Game theory

Price Discrimination

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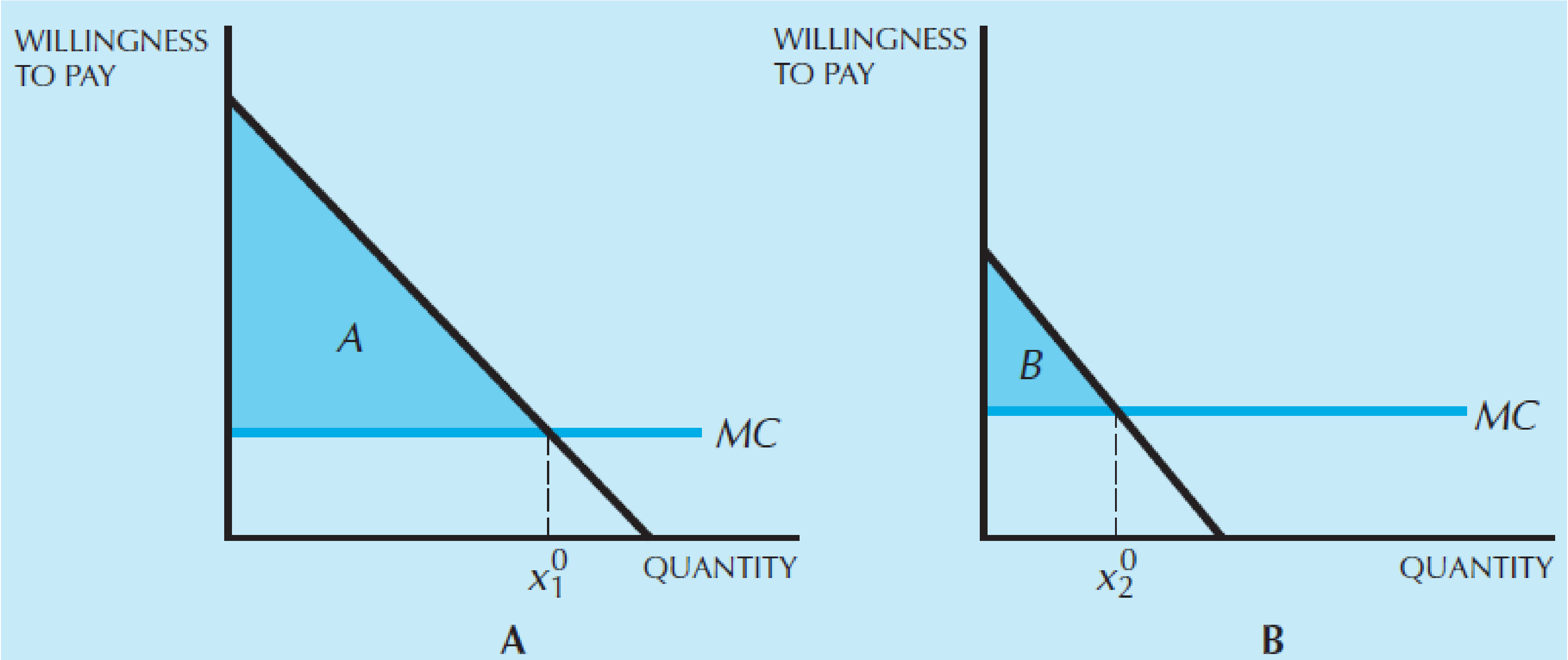
MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

Price discrimination

- **First-degree price** discrimination means that the monopolist sells different units of output for different prices and these prices may differ from person to person
- **Second-degree price** discrimination means that the monopolist sells different units of output for different prices, but every individual who buys the same amount of the good pays the same price.
- **Third-degree price** discrimination occurs when the monopolist sells output to different people for different prices, but every unit of output sold to a given person sells for the same price.

Source: Varian (2014)

First-degree price discrimination



Source: Varian (1992)

2nd degree price discrimination

- Sequential game
  - Stage 1: Monopolist creates consumption packages (quantity, quality, price) to offer
  - Stage 2: Consumer chooses one of the packages on offer

- Varian, intermediate Microeconomics, Chap 25.3

- Probability ½ for a poor person P
- Probability ½ for a rich person R
- Monopolist cannot see the difference

What packages to offer to the consumer types of R & P?

Marginal WTP

Quantity	Type P	Type R
1	7	10
2	1	5

Package 1

- ~~Type P:~~
  - ~~Quantity 2~~
  - ~~Price 8~~
- ~~Type R:~~
  - ~~Quantity 2~~
  - ~~Price 15~~

~~Monopolist profit: 23/2~~

- But is IC?
- Type R pretends to be P:
  - Quantity 2
  - Price 8
  - WTP=15
  - Profit R = 7!

Monopolist profit: 16/2

Incentive compatible

- How to differentiate packages to target them for the two different groups?

What packages to offer to the consumer types of R & P?

Marginal WTP

Quantity	Type P	Type R
1	7	10
2	1	5

Package 2

- Type P:
  - Quantity 1
  - Price 7
- Type R:
  - Quantity 2
  - Price 15

Monopolist profit: 22/2

What packages to offer to the consumer types of R & P?

Marginal WTP

Quantity	Type P	Type R
1	7	10
2	1	5

Package 2

- ~~Type P:~~
  - ~~Quantity 1~~
  - ~~Price 7~~
- ~~Type R:~~
  - ~~Quantity 2~~
  - ~~Price 15~~

~~Monopolist profit: 22/2~~

- But is IC?
- Type R pretends to be P:
  - Quantity 1
  - Price 7
  - WTP=10
  - Profit R = 3!

Monopolist profit: 14/2

Incentive compatible

Information rent

What packages to offer to the consumer types of R & P?

Marginal WTP

Quantity	Type P	Type R
1	7	10
2	1	5

- ~~Type P:~~
  - ~~Quantity 1~~
  - ~~Price 7~~
- ~~Type R:~~
  - ~~Quantity 2~~
  - ~~Price 15~~

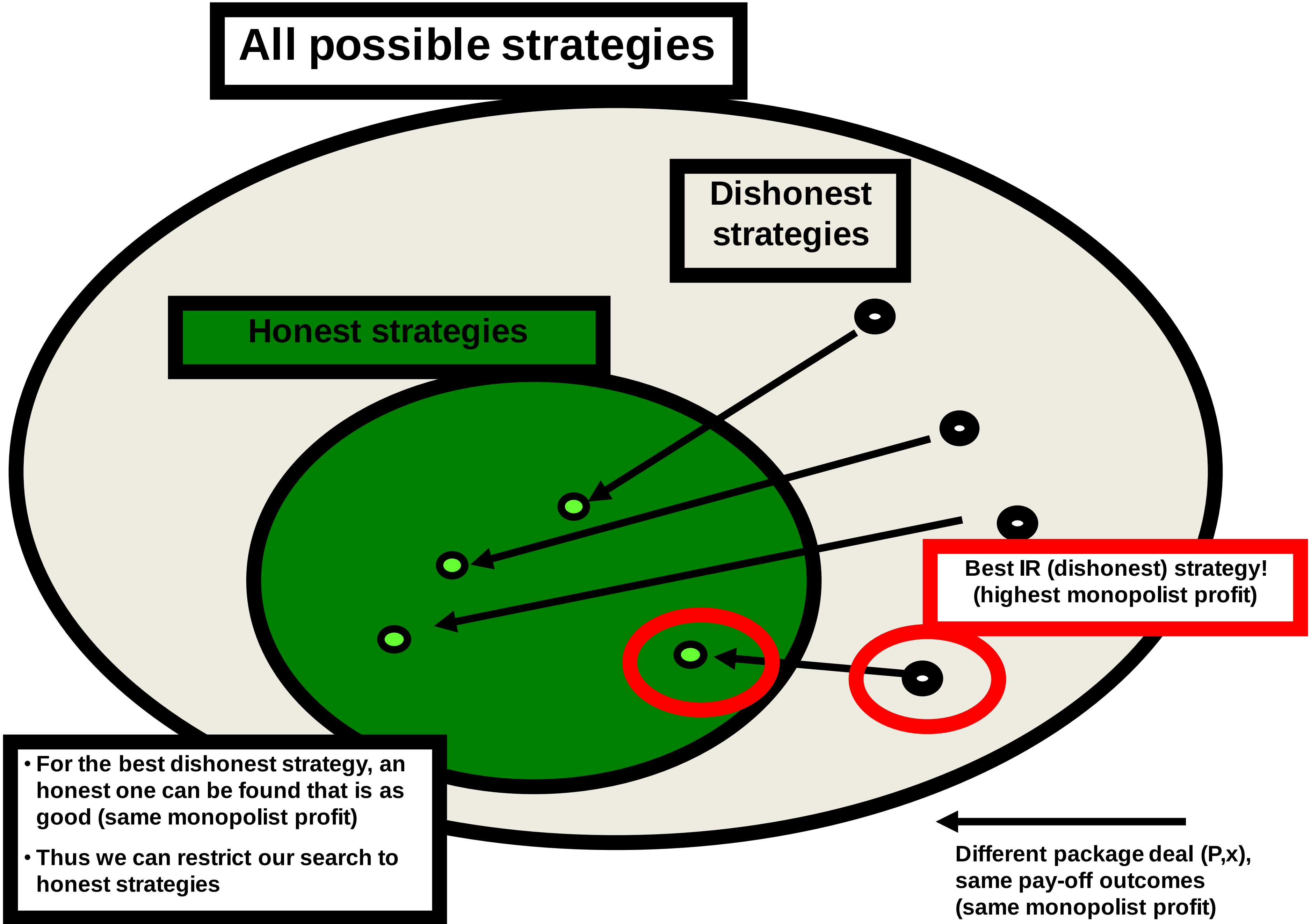
~~Monopolist profit: 22/2~~

- But…
- Type R pretends to be P:
  - Quantity 1
  - Price 7
  - WTP=10
  - Profit R = 3!

Monopolist profit: 14/2

- Type P:
  - Quantity 1
  - Price 7
- Type R:
  - Quantity 2
  - Price 15**-3=12**
    - (Profit R = 3)

Monopolist profit: 19/2


All possible strategies
Dishonest strategies
Honest strategies
Best IR (dishonest) strategy! (highest monopolist profit)
• For the best dishonest strategy, an honest one can be found that is as good (same monopolist profit)
• Thus we can restrict our search to honest strategies
Different package deal (P,x), same pay-off outcomes (same monopolist profit)

- Squeeze P
- Allow R the profit he could earn by pretending to be P
  - The **information rent**

What packages to offer to the consumer types of R & P?

