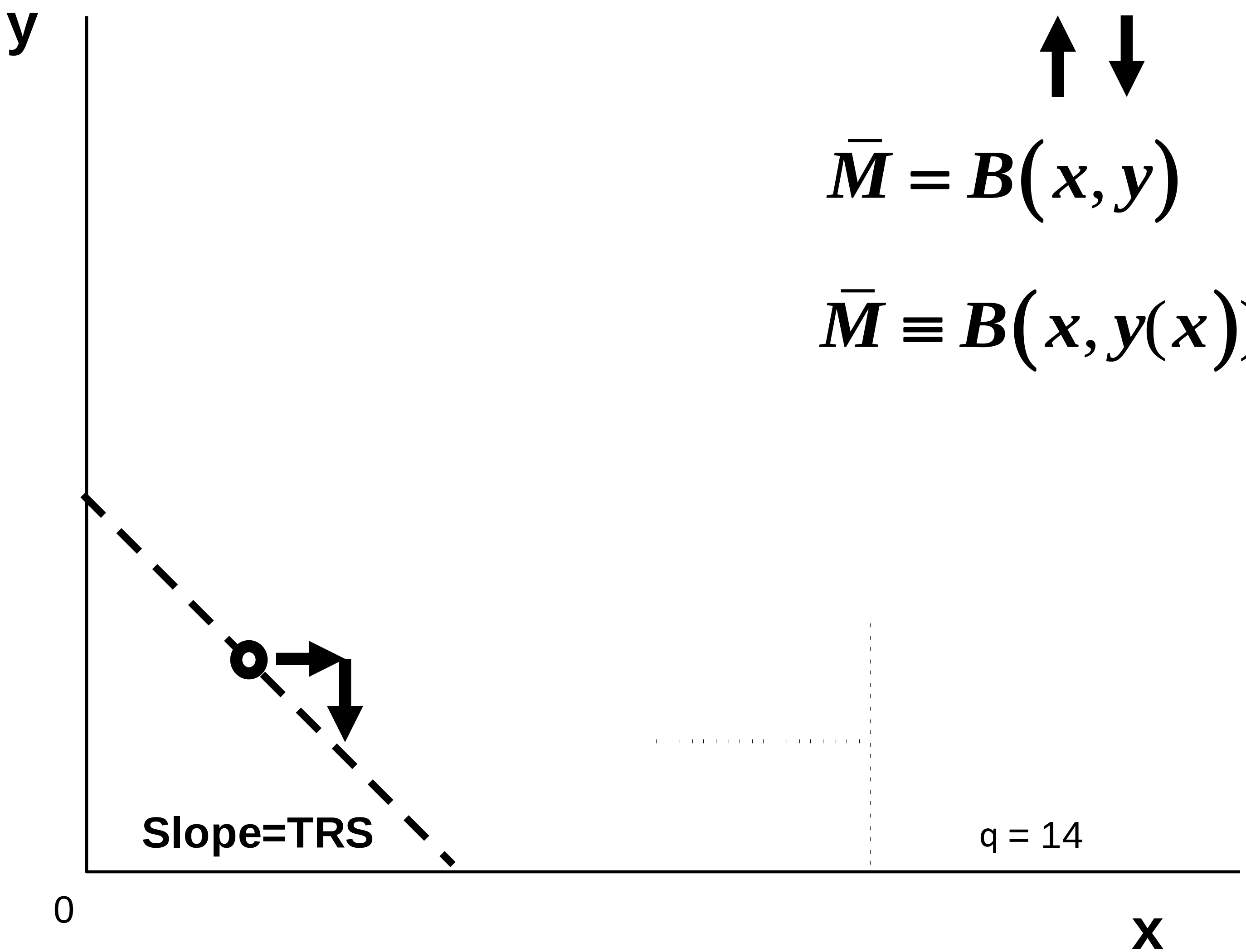
2. **Algebraic :**
   - 1* x + 33* y=184
   - 33y =184 – x
   - y = (184/33) $\boxed{– (1/33)}$ x

$$\boxed{MRT = \frac{-1}{33}}$$

3.Mathematic: $$MRT = \frac{dy}{dx}\Big|_{\bar{M}} = -\frac{\frac{\delta B}{\delta x}}{\frac{\delta B}{\delta y}} = -\frac{P_x}{P_y} = -\frac{1}{33}$$

Why is this true?

