Game Theory

Tomáš Miklánek

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

Strategy

- The most important concept in the theory of games is the notion of a strategy.

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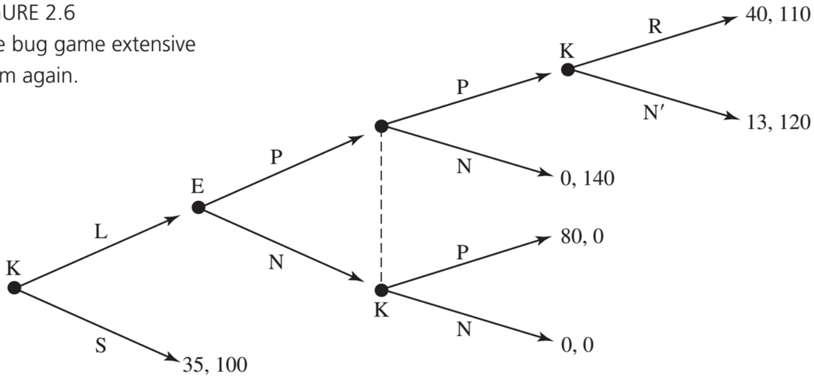
A strategy is a complete contingent plan for a player in the game

Do you remember?

FIGURE 2.6
The bug game extensive form again.

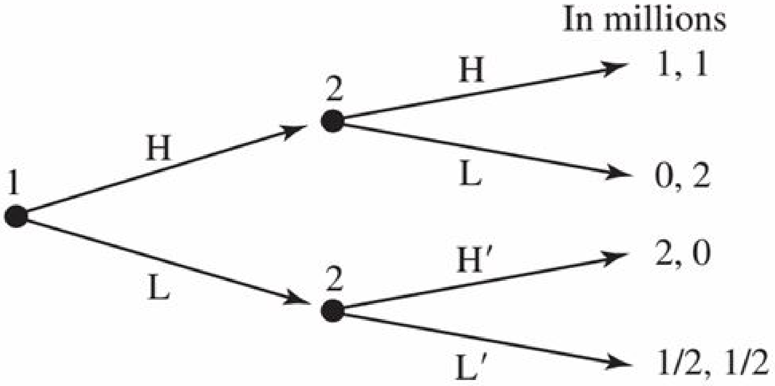


How to describe strategy?

- Consider the Katzenberg–Eisner game again and put yourself in Katzenberg's shoes.
- Your strategy must include what to do
  - at the information set given by the first node
  - at each information set
  - at the last node
- Your strategy must specify all of these things even if you plan to select "Stay" at the beginning

Another example

A price-competition game.



A price-competition game

- The easiest way of writing strategies is to put together the labels corresponding to the actions to be chosen at each information set.
- For example, for the game shown in previous slide, one strategy for player 2 is to select the high price (H) in the top information set and the low price (L') in the bottom information set; this strategy can be written HL'.
- Note that there are four strategies for player 2 in this game: HH', HL', LH',and LL'.
    - Also note that writing strategies in this way would be difficult if we did not use different labels for a player's various information sets.

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- For example, for the game shown in previous slide, one strategy for player 2 is to select the high price (H) in the top information set and the low price (L') in the bottom information set; this strategy can be written HL'.
- Note that there are four strategies for player 2 in this game: HH', HL', LH',and LL'.
    - Also note that writing strategies in this way would be difficult if we did not use different labels for a player's various information sets.
- *What would be strategies of player 2 if the game is simultaneous?*

Formally

- Let S_i denote the strategy space (also called the strategy set) of player i.
- A strategy profile is a vector of strategies, one for each player.
    - suppose we are studying a game with n players. A typical strategy profile then is a vector s $= (s_1, s_2, \ldots, s_n)$, where s_i is the strategy of player i
- s_{-i} is a strategy profile for everyone except player i: $s_{-i} = (s_1, s_2, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$.

Even more formally

Given a game, we let S_i denote the *strategy space* (also called the *strategy set)* of player i. That is, S_i is a set comprising each of the possible strategies of player i in the game. For the game shown in Figure 2.7(a), the strategy space of player 1 is $S_1 = \{H, L\}$, and the strategy space of player 2 is $S_2 = \{HH', HL', LH', LL'\}$. We use lowercase letters to denote single strategies (generic members of these sets). Thus, $s_i \in S_i$ is a strategy for player i in the game. We could thus have $s_1 = L$ and $s_2 = LH'$, for instance.