Marginal WTP

Quantity	Type P	Type R
1	7	10
2	1	5

- Type P:
  - Quantity 1
  - Price 7
- Type R:
  - Quantity 2
  - Price 15

Monopolist profit: 22/2

- But…
- Type R pretends to be P:
  - Quantity 1
  - Price 7
  - WTP=10
  - Profit R = 3!

Monopolist profit: 14/2

- Type P:
  - Quantity 1
  - Price 7
- Type R:
  - Quantity 2
  - Price 15**-3=12**
    - (Profit R = 3)

Monopolist profit: 19/2

What packages to offer to the consumer types of R & P?

Marginal WTP

Quantity	Type P	Type R
1	6	10
2	1	5

- Type P:
  - Quantity 1
  - Price 6
- Type R:
  - Quantity 2
  - Price 15

Monopolist profit: 21/2

- But…
- Type R pretends to be P:
  - Quantity 1
  - Price 6
  - WTP=10
  - Profit R = 4!

Monopolist profit: 12/2

- Type P:
  - Quantity 1
  - Price 6
- Type R:
  - Quantity 2
  - Price 15**-4=11**
    - (Profit R = 4)

Monopolist profit: 17/2

What packages to offer to the consumer types of R & P?

Marginal WTP

Quantity	Type P	Type R
1	4	10
2	1	5

- Type P:
  - Quantity 1
  - Price 4
- Type R:
  - Quantity 2
  - Price 15

Monopolist profit: 19/2

- But…
- Type R pretends to be P:
  - Quantity 1
  - Price 4
  - WTP=10
  - Profit R = 6!

Monopolist profit: 8/2

- Type P:
  - Quantity 1
  - Price 4
- Type R:
  - Quantity 2
  - Price 15**-6=9**
    - (Profit R = 6)

Monopolist profit: 13/2

Second-Degree Price Discrimination

What packages to offer to the consumer types of R & P?

Marginal WTP

Quantity	Type P	Type R
1	4	10
2	1	5

- Type P:
  - Quantity 1
  - Price 4
- Type R:
  - Quantity 2
  - Price 15**-6=9**
    - (Profit R = 6)

Monopolist profit: 13/2

- But…
- Monopolists stops catering for Type P and sells only to R:
  - Quantity 2
  - Price 15
  - Profit R = 0!

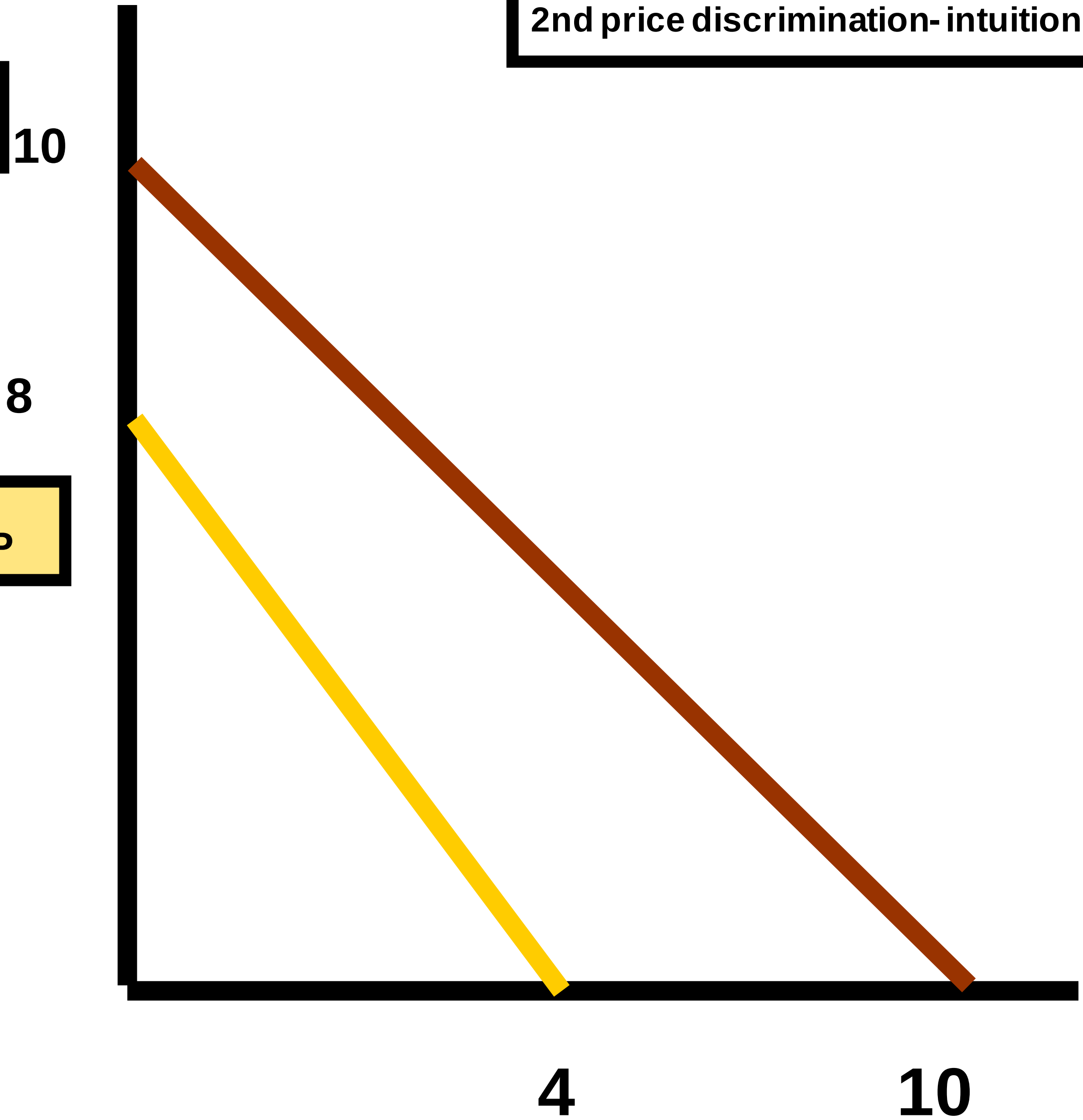
Monopolist profit: 15/2

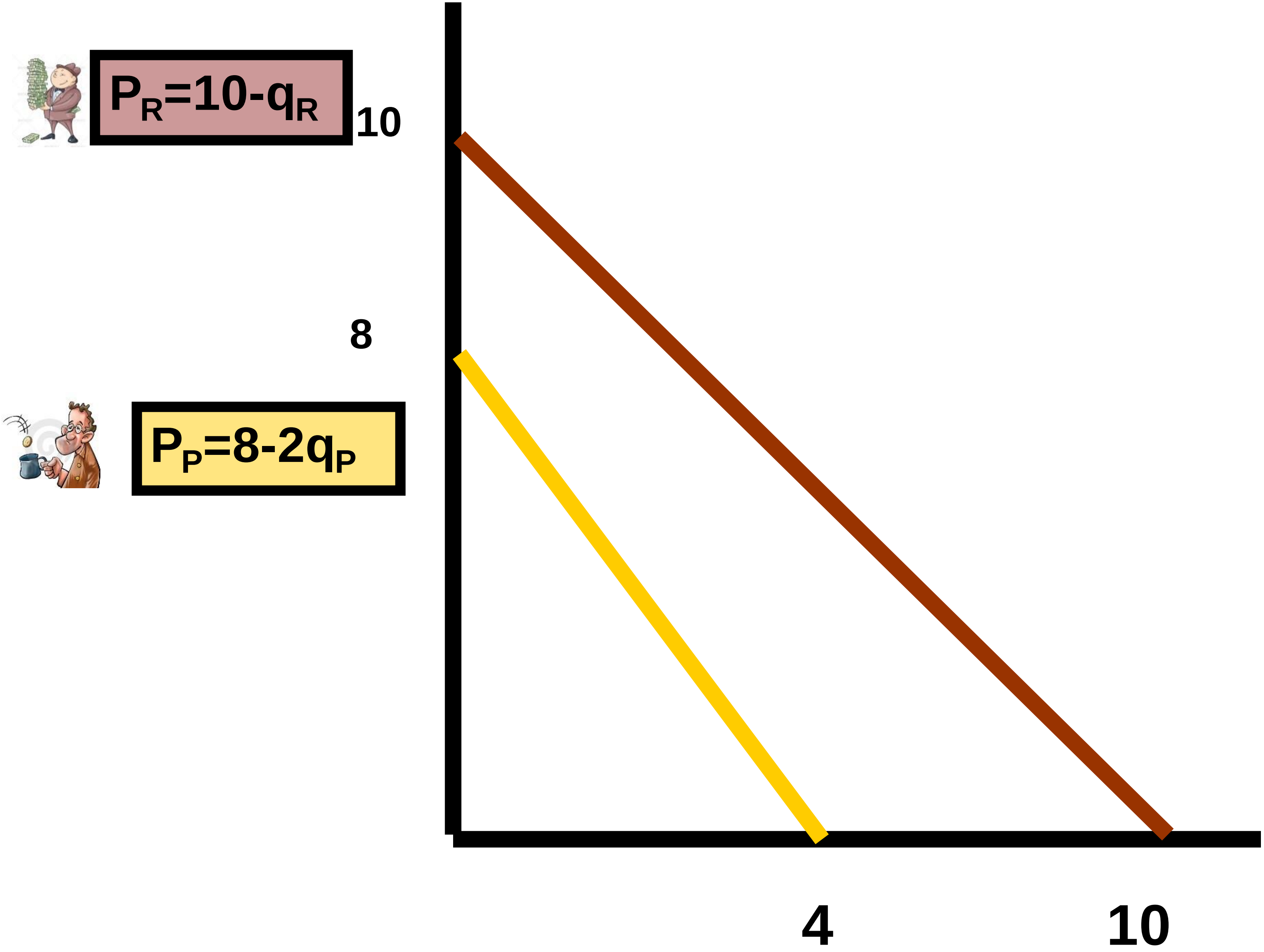
- Look at the continuous version of this problem