Budget restriction



$$\bar{M} = B(x, y)$$

$$\bar{M} \equiv B(x, y(x))$$

Derivation method 1: implicit function derivation

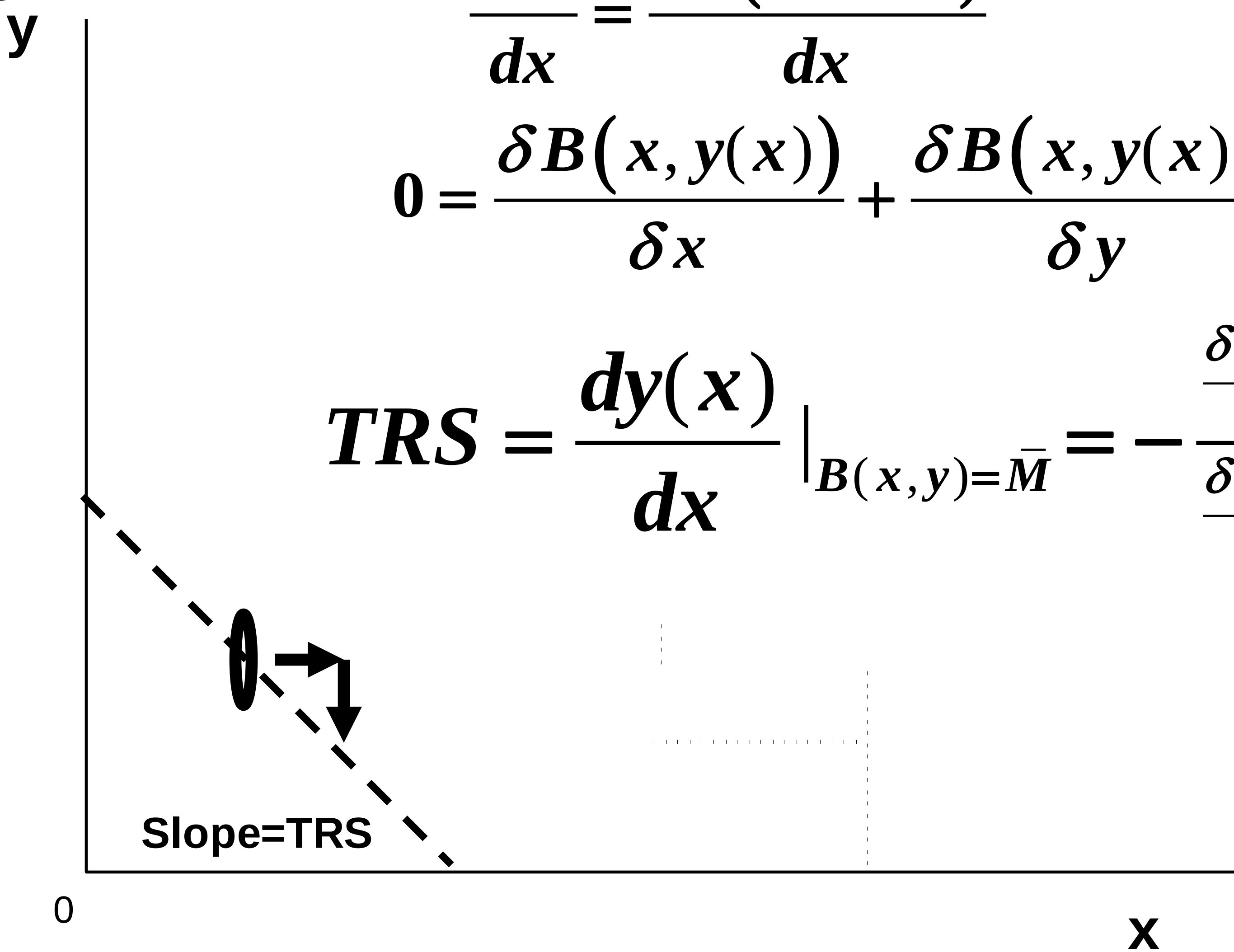
$$\bar{M} \equiv B\left(x, y(x)\right)$$

$$\frac{d\bar{M}}{dx} = \frac{dB\left(x, y(x)\right)}{dx}$$

$$0 = \frac{\delta B\left(x, y(x)\right)}{\delta x} + \frac{\delta B\left(x, y(x)\right)}{\delta y}\frac{dy}{dx}$$

$$TRS = \frac{dy(x)}{dx}\bigg|_{B(x,y)=\bar{M}} = -\frac{\frac{\delta M(x,y)}{\delta x}}{\frac{\delta M(x,y)}{\delta y}}$$

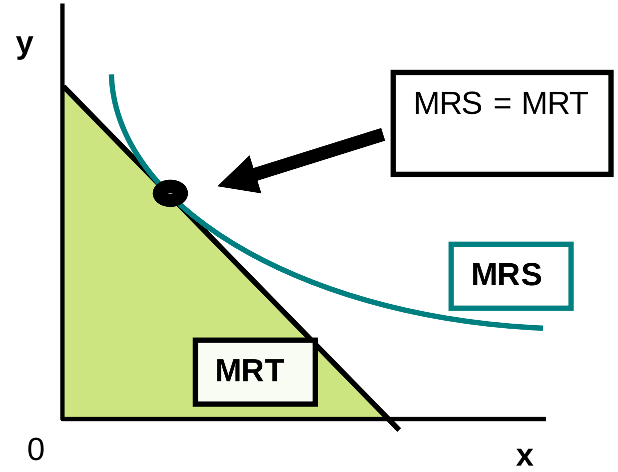
Budget restriction



Wanda's utility function is U(x, y) = x + 63y — 3y^2.

- Her income is 184,
- the price of x is 1
- the price of y is 33.

How many units of good y will Wanda demand?



$$MRT = \frac{-1}{33}$$

$$MRS = -\frac{MUx}{MUy}$$

$$MRS = -\frac{1}{63-6y}$$

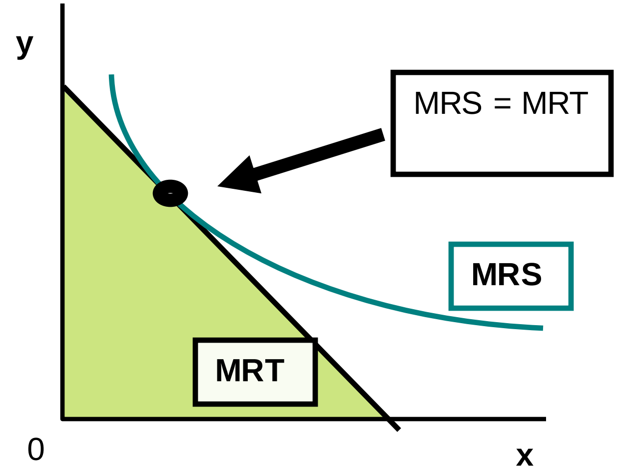
$$MUx = \frac{\Delta U}{\Delta x} = 1$$

$$MUy = \frac{\Delta U}{\Delta y} = 63-6y$$

Wanda's utility function is $U(x, y) = x + 63y - 3y^2$.

- Her income is 184,
- the price of x is 1
- the price of y is 33.

How many units of good y will Wanda demand?



$$\underbrace{MRT = \frac{-1}{33} \qquad MRS = \frac{-1}{63-6y}}$$

$$MRT = MRS$$

$$\frac{-1}{33} = \frac{-1}{63-6y}$$

$$33 = 63 - 6y$$

$$6y = 30$$

$$y = 5$$

And how many units of good x?

Budget restriction:

$$x + 33y = 184$$

$$x = 184 - 33y$$

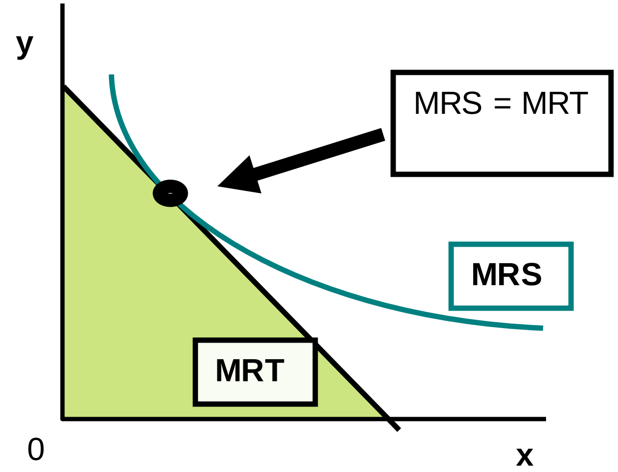
$$x = 184 - 33 \cdot 5 = 184 - 165 = 19$$

- Be careful with considering quasi-linear utility functions…

Wanda's utility function is U(x, y) = x + 63y — 3y².