A *strategy profile* is a vector of strategies, one for each player. In other words, a strategy profile describes strategies for all of the players in the game. For example, suppose we are studying a game with n players. A typical strategy profile then is a vector $s = (s_1, s_2, \ldots, s_n)$, where s_i is the strategy of player i, for $i = 1, 2, \ldots, n$. Let S denote the set of strategy profiles. Mathematically, we write $S = S_1 \times S_2 \times \ldots \times S_n$. Note that the symbol "$\times$" denotes the Cartesian product.[1]

Given a single player i, we often need to speak of the strategies chosen by all of the *other players* in the game. As a matter of notation, it will be convenient to use the term $-i$ to refer to these players. Thus, s_{-i} is a strategy profile for everyone except player i:

$$s_{-i} = (s_1, s_2, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n).$$

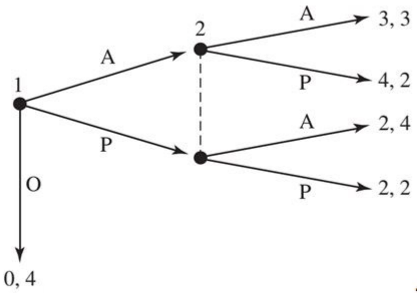
Payoffs

- For each player i, we can define a function u_i: S→R
  - a function whose domain is the set of strategy profiles and whose range is the real numbers
- So that, for each strategy profile s∈S, $u_i[s]$ is player i's payoff in the game. This function u_i is called player i's payoff function

Terminology summary

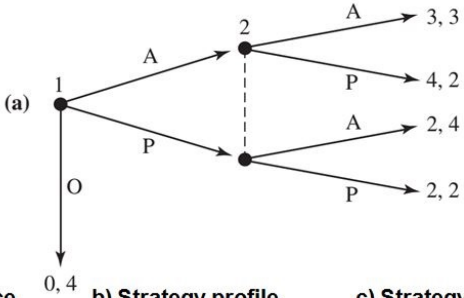
- Strategy space (set) S_i
- Strategy profile s
- Strategy set S
- Payoff function u_i [s]

Can you find...



1. Strategy space?
2. Strategy profile?
3. Strategy set?
4. Payoff function?

Solution



a) Strategy space

$$S_1 = \{A, P, O\}$$

$$S_2 = \{A, P\}$$

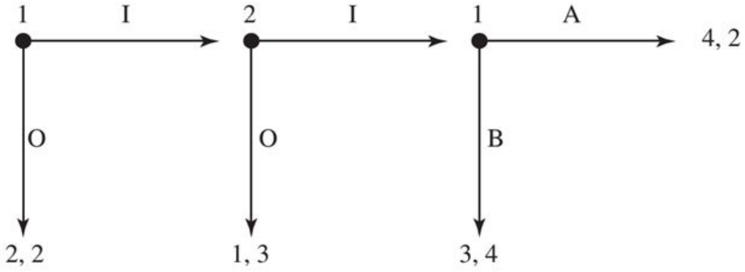
b) Strategy profile

$$s = \{A, A\}$$
$$s = \{A, P\}$$
$$s = \{P, A\}$$
$$s = \{P, P\}$$
$$s = \{O, A\}$$
$$s = \{O, P\}$$

c) Strategy set S

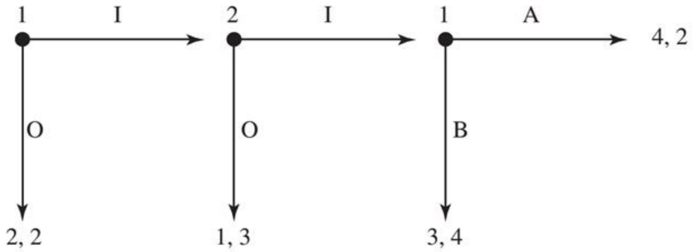
$$S = \left\{ \begin{array}{l} \{A, A\}, \{A, P\}, \\ \{P, A\}, \{P, P\}, \\ \{O, A\}, \{O, P\} \end{array} \right\}$$

Task 2



- Strategy space
- Strategy profile
- Strategy set
- Payoff function

Task 2 solution



a) Strategy space

$S_1 = \{OA, OB, IA, IB\}$

$S_2 = \{O, I\}$

b) Strategy profile

$$s = \{OA, O\} \quad s = \{OA, I\}$$
$$s = \{OB, O\} \quad s = \{OB, I\}$$
$$s = \{IA, O\} \quad s = \{IA, I\}$$
$$s = \{IB, O\} \quad s = \{IB, I\}$$

Important note to the previous task

- We need "complete contingent plans" (not just a "plan")
- Can't we just say that player 1 has three different strategies O, IA, and IB?