2nd price discrimination- intuition

$P_R=10-q_R$

$P_P=8-2q_P$



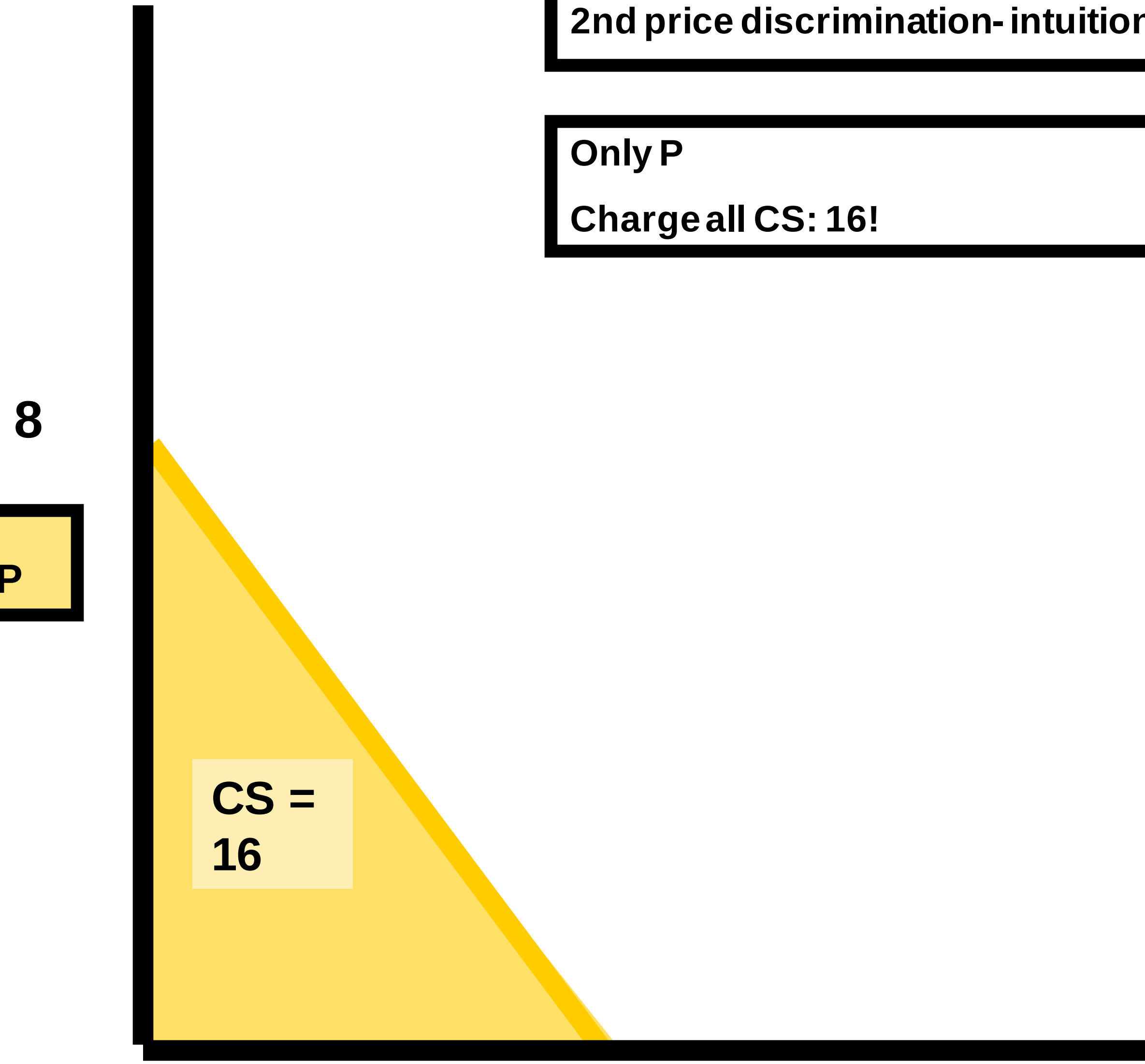

$P_R=10-q_R$
10
8
$P_P=8-2q_P$
4
10

2nd price discrimination- intuition

Only P

Charge all CS: 16!

$P_P=8-2q_P$

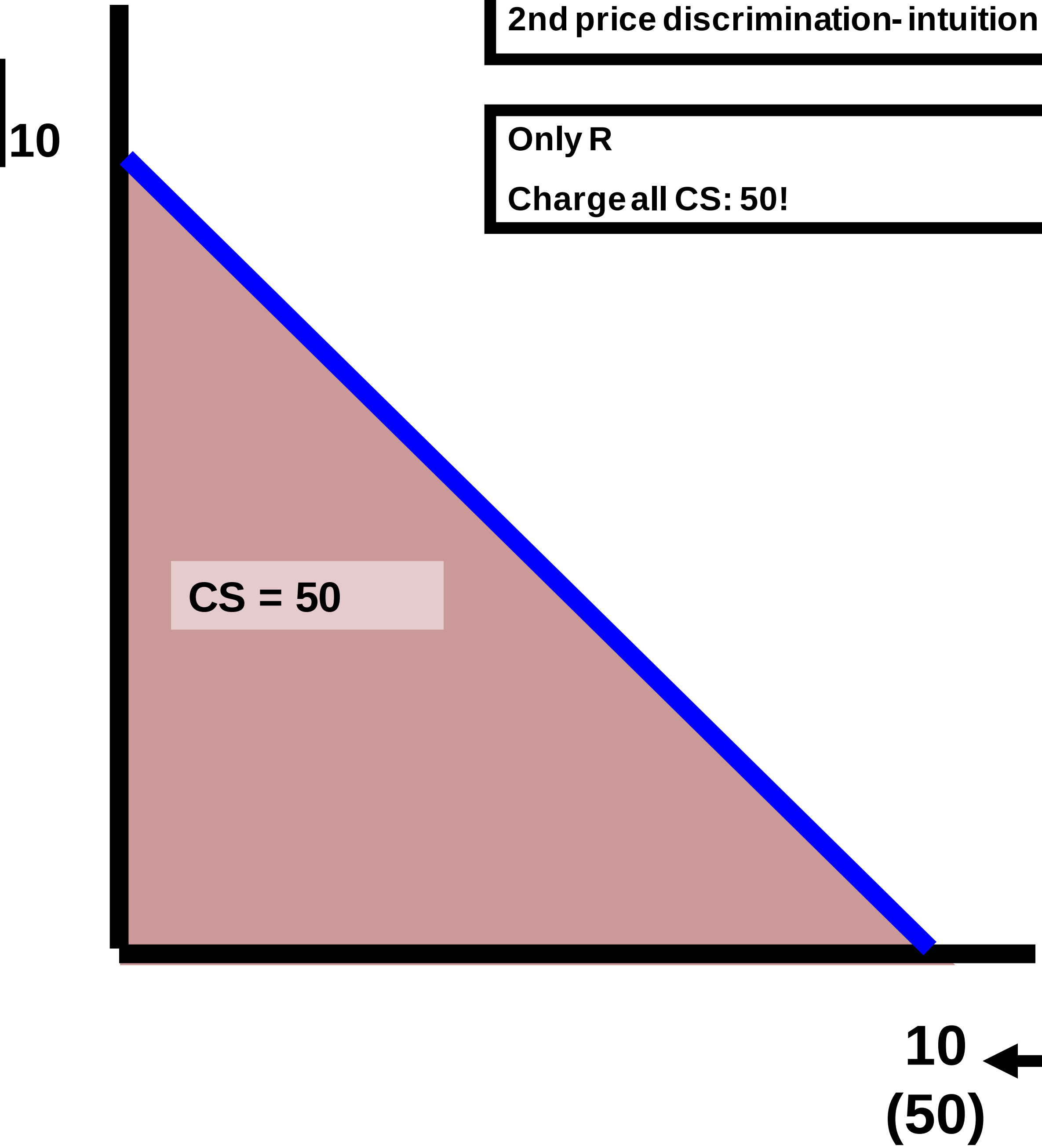


2nd price discrimination- intuition

Only R

Charge all CS: 50!