- Her income is **<u>100</u>**,
- the price of x is 1
- the price of y is 33.

How many units of good y will Wanda demand?



$$\underbrace{MRT = \frac{-1}{33} \qquad MRS = \frac{-1}{63-6y}}$$

$$MRT = MRS$$

$$\frac{-1}{33} = \frac{-1}{63-6y}$$

$$33 = 63 - 6y$$

$$6y = 30$$

$$y = 5$$

And how many units of good x?

Budget restriction:

$$x + 33y = 100$$

$$x = 100 - 33y$$

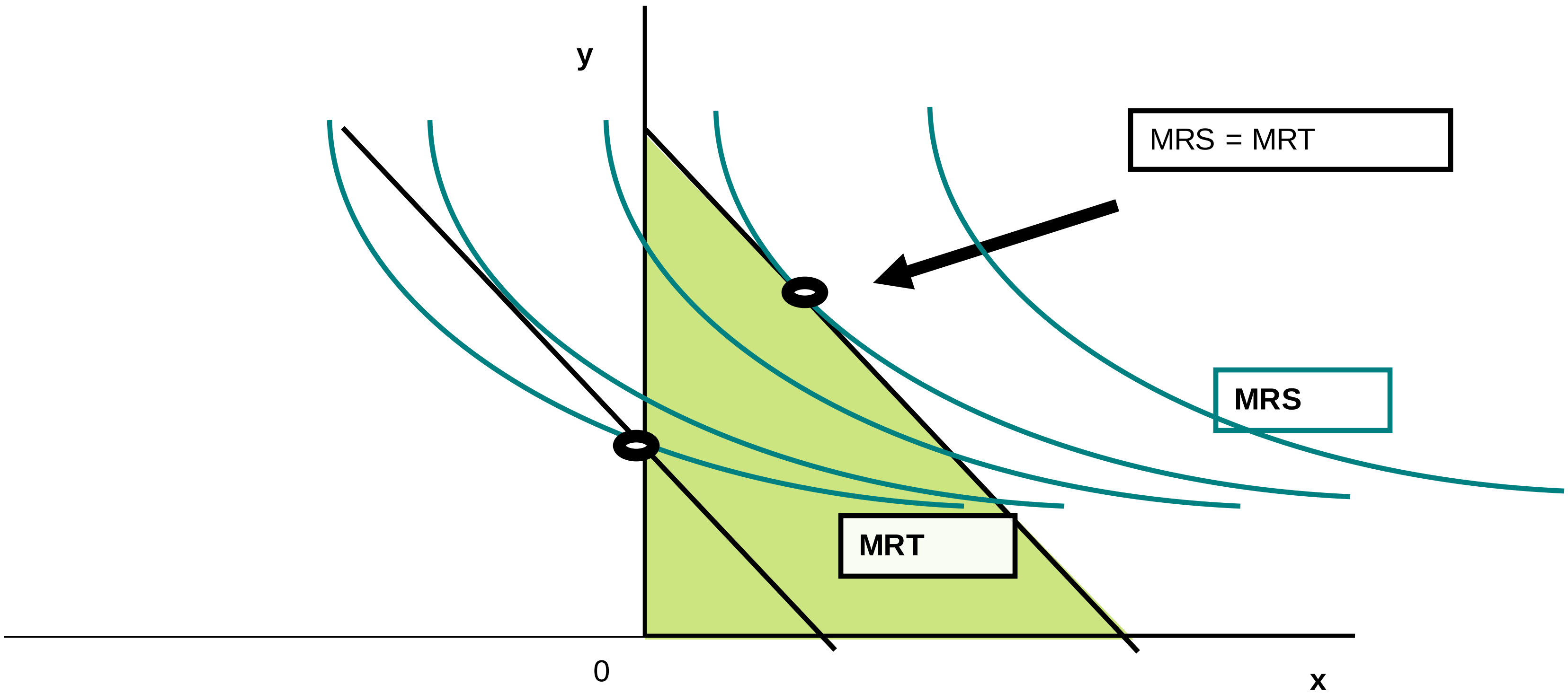
?!

$$x = 100 - 33 \cdot 5 \;= 100 - 165 = -65$$

Wanda's utility function is $U(x, y) = x + 63y - 3y^2$.

- Her income is **100**,
- the price of x is 1
- the price of y is 33.

How many units of good y will Wanda demand?