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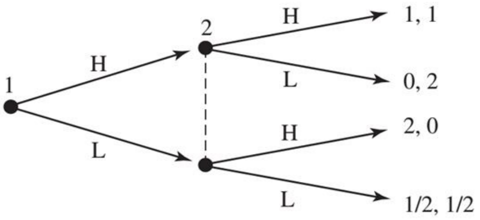
- The definition of a strategy requires a specification of player 1's choice at the second information set even in the situation in which he plans to select O at his first information set.
- We need to keep track of behavior at all information sets to fully analyze any game.
- Our study of rationality will require the evaluation of players' optimal moves starting from arbitrary points in a game.
- People might make mistakes in the course of the game.

Normal form

- The extensive form is one straightforward way of representing a game.
- Another way of formally describing games is based on the idea of strategies.
- It is called the normal form (or strategic form) representation of a game.
- This alternative representation is more compact than the extensive form in some settings.

How to do it?

How in the normal form?



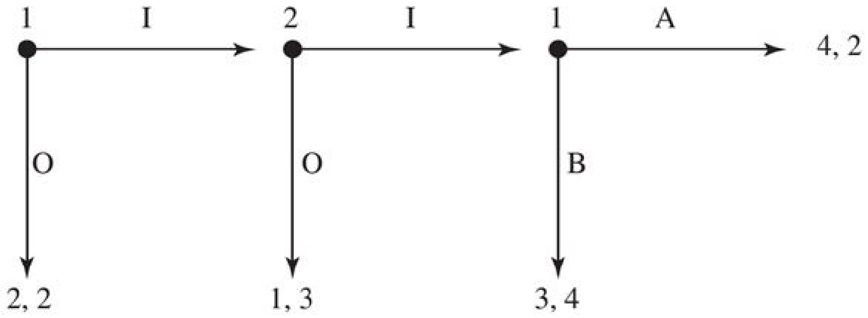
1. Get the Strategy sets

$$S_1 = \{H, L\}$$
$$S_2 = \{H, L\}$$

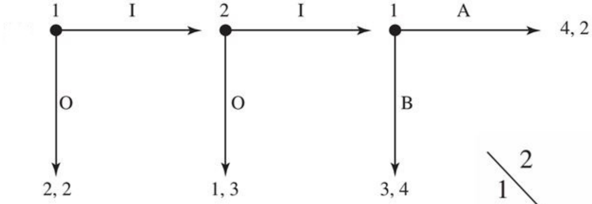
2. Combine the Strategy sets into matrix form:

		2	
		H	L
1	H	(1, 1)	(0, 2)
	L	(2 , 0)	(½ , ½)

Try this...



Solution



1. Get the Strategy sets

$$S_1 = \{OA, OB, IA, IB\}$$

$$S_2 = \{O, I\}$$

2. Combine the Strategy sets into matrix form:

1 \ 2	I	O
OA	2, 2	2, 2
OB	2, 2	2, 2
IA	4, 2	1, 3
IB	3, 4	1, 3

Classic normal form games

Matching Pennies

1 \ 2	H	T
H	1, −1	−1, 1
T	−1, 1	1, −1

Prisoners' Dilemma

1 \ 2	C	D
C	2, 2	0, 3
D	3, 0	1, 1

Battle of the Sexes

1 \ 2	Opera	Movie
Opera	2, 1	0, 0
Movie	0, 0	1, 2

Hawk-Dove/Chicken

1 \ 2	H	D
H	0, 0	3, 1
D	1, 3	2, 2

Coordination

1 \ 2	A	B
A	1, 1	0, 0
B	0, 0	1, 1

Pareto Coordination

1 \ 2	A	B
A	2, 2	0, 0
B	0, 0	1, 1

Pigs

D \ S	P	D
P	4, 2	2, 3
D	6, −1	0, 0

Problem with normal form games?