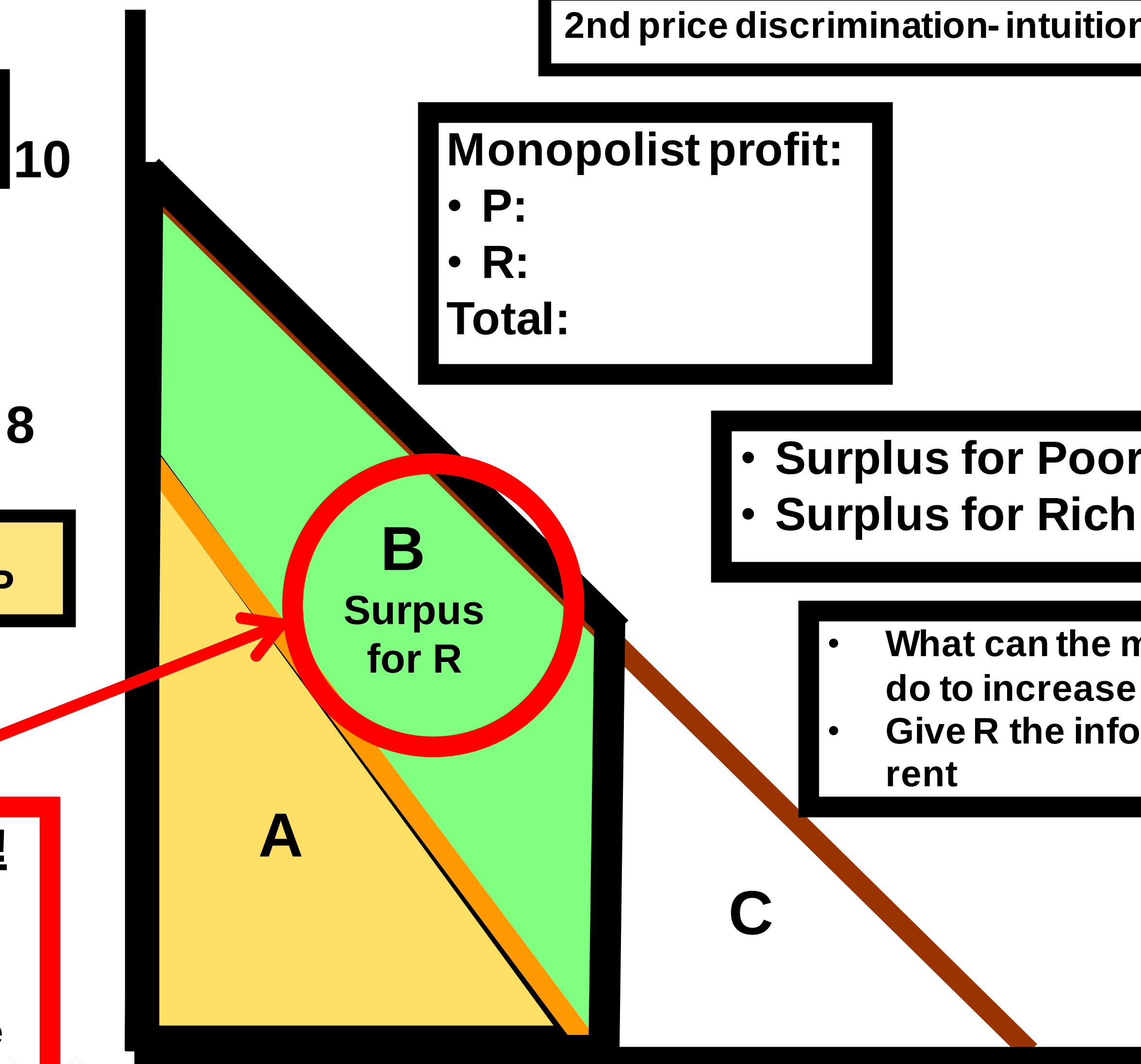
$P_R=10-q_R$



2nd price discrimination- intuition

$P_R=10-q_R$

$P_P=8-2q_P$



Monopolist profit:

- **P:**
- **R:**

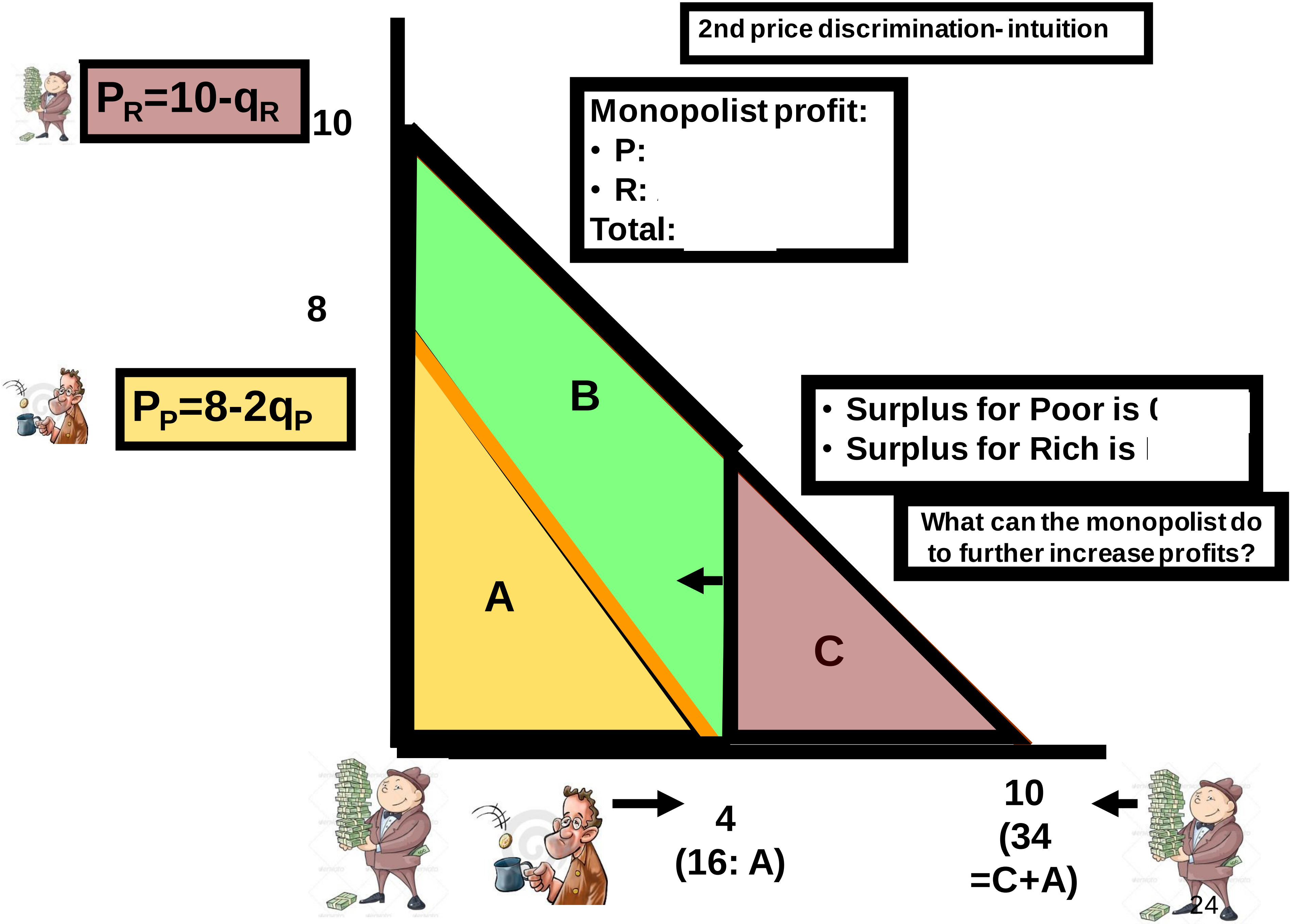
Total:

- **Surplus for Poor is**
- **Surplus for Rich is**

- What can the monopolist do to increase profits?
- Give R the information rent

<u>Information rent!</u>

Rent R can earn by pretending to be poor (hiding the info that he is rich)


2nd price discrimination- intuition
$P_R=10-q_R$
10
Monopolist profit:
• P:
• R:
Total:
8
$P_P=8-2q_P$
B
• Surplus for Poor is
• Surplus for Rich is
What can the monopolist do to further increase profits?
A
C
4
(16: A)
10
(34
=C+A)

2nd price discrimination- intuition

$P_R=10-q_R$

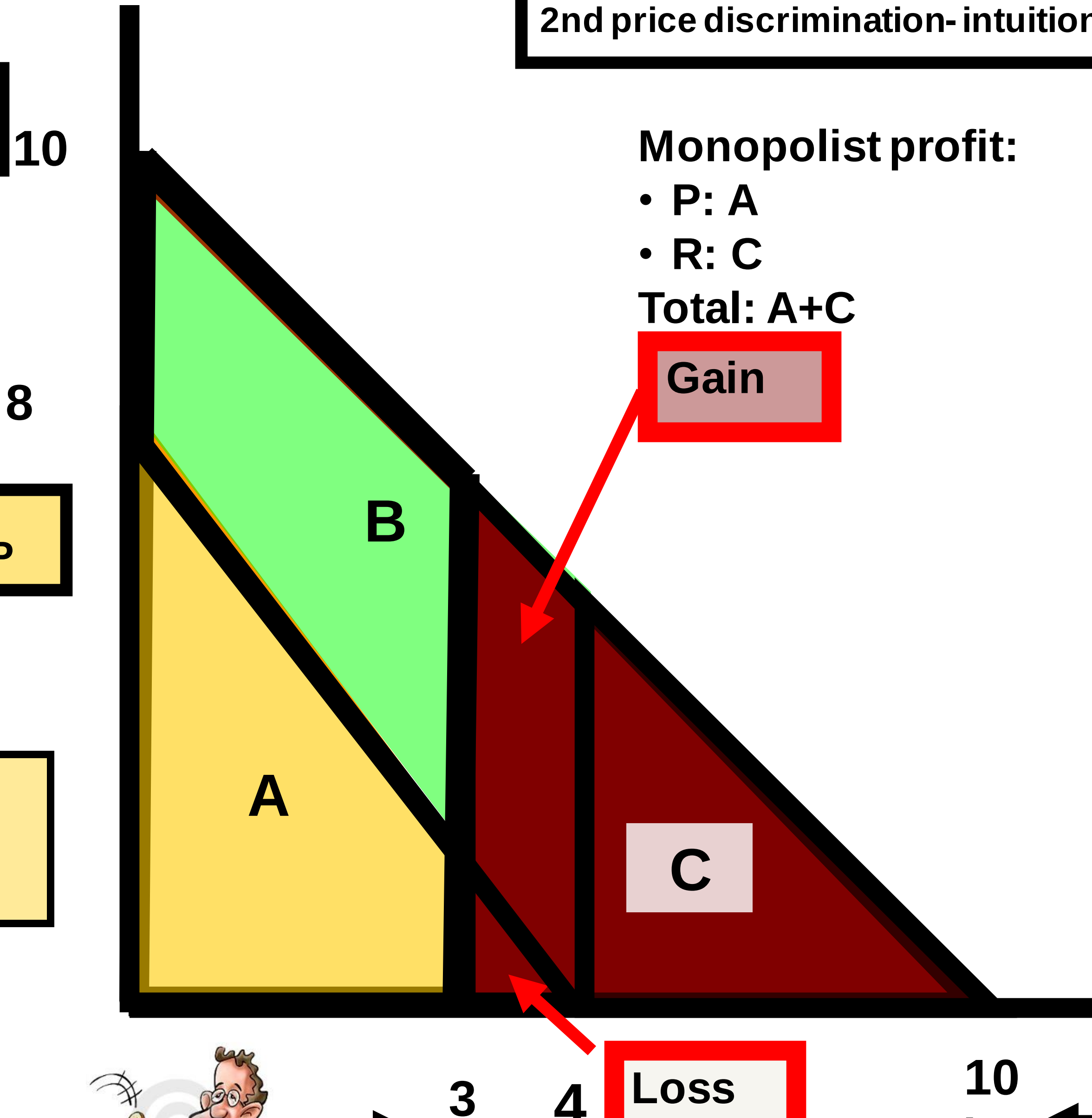
$P_P=8-2q_P$

Monopolist profit:

- P: A
- R: C

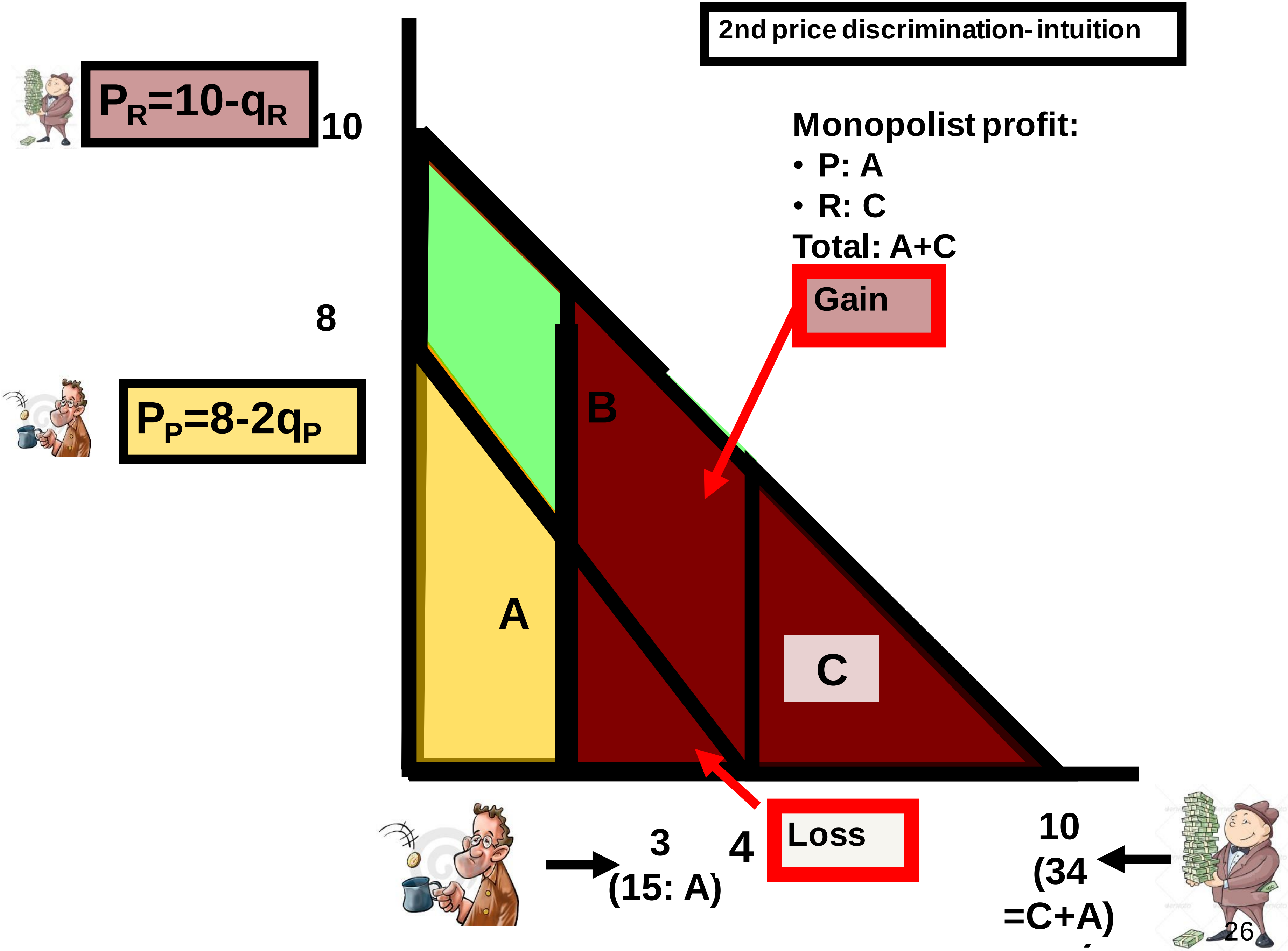
Total: A+C

A
(16-1= 15)



→ 3 (15: A)

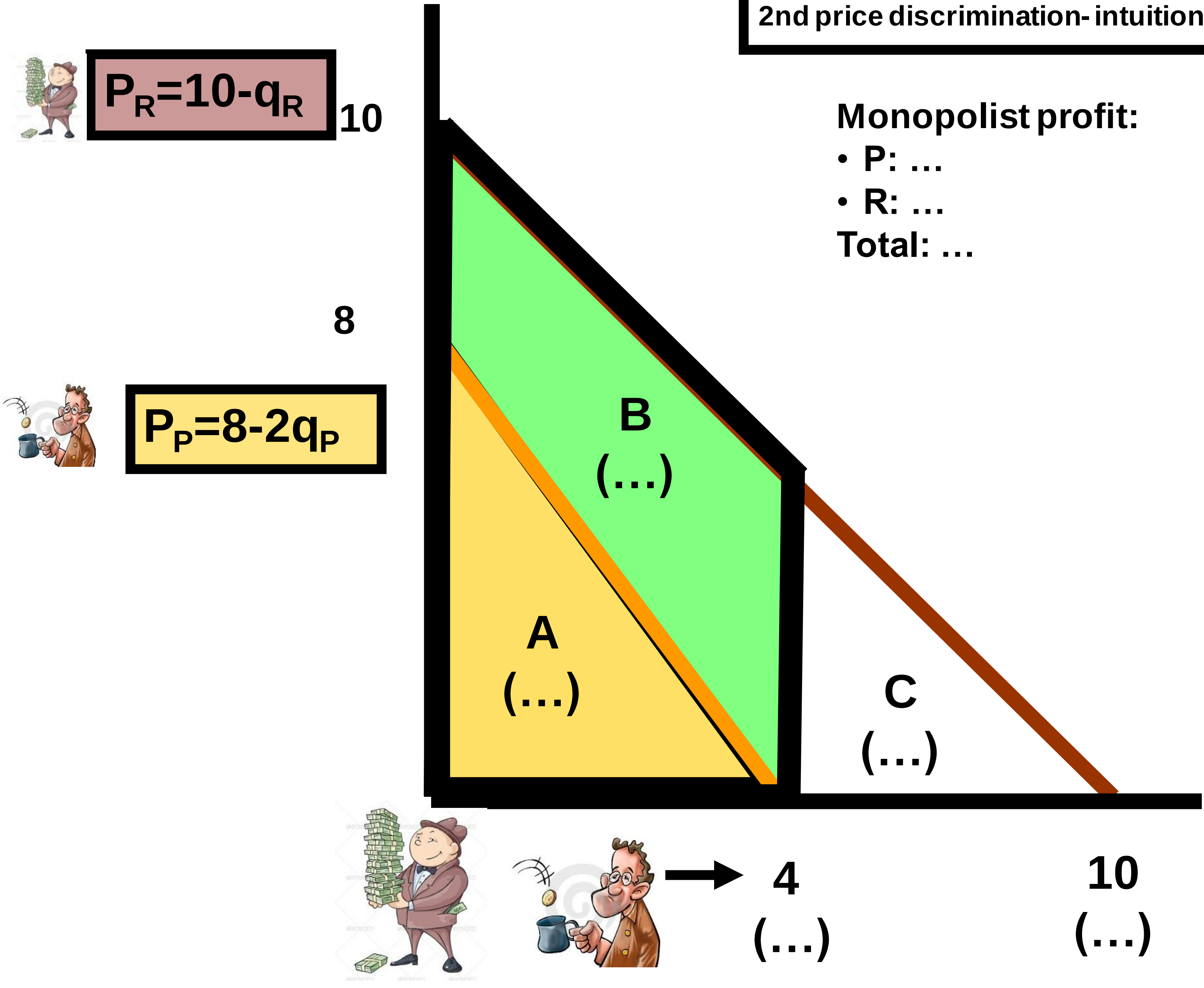
← 10 (34 =C+A)


2nd price discrimination- intuition
$P_R=10-q_R$
10
8
$P_P=8-2q_P$
Monopolist profit:
• P: A
• R: C
Total: A+C
Gain
B
A
C
Loss
3
(15: A)
4
10
(34
=C+A)

What has the shortage of space for your knees to do with 2nd price discrimination?


TERMINAL 2
AAA
14014

2nd price discrimination- intuition



Monopolist profit:

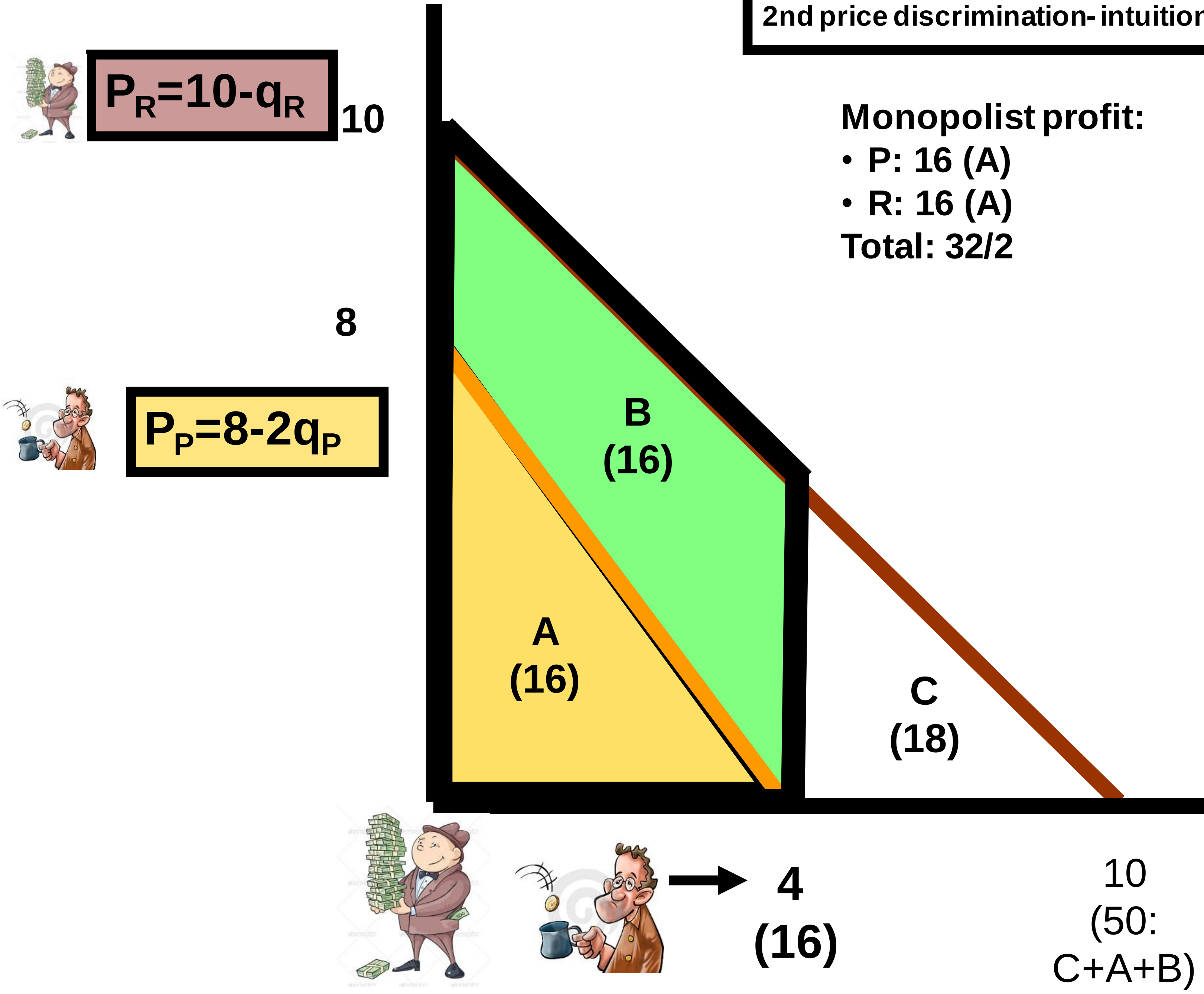
- **P: …**
- **R: …**

Total: …

2nd price discrimination- intuition

Monopolist profit:
- **P: 16 (A)**
- **R: 16 (A)**

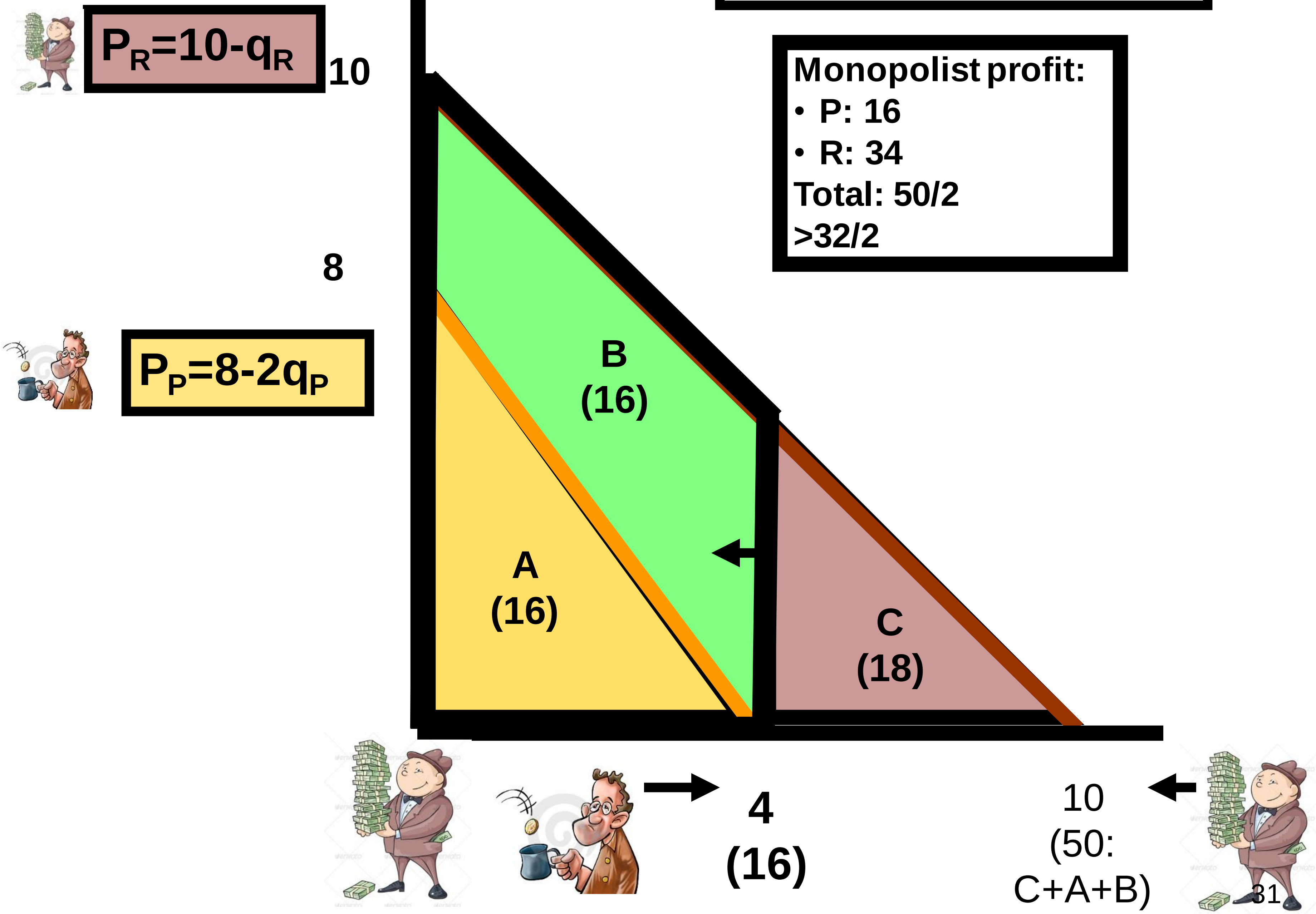
Total: 32/2



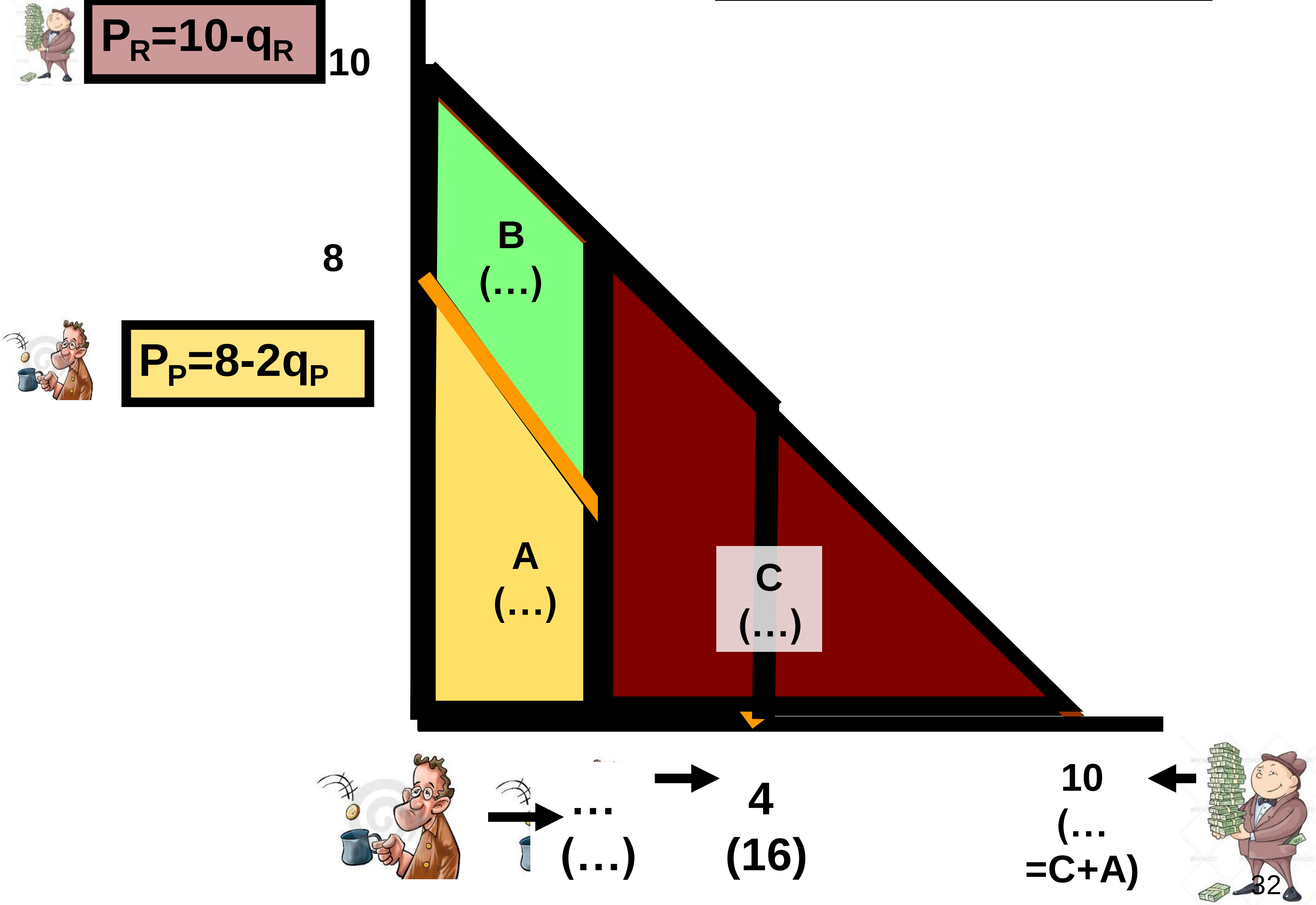
2nd price discrimination- intuition

Monopolist profit:
- **P: 16**
- **R: 34**

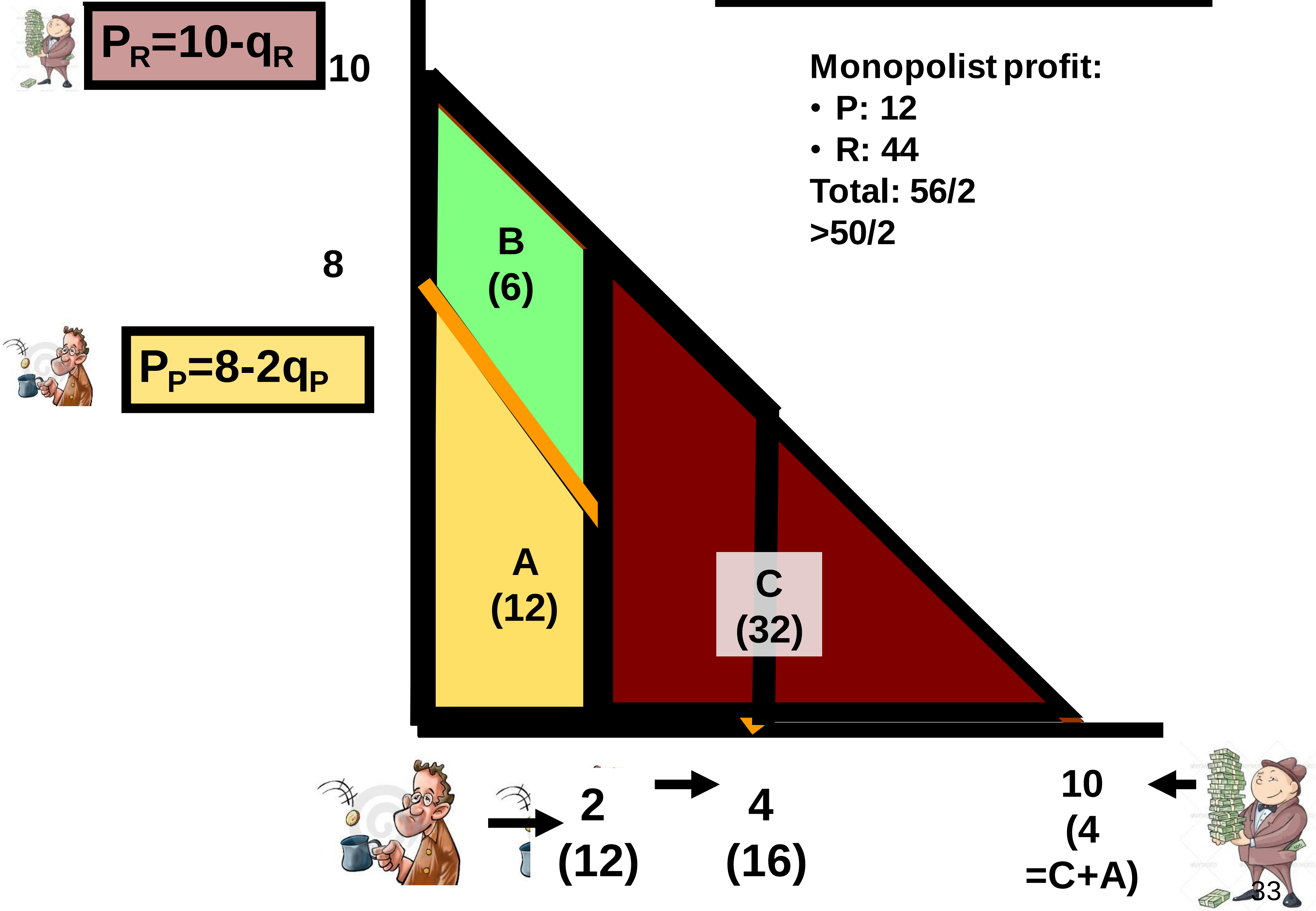
Total: 50/2
>32/2



2nd price discrimination- intuition



2nd price discrimination- intuition



$$Max_q \tfrac{1}{2}\cdot \text{Profit from poor} + \tfrac{1}{2}\cdot \text{Profit from rich}$$

$$Max_q \tfrac{1}{2}\cdot \text{CS of poor} + \tfrac{1}{2}\cdot \left(\text{CS of rich} - \text{Information Rent}\right)$$

$$Max_q \tfrac{1}{2}\cdot \int_0^q (8-2x)\,dx + \tfrac{1}{2}\cdot \left(\int_0^{10} (10-x)\,dx - \int_0^q \left(10-x-(8-2x)\right)dx\right)$$

$$Max_q \tfrac{1}{2}\cdot \int_0^q (8-2x)\,dx + \tfrac{1}{2}\cdot \int_0^{10} (10-x)\,dx - \tfrac{1}{2}\cdot \int_0^q (2+x)\,dx$$

$$Max_q \tfrac{1}{2}\cdot [8x-x^2]_0^q + \tfrac{1}{2}\cdot [10x-\tfrac{1}{2}x^2]_0^{10} - \tfrac{1}{2}\cdot [2x+\tfrac{1}{2}x^2]_0^q$$

$$Max_q \tfrac{1}{2}\cdot \left(8q-q^2\right) + \tfrac{1}{2}\cdot (100-50) - \tfrac{1}{2}\cdot \left(2q+\tfrac{1}{2}q^2\right)$$

$$Max_q 25 + \tfrac{1}{2}\cdot \left(6q - 1\tfrac{1}{2}q^2\right)$$

$$0 = 6-3q$$

$$q = 2$$

$$Max_q (1-a)\cdot \text{Profit from poor} + a\cdot \text{Profit from rich}$$