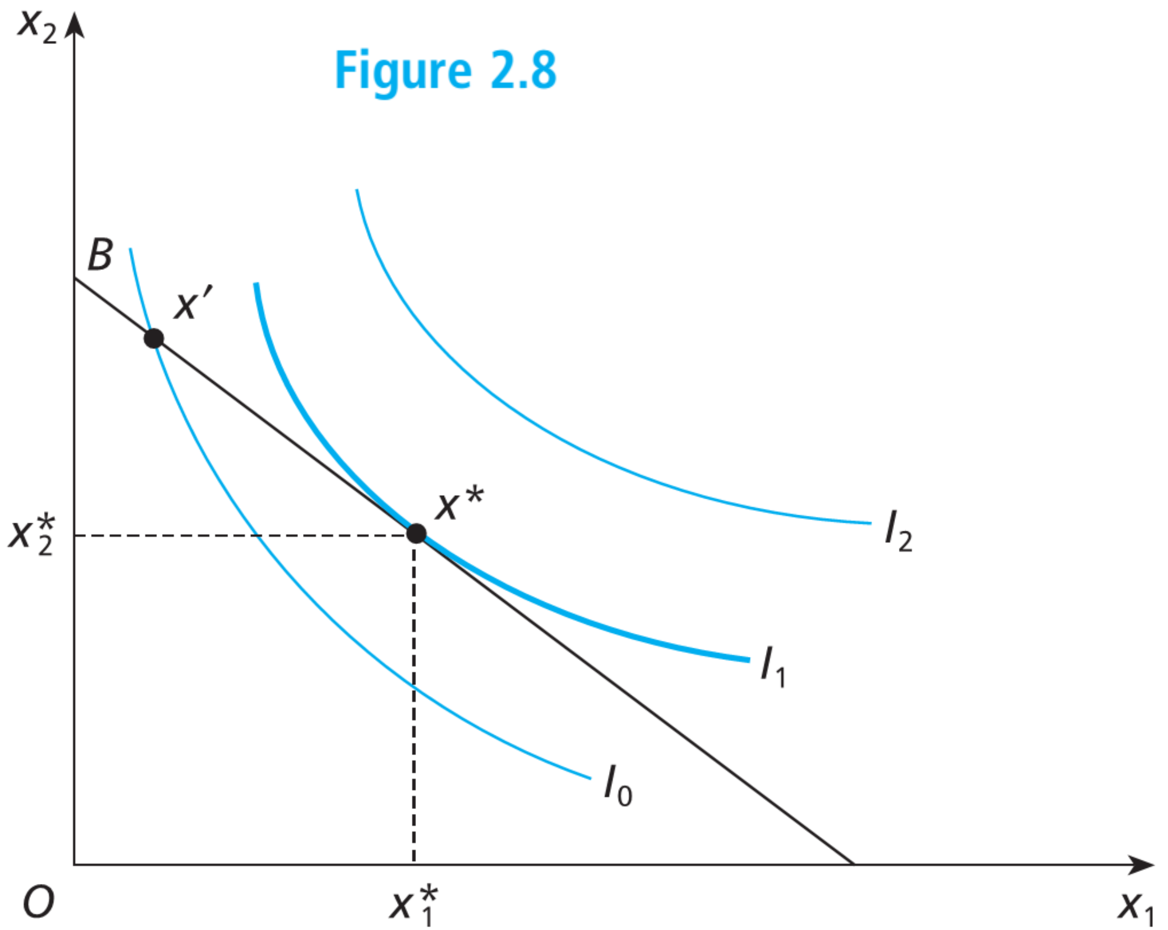


Only y is consumed $$y = \frac{M}{p_y} = \frac{100}{33} = 3\tfrac{1}{3}$$

- Gravelle & Rees, Chap 2.B

$$\max_{x_1,\ldots,x_n} u(x_1, x_2, \ldots, x_n) \quad \text{s.t.} \sum_i p_i x_i \leqslant M, \qquad x_i \geqslant 0, \qquad i = 1, \ldots, n$$

Figure 2.8



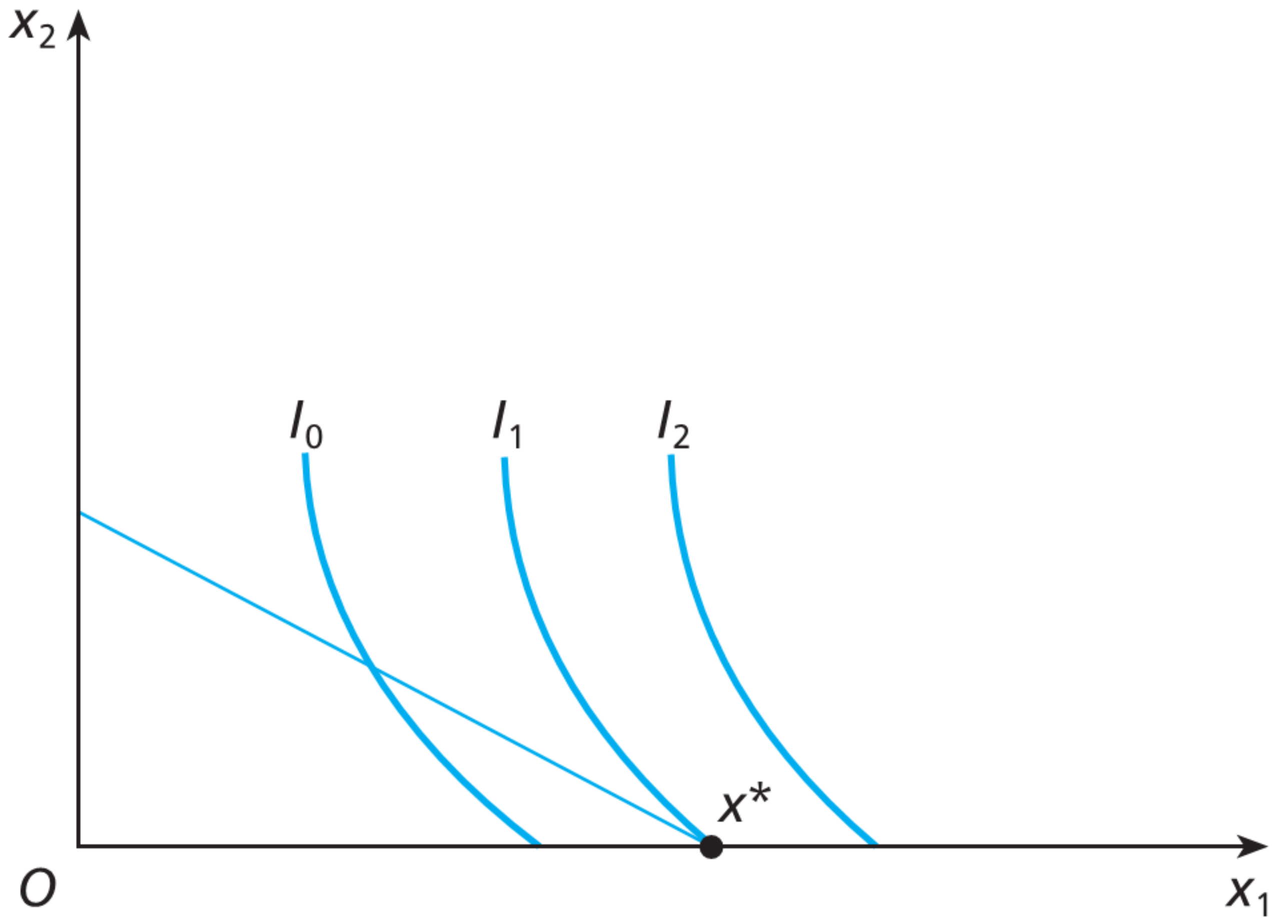
In Fig. 2.8 there is a tangency solution