- One way of viewing the normal form is that it models a situation in which players simultaneously and independently select complete contingent plans for an extensive-form game.

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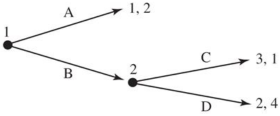

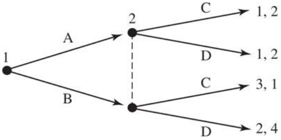

1 \ 2	C	D
A	1, 2	1, 2
B	3, 1	2, 4

Problem with normal form games?

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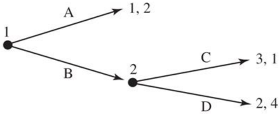
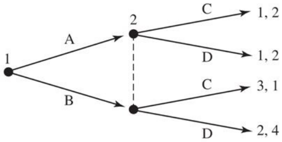




1 \ 2	C	D
A	1, 2	1, 2
B	3, 1	2, 4

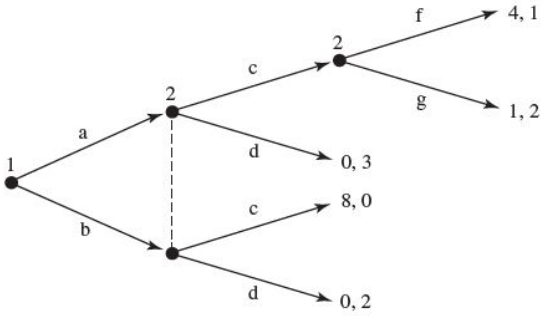
- Not always possible to go from normal to the extensive form

Summary

- Strategies –> Strategy space –> Strategy profiles –> Strategy set
- Payoff function
- Normal form games

Review task 1

- How many strategies does player 2 have?



Review task 1

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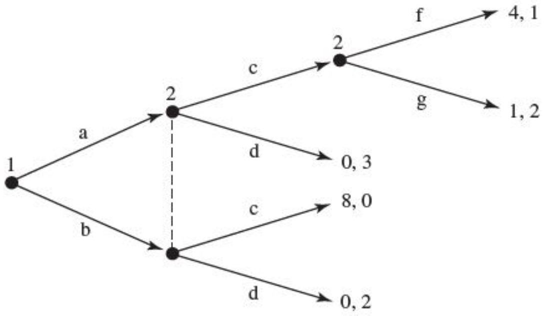

- What about player 1?
- Draw normal form of this game.

Review task 2

- Draw an extensive-form representation of each game. Is there only one unique solution?

1 \ 2	HC	HD	LC	LD
H	3, 3	3, 3	0, 4	0, 4
L	4, 0	1, 1	4, 0	1, 1

Rationality

- In its mildest form, rationality implies that every player is motivated by maximizing his own payoff.
- In a stricter sense, it implies that every player always maximizes his utility, thus being able to perfectly calculate the probabilistic result of every action.
- Preferences are rational if they are complete and transitive (Microeconomics).
- Rationality is not the same thing as sensibility.
- You can chart rational preferences with an ordering, which is simply a list that has the most preferred outcome on top and progressively less preferred outcomes running down.
- Be careful with group preferences: aggregating individuals who each have rational preferences may result in irrational group preferences.

Common knowledge

- A particular fact F is said to be common knowledge between the players if each player knows F, each player knows that the others know F, each player knows that every other player knows that each player knows F, and so on.
- ...or watch this VIDEO

Reminder

- Do you remember beauty contest from the last week?

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- There is only one **non-dominated** strategy
  - Everybody giving an estimate of 0!
- If a strategy is non-dominated for all players, it is a Nash-equilibrium

Reminder

- Do you remember beauty contest from the last week?
- There is only one **non-dominated** strategy
  - Everybody giving an estimate of 0!
- If a strategy is non-dominated for all players, it is a Nash-equilibrium
- *What was the problem?*

Common knowledge in beauty contest

- The equilibrium of 0 is only achieved when the assumption of common knowledge holds
- In experiments, such as in our class, we could see it wasn't fulfilled.
- Infinite repetition of me knowing you know me knowing you know. . .
    - Each repetition gets us close to zero!