$$Max_q (1-a)\cdot \text{CS}_q \text{ of poor} + a\cdot \left(\text{Total CS of rich} - \text{Information Rent}\right)$$

$$Max_q (1-a)\cdot \text{CS}_q \text{ of poor} + a\cdot \left(\text{Total CS of rich} - \left(\text{CS}_\text{q} \text{ of rich} - \text{CS}_\text{q} \text{ of poor}\right)\right)$$

$$Max_q 1\cdot \text{CS}_q \text{ of poor} + a\cdot \left(\text{Total CS of rich} - \text{CS}_\text{q} \text{ of rich}\right)$$

$$Max_q \text{CS}_q \text{ of poor} - a\cdot \text{CS}_\text{q} \text{ of rich} + a\cdot \left(\text{Total CS of rich}\right)$$

$$Max_q \int_0^q \left(8-2x\right)dx - a\cdot \int_0^q \left(10-x\right)dx + a\cdot \int_0^{10} \left(10-x\right)dx$$

$$Max_q [8x-x^2]_0^q - a\cdot [10x-\tfrac{1}{2}x^2]_0^q + a\cdot [10x-\tfrac{1}{2}x^2]_0^{10}$$

$$Max_q \left(8q-q^2\right) - a\cdot \left(10q-\tfrac{1}{2}q^2\right) + a\cdot \left(100-50\right)$$

$$0 = 8-2q-a\cdot(10-q)$$

$$(2-a)q = 8-10a \qquad q = \frac{8-10a}{2-a}$$

$$a = \tfrac{1}{2} \Leftrightarrow q = \frac{8-5}{2-\frac{1}{2}} = \frac{3}{1\frac{1}{2}} = 2$$

- Varian, Microeconomic Analysis, Chap 14.7

More general problem

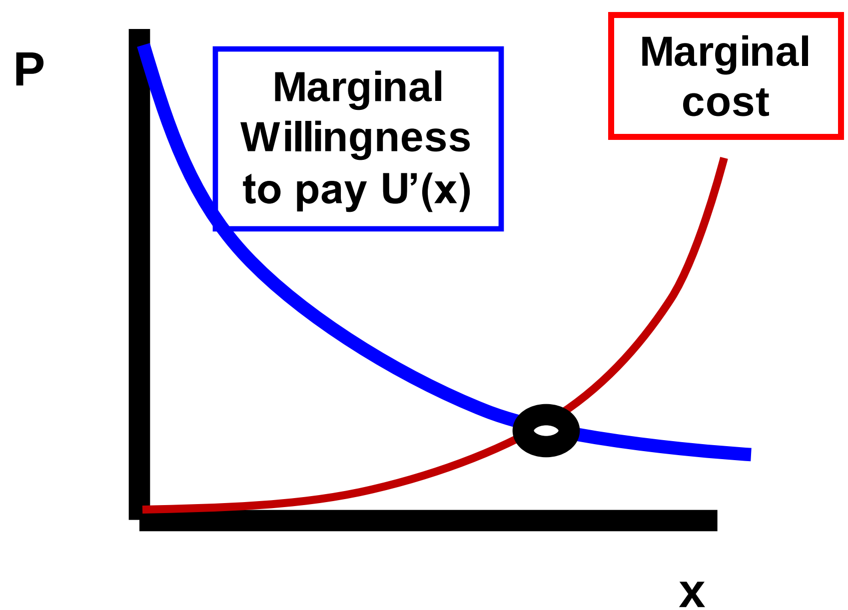
Consumer R and P have utility functions expressing the willingness to pay (in $)

$$u_P(x)$$
$$u_R(x)$$

Cost of production

$$C(x)$$

What would be the efficient way of production?