$$\left.\frac{dx_2}{dx_1}\right|_{u\text{ constant}} = \left.\frac{dx_2}{dx_1}\right|_{M\text{ constant}}$$

$$MRS_{21} = \frac{u_1}{u_2} = \frac{p_1}{p_2}$$

$$\frac{u_1}{p_1} = \frac{u_2}{p_2}$$

$$\frac{u_1}{p_1} = \frac{u_2}{p_2} = u_M$$


x_2
I_0
I_1
I_2
x^*
O
x_1

$$\left.\frac{dx_2}{dx_1}\right|_{u\ \text{constant}} < \left.\frac{dx_2}{dx_1}\right|_{M\ \text{constant}}$$

$$\left.\frac{-dx_2}{dx_1}\right|_{u\ \text{constant}} = MRS_{12} = \frac{u_1}{u_2} > \frac{p_1}{p_2} = \left.\frac{-dx_2}{dx_1}\right|_{M\ \text{constant}}$$

$$u_M = \frac{u_1}{p_1} > \frac{u_2}{p_2}$$

$$L = u(x_1, \ldots, x_n) + \lambda[M - \sum p_i x_i]$$

$$\frac{\partial L}{\partial x_i} = u_i - \lambda p_i = 0 \qquad i = 1, \ldots, n$$

$$\frac{\partial L}{\partial \lambda} = M - \sum p_i x_i^* = 0$$

$$\frac{u_i}{u_j} = \frac{p_i}{p_j}$$

$$\frac{u_1}{p_1} = \frac{u_2}{p_2} = \ldots = \frac{u_n}{p_n} = \lambda$$

$$\lambda = \frac{du^*}{dM} = u_M$$

λ is the utility of money!

Corner solutions

$$x_i^* > 0$$

$$\frac{\partial L}{\partial x_i} = u_i - \lambda^* p_i \leqslant 0, \qquad x_i^* \geqslant 0, \qquad x_i^*(u_i - \lambda^* p_i) = 0$$

$$i = 1, 2, \ldots, n$$

$$\lambda^* \geqslant \frac{u_i}{p_i}, \qquad x_i^* \geqslant 0, \qquad x_i^*\left(\frac{u_i}{p_i} - \lambda^*\right) = 0$$

if the marginal utility of expenditure on good i, (ui/pi), is less than the marginal utility of money at the optimal point, λ*, then good I will not be bought since the consumer will get greater utility by expenditure on other goods.

- What to do?
- Use KKT!

$$U[x,y] = x + 63y - 3y^2 \qquad s.t. \quad p_x x + p_y y = M$$

$$\boxed{\begin{aligned} p_x &= 1 \\ p_y &= 33 \end{aligned}}$$

$$L[x,y,\lambda] = x + 63y - 3y^2 + \lambda\left(M - p_x x - p_y y\right)$$

KKT-FOCs:

$$0 = L_1 = x \cdot \left(1 - \lambda p_x\right) \,\&\, \left(1 - \lambda p_x\right) \le 0$$

$$0 = L_2 = y \cdot \left(63 - 6y - \lambda p_y\right) \,\&\, \left(63 - 6y - \lambda p_y\right) \le 0$$

$$0 = L_3 = \lambda \cdot \left(M - p_x x - p_y y\right)$$

Do I have to use KKT for the budget restriction here?

Case 1: $x > 0, \; y > 0$

$$\begin{cases} \lambda = \frac{1}{p_x} \\ y = \frac{1}{6}\left(63 - \lambda p_y\right) = \frac{1}{6}\left(63 - \dfrac{p_y}{p_x}\right) = \frac{1}{6}\left(63 - 33\right) = 5 \\ p_x x = x = M - p_y y = M - 33 \cdot 5 = M - 165 \end{cases}$$

Thus this case can only happen when $M > 165$

$$U[x,y] = x + 63y - 3y^2 \qquad s.t. \quad p_x x + p_y y = M$$

$$\boxed{\begin{aligned} p_x &= 1 \\ p_y &= 33 \end{aligned}}$$

$$L[x,y,\lambda] = x + 63y - 3y^2 + \lambda\left(M - p_x x - p_y y\right)$$

KKT-FOCs:

$$0 = L_1 = x\cdot\left(1 - \lambda p_x\right) \ \& \left(1 - \lambda p_x\right) \le 0$$

$$0 = L_2 = y\cdot\left(63 - 6y - \lambda p_y\right) \ \& \left(63 - 6y - \lambda p_y\right) \le 0$$

$$0 = L_3 = \lambda\cdot\left(M - p_x x - p_y y\right)$$

Do I have to use KKT for the budget restriction here?

OK!

Case 2: $x = 0, \ y > 0$

$$\left.\begin{cases} x = 0 \\ \lambda = \frac{1}{p_y}\left(63 - 6y\right) \\ y = \dfrac{M}{p_y} = \dfrac{M}{33} \end{cases}\right\} \lambda = \tfrac{1}{33}\left(63 - 6\cdot\frac{M}{33}\right)$$

Let's check that $\left(1 - \lambda p_x\right) \le 0$

Case 2: $x = 0,\ y > 0$

$$\left.\begin{cases} \lambda = \frac{1}{p_y}\left(63 - 6y\right) \\ y = \dfrac{M}{p_y} = \dfrac{M}{33} \end{cases}\right\} \lambda = \frac{1}{33}\left(63 - 6\cdot\frac{M}{33}\right)$$

OK!