Solutions

- Why do we "solve" games?
- To know which one to play!
    - How do internal corporate changes impact the outcome of strategic interaction?
- Some strategies are better than others

Example

1 \ 2	L	R
U	2, 3	5, 0
D	1, 0	4, 3

- Notice there is something interesting about strategy U

Example

1 \ 2	L	R
U	2, 3	5, 0
D	1, 0	4, 3

- Notice there is something interesting about strategy U
- Regardless of player 2's choice, U gives you a strictly higher payoff than does D.
  - We say that strategy D is dominated by strategy U => D should never be played by a rational player 1.

Dominant strategies

- Why do we care about dominant and dominated strategies?
- We expect that a rational person will never choose a dominated strategy!
- Thus helps us solving games

Strict dominance

A pure strategy s_i of player i is dominated if there is a strategy (pure or mixed) $\sigma_i \in S_i$ such that $u_i(\sigma_i, s_{-i}) > u_i(s_i, s_{-i})$, for all strategy profiles $s_{-i} \in S_{-i}$ of the other players.

Solution concept and equilibrium

- Trying to maximize their payoffs, the players will devise plans known as strategies that pick actions depending on the information that has arrived at each moment.
- The combination of strategies chosen by each player is known as the equilibrium.
- Given an equilibrium, the modeler can see what actions come out of the conjunction of all the players' plans, and this tells him the outcome of the game.

Let's play a game

- You can play "to myself" or "to other"
  - "to myself" gives yourself +1 point
  - "to other" gives the other +2 point
- Write down your move on a paper
- I will randomly pick up one pair of players and their decisions will influence their score for this course

Outcome

	Colleague	
You	other	myself
other	(2, 2)	(0, 3)
myself	(3, 0)	(1, 1)

- This is a version of the prisoner's game
- The social optimum is (other, other)
    - Each gets 2; total 4
- But it is a dominant strategy to choose myself

Prisoner's dilemma

- Prisoners game is very general
- Payoff is years in jail!

		Buddy	
		Talk	Quiet
You	Talk	(-20, -20)	(0, -25)
	Quiet	(-25, 0)	(-1, -1)

General form of PD

	Buddy	
You	Talk	Quiet
Talk	**(c, c)**	(a, d)
Quiet	(d, a)	(b, b)

- *Can you find general relationships for a,b, c, d?*

General form of PD

	Buddy	
You	**Talk**	**Quiet**
Talk	**(c, c)**	(a, d)
Quiet	(d, a)	(b, b)

- *Can you find general relationships for a,b, c, d?*
- a>b>c>d

Consequences

- The result is even stronger than it seems, because it is robust to substantial changes in the model.
- Because the equilibrium is a dominant strategy equilibrium, the information structure of the game does not matter.
    - If Column is allowed to know Row's move before taking his own, the equilibrium is unchanged. Row still chooses Confess, knowing that Column will surely choose Confess afterwards.
- Oligopoly pricing, auction bidding, salesman effort, political bargaining, and arms races, advertisement

Application: International trade

- Both countries no tariff: each earn 50
- Only one country a tariff: earn 80
- Both have tariff: earn 20

		Country 2	
		Tariff	No tariff
Country 1	Tariff	(20, 20)	(80, 0)
	No tariff	(0, 80)	(50, 50)

Application: Oligopoly

Two firms

- Both colluding = $50mil each
- Both cheating = $20mil each
- Cheating alone = $80mil for cheater, 0 for other

		Firm 2	
		Cheat	Collude
Firm 1	Cheat	(20, 20)	(80, 0)
	Collude	(0, 80)	(50, 50)

Using the prisoner's dilemma to your advantage

- How to change prisoner's dilemma?
- How to use the prisoner's dilemma to your advantage?
- If you can't change the game or use it to your advantage, are there other ways to escape the prisoner's dilemma?

How to change prisoner's dilemma?

- Change the payoff by cross-shareholding.
- *Calculate the payoffs if firm acquires 40% of the other in our oligopoly example.*

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	Buddy		
Firm 1		Cheat	Collude
	Cheat	(20,20)	(48,32)
	Collude	(32, 48)	(50, 50)

How to change prisoner's dilemma?