$$P_P = u'_P(x_P) = C'(x_P + x_R)$$
$$P_R = u'_R(x_R) = C'(x_P + x_R)$$

$\left(p_P, x_P\right)$

Package Deal for Poor

$$u_P(x) < u_R(x)$$
$$u_P'(x) < u_R'(x)$$

$u_R(x)$

$\left(p_R, x_R\right)$

Package Deal for Rich

Single-crossing condition

1. Individual Rationality (IR):

IR$_P$**:** $u_P(x_P) - P_P \geq 0$

IR$_R$**:** $u_R(x_R) - P_R \geq 0$

2. Incentive Compatabillity (IC) = self-selection constraints for true revelation:

IC$_P$**:** $u_P(x_P) - P_P \geq u_P(x_R) - P_R$

IC$_R$**:** $u_R(x_R) - P_R \geq u_R(x_P) - P_P$

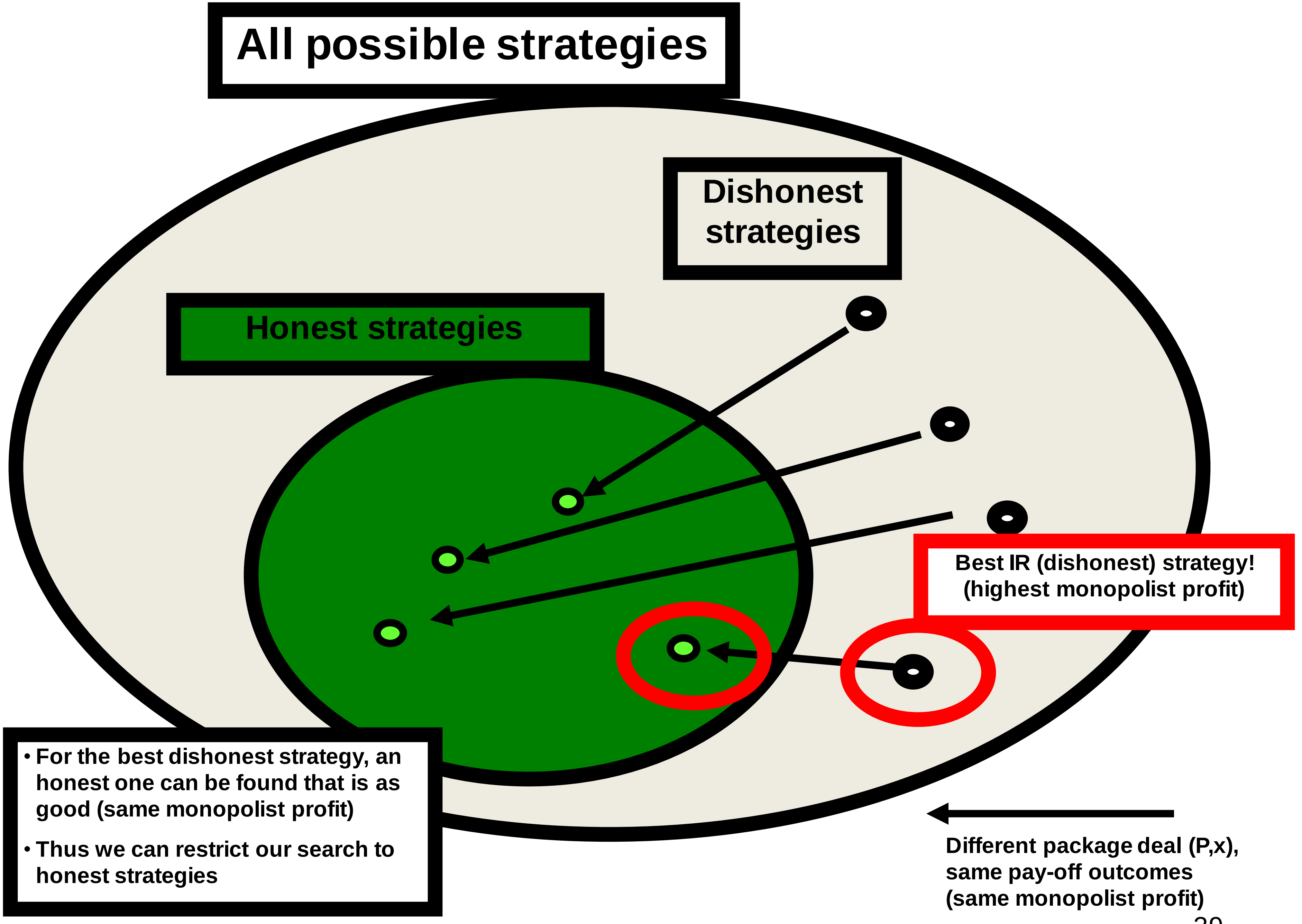
Why we want true revelation?

<u>!!! The revelation principle !!!</u>

-For the outcomes of **any** (dishonest) IR strategy that P & R can think up in reaction to our package deal {(P_P, x_P), (P_R, x_R)}

-There is a (different) package deal {(P_P', x_P'), (P_R', x_R')}, with the same outcomes, and strategies that are honest (true revelations)

Gibbard (1973)


All possible strategies
Dishonest strategies
Honest strategies
Best IR (dishonest) strategy!
(highest monopolist profit)
• For the best dishonest strategy, an honest one can be found that is as good (same monopolist profit)
• Thus we can restrict our search to honest strategies
Different package deal (P,x), same pay-off outcomes (same monopolist profit)

(p_P, x_P)

Package Deal for Poor

$$u_P(x) < u_R(x)$$
$$u'_P(x) < u'_R(x)$$

$u_R(x)$

(p_R, x_R)

Package Deal for Rich

1. Individual Rationality (IR):

IR$_P$: $u_P(x_P) - P_P \geq 0$

IR$_R$: $u_R(x_R) - P_R \geq 0$

2. Incentive Compatabillity (IC) = self-selection constraints for true revelation:

IC$_P$: $u_P(x_P) - P_P \geq u_P(x_R) - P_R$

IC$_R$: $u_R(x_R) - P_R \geq u_R(x_P) - P_P$

IR$_P$: $\boxed{P_P \leq u_P(x_P)}$

IC$_P$: $\cancel{P_P \leq u_P(x_P) + (P_R - u_P(x_R))}$

$\boxed{>0}$

IR$_R$: $\cancel{P_R \leq u_R(x_R)}$

IC$_R$: $\boxed{P_R \leq u_R(x_R) + (P_P - u_R(x_P))}$

$\boxed{<0}$

Information rent

(p_P, x_P)

Package Deal for Poor

$$u_P(x) < u_R(x)$$
$$u'_P(x) < u'_R(x)$$

Given:

(p_R, x_R)

Package Deal for Rich

IR$_P$: $P_P = u_P(x_P)$

IC$_R$: $P_R = u_R(x_R) + \left(P_P - u_R(x_P)\right)$

Maximize profit

$$E\pi = \tfrac{1}{2}\left([P_P] - C[x_P]\right) + \tfrac{1}{2}\left([P_R] - C[x_R]\right)$$

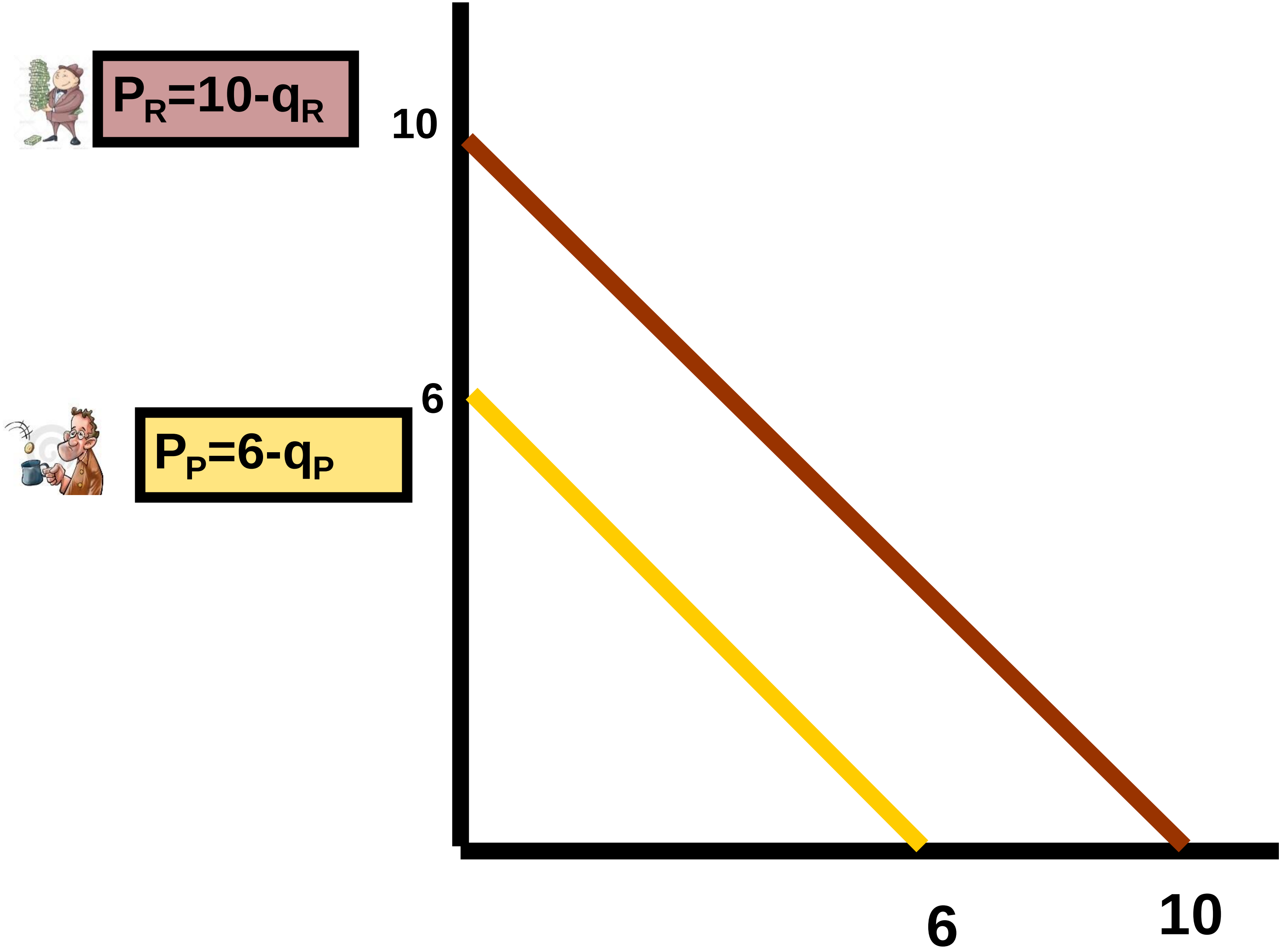
$$E\pi = \tfrac{1}{2}\left([u_P(x_P)] - C[x_P]\right) + \tfrac{1}{2}\left(\left[u_R(x_R) + \left(u_P(x_P) - u_R(x_P)\right)\right] - C[x_R]\right)$$

$$0 \equiv \frac{d\pi}{dx_R} = u'_R(x_R) - C'[x_R]$$

$$\frac{d\pi}{dx_P} = u'_P(x_P) + u'_P(x_P) - u'_R(x_P) - C'[x_P$$

$$u'_R(x_R) = C'[x_R]$$

$$u'_P(x_P) = \left(u'_R(x_P) - u'_P(x_P)\right) + C'[x_P]$$


$P_R=10-q_R$
$P_P=6-q_P$
10
6
6
10

Perfect information

- Calculate the optimal price-quantity offer for

1. the poor
2. the rich

Imperfect information

- Calculate the optimal price-quantity offer for

3. the poor
4. the rich
5. Calculate the information rent of the rich

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MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

Národohospodářská fakulta VŠE v Praze