Game Theory

Tomáš Miklánek

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

Lecturer

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  - Tomáš Miklánek (tomas.miklanek@vse.cz)

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- Research interests:
  - Experimental and behavioral economics
  - Social preferences

Basic logistics

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- Lectures: Monday 16:15 – 17:45, 18:00- 19:30, room SB 212

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Suggested topics

1. Intro: Notation, Definitions, Math review
2. Dominant, mixed, iterated strategies
3. Applications of BR
4. Location models
5. Sequential moves
6. Game theory in theory of the firm
7. Financial markets and price discrimination
8. Repeated games
9. Bargaining
10. Bayesian games
11. Auctions (3-4 classes)

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- **No class on Apr 2**

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  - Midterm Exam 35%
  - Final Exam 45% (non-cumulative)
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- Midterm (March 12)
  - **mandatory**
  - non-participation possible if supported by medical evidence (then it is written right after your final exam)
- Final exam (last week of the semester? + one or two exam dates in the exam period)

Readings

- Carmichael, F. (2005). A guide to game theory. Pearson Education.
- Watson, J. (2013). Strategy: an introduction to game theory.
- Osborne, M. J. (2004). An introduction to game theory (Vol. 3, No. 3). New York: Oxford university press.
- Varian, H. R. (1992). Microeconomic analysis. Norton & Company.
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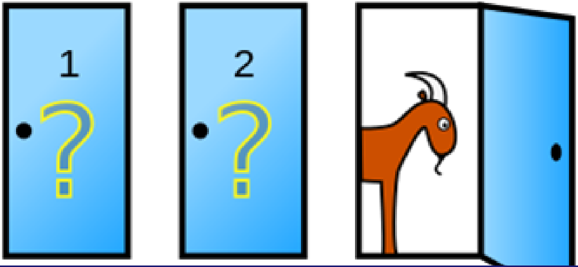
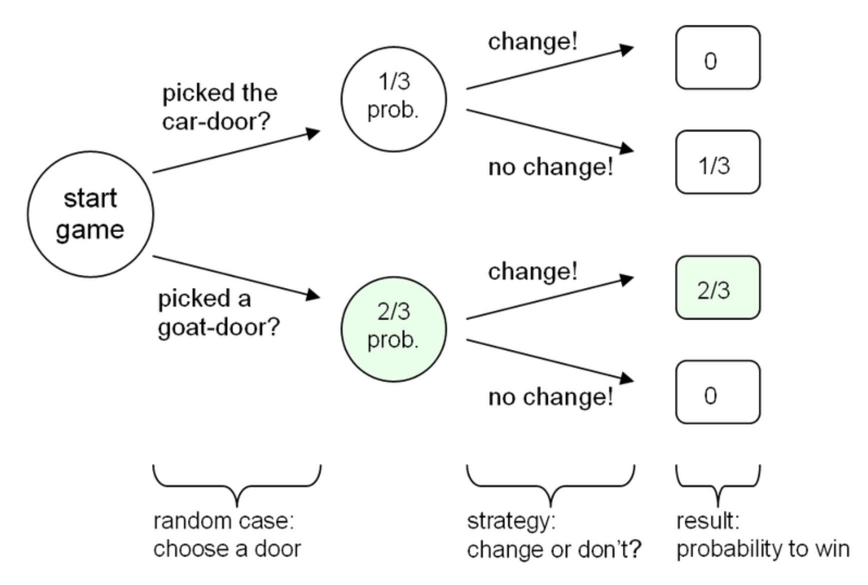
- Topic related readings - will be announced with the topics

Monty Hall problem

- There are 3 doors, behind which are two goats and a car.
- You pick a door (call it door A). You're hoping for the car of course.
- Monty Hall, the game show host, examines the other doors (B & C) and always opens one of them with a goat (Both doors might have goats; he'll randomly pick one to open)
- Here's the game: *Do you stick with door A (original guess) or switch to the other unopened door? Does it matter?*



Monty Hall Solution



Instructions

Instructions

- Please, write on the paper (legibly) any number **between 0 and 100**.
- If your number is the closest to the **1/2 of the average** of all numbers in the group, you are the winner.
- Prize is 2 points from this course.

Beauty contest

- Giving an estimate higher than $\frac{1}{2}$* the average is **dominated**
- There is only one non-dominated strategy
- Everybody giving an estimate of 0! If a strategy is non-dominated for all players, it is a **Nash-equilibrium**

Game theory

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- Applications
    - The study of oligopolies (industries containing only a few firms)
    - The study of cartels; e.g. OPEC
    - The study of externalities; e.g. using a common resource such as a fishery.
    - The study of military strategies.
    - The study of individual economic interactions.