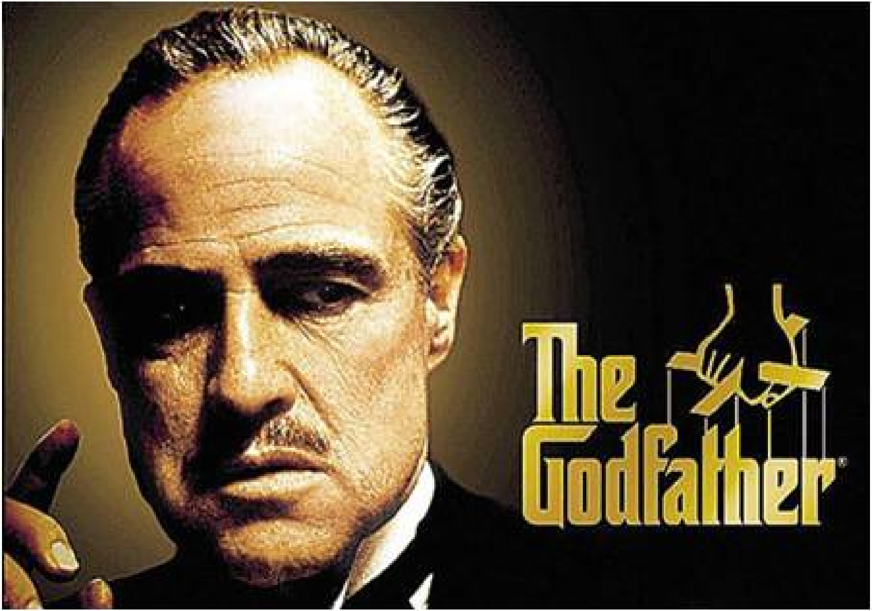
- Change the payoff by cross-shareholding.
- *Calculate the payoffs if firm acquires 40% of the other in our oligopoly example.*

	Buddy	
Firm 1		Cheat	Collude
	Cheat	(20,20)	(48,32)
	Collude	(32, 48)	(50, 50)

- *What would happen when firms acquire only 30% of each other?*

How to change prisoner's dilemma?

- Change the payoffs by hostages
- Promises can sometimes be credible through a contract with the party to whom you are making the promise.
- Must contract with third party
    - Sometimes the state
    - Or private 3rd parties


The Godfather

Change the payoff by hostages

The Bocchicchio Family:

> *"Once a particularly ferocious branch of the Mafia in Sicily, it had become an instrument of peace in America."*

- How can Solozzo invite Michael for a meeting and guarantee that Michael will not be harmed?

Customers as Hostages:

"I will punish you if you lower prices"

- Price Matching Guarantee
  - If any competitor offers a lower price, I will match it
- *Draw a new game now (for our oligopoly case)*

How to use the prisoner's dilemma to your advantage?

- The battle for Federated (1988)
  - Parent of Bloomingdales
- Current share price $\approx$ \$60
- Expected post-takeover share price $\approx$ \$60

How to use the prisoner's dilemma to your advantage?

- The battle for Federated (1988)
  - Parent of Bloomingdales
- Current share price $\approx$ \$60
- Expected post-takeover share price $\approx$ \$60
- Macy's offers \$70/share
  - contingent on receiving 50% of the shares
- *Do you tender your shares to Macy's?*

How to use the prisoner's dilemma to your advantage?

- Robert Campeau bids $74 per share not contingent on amount acquired

How to use the prisoner's dilemma to your advantage?

- Robert Campeau bids $74 per share not contingent on amount acquired
- Campeau's Mixed Scheme:
    - If less than 50% tender their shares, each receives: $74 per share
    - If more than 50% tender their shares, (if X% tender), each receives:

$$\left(\frac{50\%}{X\%}\right) * 74 + \left(\frac{X\% - 50\%}{X\%}\right) * 60$$

How to use the prisoner's dilemma to your advantage?

	Majority of Others		
You		Macy's	Campeau
	Macy's	70 USD	60 USD
	Campeau	74 USD	67+ USD

- To whom do you tender your shares?

How to use the prisoner's dilemma to your advantage?

- Each player has a dominant strategy: Tender shares to Campeau
- Resulting Price: $(\frac{1}{2}$ x $74) + (\frac{1}{2}$ x $60) = \$67$
- BUT: Macy's offered $70 !

Escape the prisoner's dilemma

- Firms do collude
- Experimental evidence
  - people cooperate about half of the time (Camerer, 2003: 46)!
  - Communication helps
- The prisoner's dilemma can be escaped
  - In contradiction with economic theory!
- Repeated play

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Národohospodářská fakulta VŠE v Praze