Let's check that $\left(1 - \lambda p_x\right) \le 0$

$$\left(1 - \lambda p_x\right) < 0 \Leftrightarrow \frac{1}{33}\left(63 - 6\cdot\frac{M}{33}\right) > 1 \Leftrightarrow \left(63 - 6\cdot\frac{M}{33}\right) > 33$$

$$\Leftrightarrow -6\cdot\frac{M}{33} > -30 \Leftrightarrow \frac{M}{33} < 5 \Leftrightarrow M < 165$$

Thus:

$$x[p_x = 1, p_y = 33, M]$$

$$= 0 \qquad \text{if } M \leq 165$$

$$= M - 165 \qquad \text{if } M > 165$$

$$y[p_x = 1, p_y = 33, M]$$

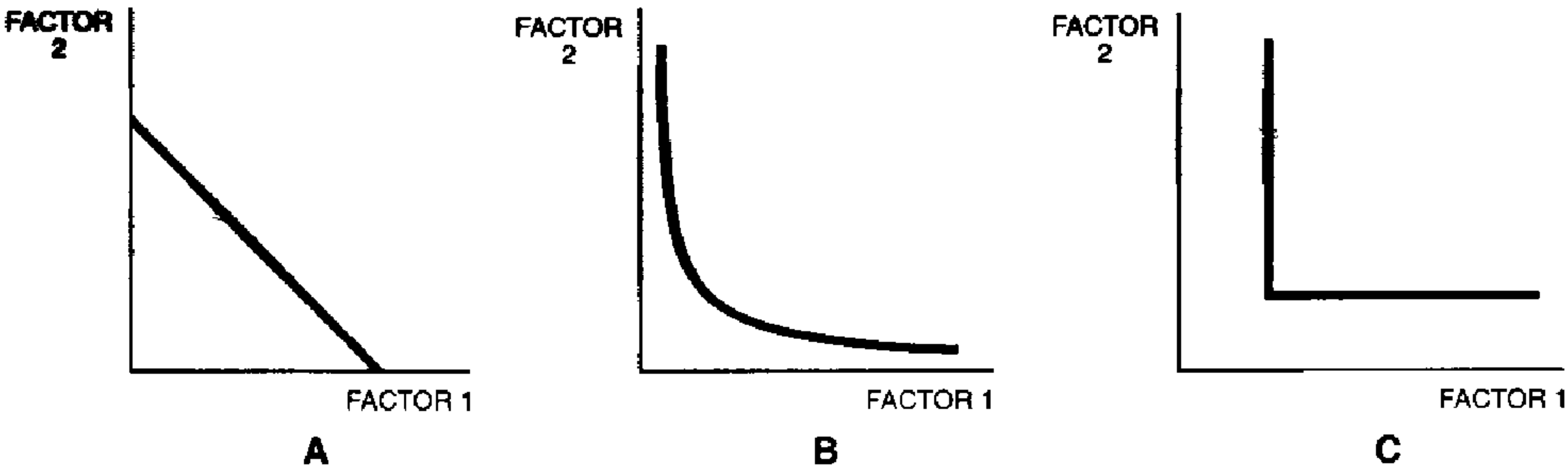
$$= \frac{M}{33} \qquad \text{if } M \leq 165$$

$$= 5 \qquad \text{if } M > 165$$

- Basic U-functions
  - Quasi-Linear
  1. Cobb-douglas
  2. Linear
  3. Leontiev
     - The 3 are special cases of CES!

- The CES production function

$$y = \left[x_1^p + x_2^p \right]^{\frac{1}{p}}$$

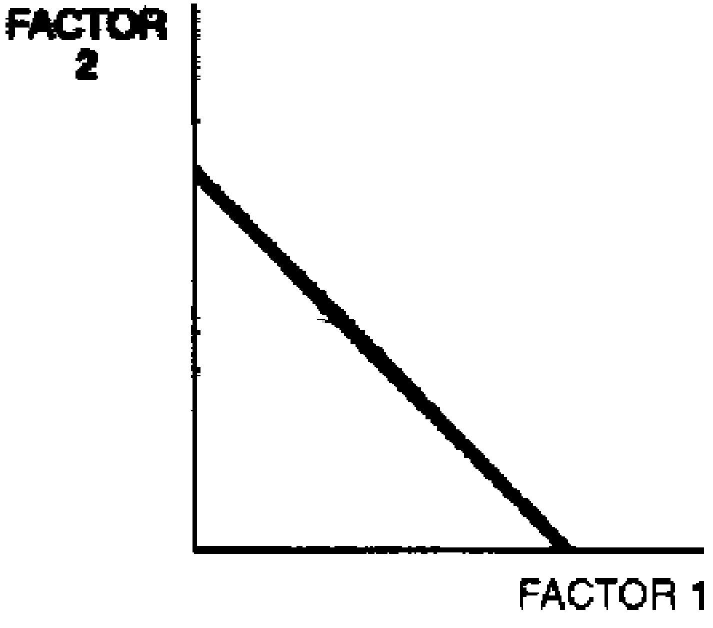


Linear pf | **Cobb-Douglas pf** | **Leontief pf**

p=1 | **p=0** | **p=-∞**

- The CES production function

$$y = \left[x_1^p + x_2^p \right]^{\frac{1}{p}}$$



A

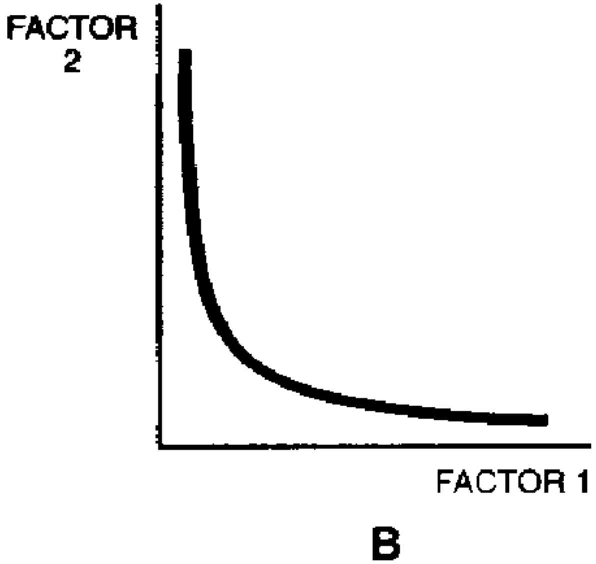
Linear pf **p=1**

$$\boxed{y = \left[x_1^1 + x_2^1 \right]^{\frac{1}{1}} = x_1 + x_2}$$

- The CES production function

$$y = \left[x_1^p + x_2^p \right]^{\frac{1}{p}}$$

Cobb-Douglas pf p=0



$$\frac{d\left[x_1^p + x_2^p \right]^{\frac{1}{p}}}{dx_1^p} = \frac{1}{p}\left[x_1^p + x_2^p \right]^{\frac{1}{p}-1} \cdot px_1^{p-1}$$

$$TRS = -\frac{\frac{1}{p}\left[x_1^p + x_2^p \right]^{\frac{1}{p}-1} \cdot px_1^{p-1}}{\frac{1}{p}\left[x_1^p + x_2^p \right]^{\frac{1}{p}-1} \cdot px_2^{p-1}} = -\frac{px_1^{p-1}}{px_2^{p-1}} = -\left(\frac{x_1}{x_2} \right)^{p-1}$$

$$\lim_{p->0} \left(\frac{x_1}{x_2} \right)^{p-1} = \left(\frac{x_1}{x_2} \right)^{-1} = \left(\frac{x_2}{x_1} \right)$$

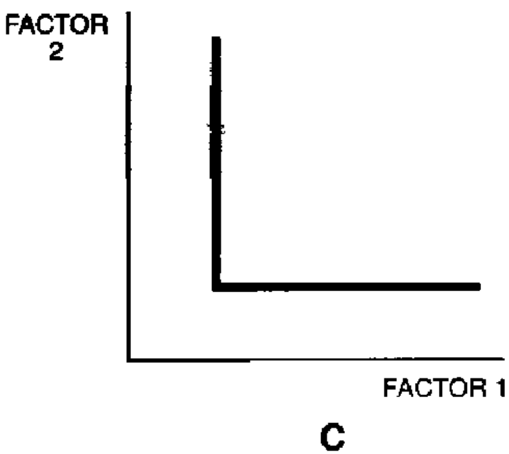
$$TRS = -\frac{x_2}{x_1}$$

Same TRS as Cobb-Douglas pf!

- The CES production function

$$y = \left[x_1^p + x_2^p \right]^{\frac{1}{p}}$$

Leontief pf **p=-∞**



$$TRS = -\left(\frac{x_1}{x_2} \right)^{p-1}$$

$$= -\left(\frac{x_1}{x_2} \right)^{-\infty} = -\left(\frac{x_2}{x_1} \right)^{\infty}$$

$x_2 > x_1 \Rightarrow TRS = -\infty$ **vertical**

$x_2 < x_1 \Rightarrow TRS = 0$ **horizontal**

Same TRS as Leontief pf!

- The elasticity of substitution of the CES pf

$$\sigma = \frac{\frac{d(x_2/x_1)}{(x_2/x_1)}}{\frac{dTRS}{TRS}} = \frac{d(x_2/x_1)}{dTRS}\frac{TRS}{(x_2/x_1)}$$

$$TRS = -\left(\frac{x_1}{x_2}\right)^{p-1}$$

$$\frac{x_2}{x_1} = |TRS|^{\frac{1}{1-\rho}}$$

$$\ln\frac{x_2}{x_1} = \frac{1}{1-\rho}\ln|TRS|$$

$$\sigma = \frac{d\ln x_2/x_1}{d\ln|TRS|} = \frac{1}{1-\rho}$$

CES utility

The three specific utility functions illustrated so far are special cases of the more general constant elasticity of substitution function (CES), which takes the form

$$\textbf{utility} = U(x, y) = \frac{x^{\delta}}{\delta} + \frac{y^{\delta}}{\delta}, \tag{3.29}$$

where $\delta \le 1, \delta \ne 0$, and

$\delta = 0.$ Cobb-Douglass $\textbf{utility} = U(x, y) = \ln x + \ln y$ **(3.30)**

$\delta = 1$ Linear preferences

$\delta = -\infty$ Leontief preferences

CES utility

The three specific utility functions illustrated so far are special cases of the more general constant elasticity of substitution function (CES), which takes the form

$$\textbf{utility} = U(x, y) = \frac{x^{\delta}}{\delta} + \frac{y^{\delta}}{\delta}, \tag{3.29}$$

where $\delta \leq 1, \delta \neq 0$, and

Case 1: $\delta = \mathbf{0.5}$. In this case, utility is

$$U(x, y) = x^{0.5} + y^{0.5}.$$

Setting up the Lagrangian expression

$$\mathscr{L} = x^{0.5} + y^{0.5} + \lambda(I - p_x x - p_y y) \tag{4.29}$$

yields the following first-order conditions for a maximum:

$$\begin{aligned} \partial\mathscr{L}/\partial x &= 0.5x^{-0.5} - \lambda p_x = 0, \\ \partial\mathscr{L}/\partial y &= 0.5y^{-0.5} - \lambda p_y = 0, \\ \partial\mathscr{L}/\partial \lambda &= I - p_x x - p_y y = 0. \end{aligned} \tag{4.30}$$

Division of the first two of these shows that

$$(y/x)^{0.5} = p_x/p_y. \tag{4.31}$$

By substituting this into the budget constraint and doing some messy algebraic manipulation, we can derive the demand functions associated with this utility function:

$$x^* = I/p_x[1 + (p_x/p_y)], \tag{4.32}$$

$$y^* = I/p_y[1 + (p_y/p_x)]. \tag{4.33}$$

Price responsiveness. In these demand functions notice that the share of income spent on, say, good x—that is, $p_x x/I = 1/[1 + (p_x/p_y)]$—is not a constant; it depends on the price ratio p_x/p_y. The higher is the relative price of x, the smaller will be the share of income spent on that good. In other words, the demand for x is so responsive to its own price that a rise in the

CES utility

The three specific utility functions illustrated so far are special cases of the more general constant elasticity of substitution function (CES), which takes the form

$$\text{utility} = U(x, y) = \frac{x^{\delta}}{\delta} + \frac{y^{\delta}}{\delta}, \tag{3.29}$$

where $\delta \leq 1, \delta \neq 0$, and

Case 2: $\delta = -1$.

$$U(x, y) = -x^{-1} - y^{-1}, \tag{4.34}$$

and it is easy to show that the first-order conditions for a maximum require

$$y/x = (p_x/p_y)^{0.5}. \tag{4.35}$$

Again, substitution of this condition into the budget constraint, together with some messy algebra, yields the demand functions

$$\begin{aligned} x^* &= I/p_x[1 + (p_y/p_x)^{0.5}], \\ y^* &= I/p_y[1 + (p_x/p_y)^{0.5}]. \end{aligned} \tag{4.36}$$

That these demand functions are less price responsive can be seen in two ways. First, now the share of income spent on good x—that is, $p_x x/I = 1/[1 + (p_y/p_x)^{0.5}]$—responds positively to increases in p_x. As the price of x rises, this individual cuts back only modestly on good x, so total spending on that good rises. That the demand functions in Equations 4.36 are less price responsive than the Cobb-Douglas is also illustrated by the relatively small exponents of each good's own price (-0.5).

CES utility

The three specific utility functions illustrated so far are special cases of the more general constant elasticity of substitution function (CES), which takes the form

$$\textbf{utility} = U(x, y) = \frac{x^{\delta}}{\delta} + \frac{y^{\delta}}{\delta}, \tag{3.29}$$

where $\delta \leq 1, \delta \neq 0$, and

Case 3: $\delta = -\infty$.

(3) The Leontief production function ($\text{p} = -\infty$). We have just seen that the TRS of the CES production function is given by equation (1.1). As ρ approaches $-\infty$, this expression approaches

$$TRS = -\left(\frac{x_1}{x_2}\right)^{-\infty} = -\left(\frac{x_2}{x_1}\right)^{\infty}.$$

If $x_2 > x_1$ the TRS is (negative) infinity; if $x_2 < x_1$ the TRS is zero. This means that as ρ approaches $-\infty$, a CES isoquant looks like an isoquant associated with the Leontief technology. ∎

5.6 Choosing Taxes

Better to levy tax on a good (x1) or on income (M)?

On a good:

$$p_1 x_1 + p_2 x_2 = m.$$

$$(p_1 + t)x_1 + p_2 x_2 = m.$$

$$(p_1 + t)x_1^* + p_2 x_2^* = m.$$

$$R^* = t x_1^*$$

5.6 Choosing Taxes

Better to levy tax on a good (x1) or on income (M)?

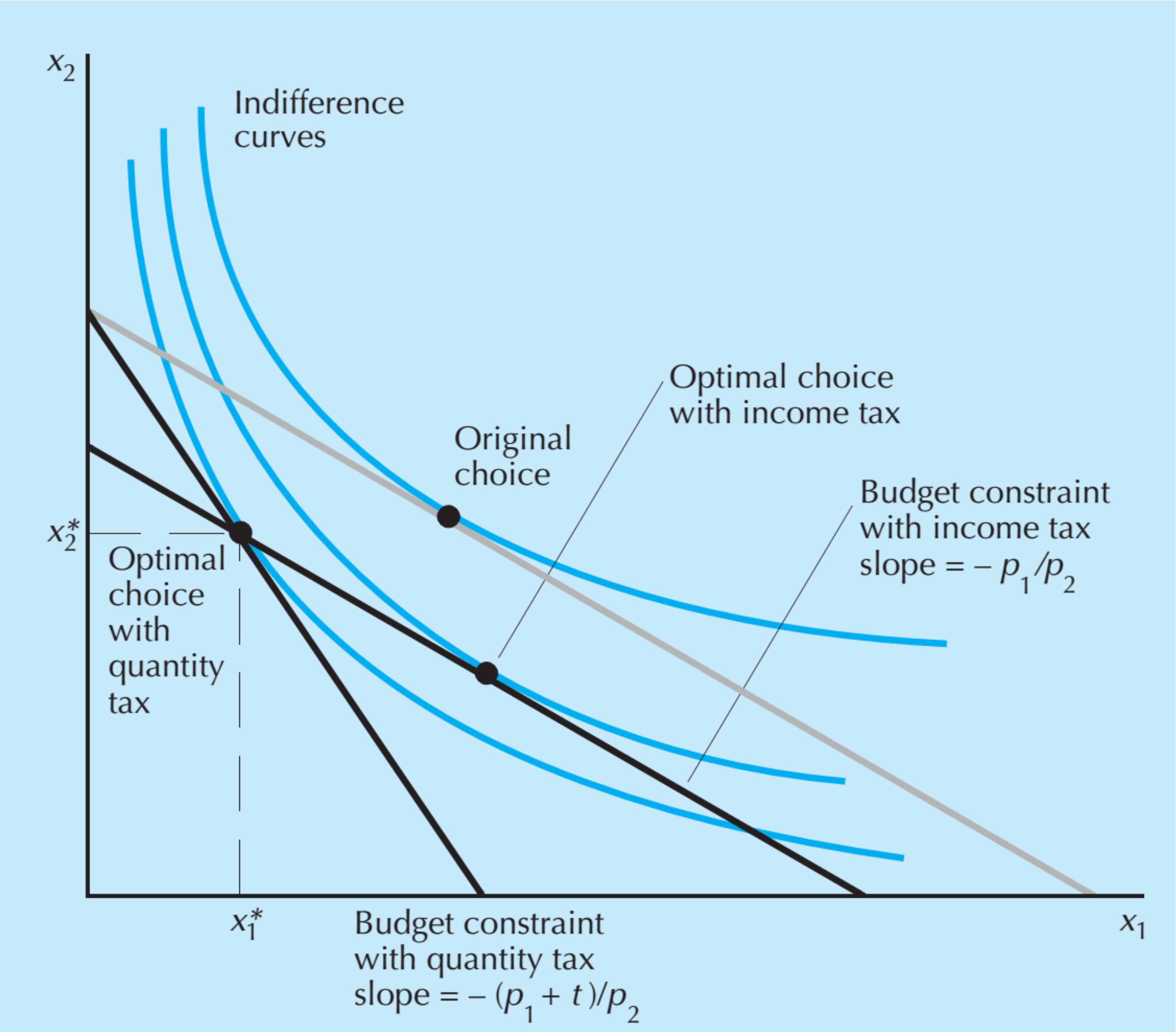
On income:

$$p_1x_1 + p_2x_2 = m - R^*$$

$$p_1x_1 + p_2x_2 = m - tx_1^*$$

$$p_1x_1^* + p_2x_2^* = m - tx_1^*?$$

$$(p_1 + t)x_1^* + p_2x_2^* = m.$$



x_2
Indifference curves
Optimal choice with income tax
Original choice
Budget constraint with income tax slope = $-p_1/p_2$
x_2^*
Optimal choice with quantity tax
x_1^*
Budget constraint with quantity tax slope = $-(p_1+t)/p_2$
x_1

Steps

- First consider quantity tax on good 1
  - What is the revenue R?
- Now look what would be the budget restriction that brings the same revenue R but is levied on income
- Compare outcomes

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MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

Národohospodářská fakulta VŠE v Praze