What is a Game?

1. List of players
2. Actions
   - complete description of what the players can do
3. Knowledge description
   - what the players know when they act
4. Outcomes
   - a specification of how the actions lead to outcomes
5. Preferences over outcomes

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- *Apply it to the guessing game we played.*

Chess?

Chess?

1. There are two players
2. The players alternate in moving pieces on the game board, subject to rules about what moves can be made in any given configuration of the board
3. Players observe each other's moves, so each knows the entire history of play as the game progresses
4. A player who captures the other player's king wins the game, and otherwise, a draw is declared
5. Players prefer winning over a draw and a draw over losing

Extensive form of a game

- The extensive form is a way of describing a game using a game tree
- It's simply a diagram that shows that choices are made at different points in time (corresponding to each node).
- The payoffs are represented at the end of each branch.

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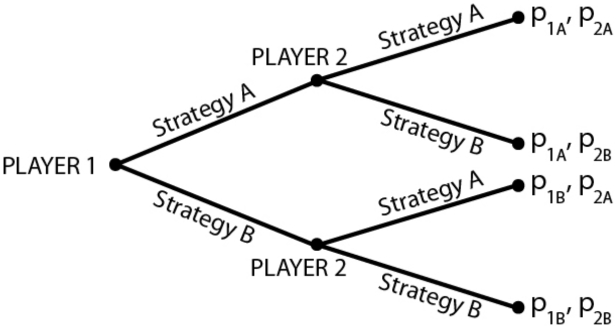

Extensive form - example

- Two firms share the market, colluding and maintaining high prices.
- Each firm can decide to stop colluding and start a price war, in order to increase their market share, even force the other to quit the market.
- Firm 1 can either keep colluding with firm 2, or start a price war.
- If firm 1 decides to keep colluding, firm 2 will need to make a decision.
- If they both agree to collude, they will get 5,5.
- However, if one of them decides to start a price war, the set of payoffs will be either 4,3 or 3,4, depending on which one starts the war (and therefore acquires a greater market share).

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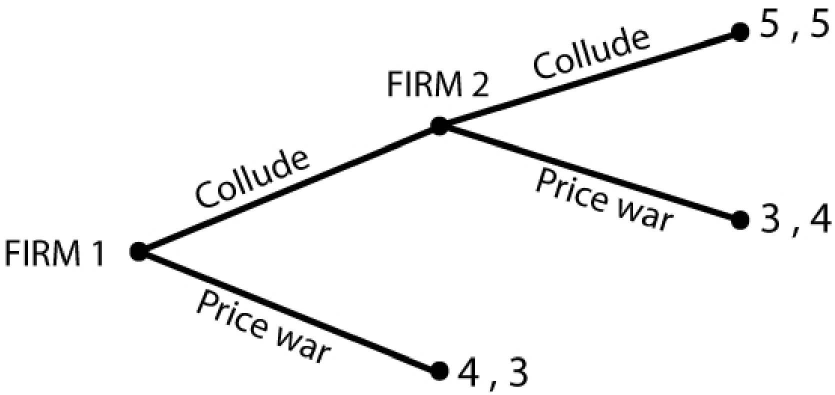
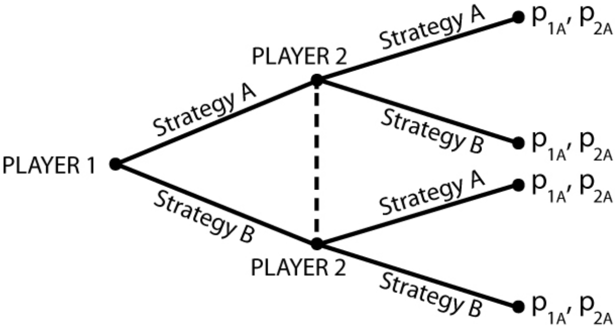
- However, if one of them decides to start a price war, the set of payoffs will be either 4,3 or 3,4, depending on which one starts the war (and therefore acquires a greater market share).
- *Can you draw extensive form of this game? (For simplicity, assume that Firm 1 is the first decision-maker)*

Example



Simultaneous games

- It's worth mentioning that the extensive form can be used also to describe simultaneous games, by using information sets.
- These information sets, usually represented by a dashed line uniting two nodes or by encircling them, mean that the player does not know in which node he is, which implies imperfect information



Normal form

- The normal form is a matrix representation of a simultaneous game.
- For two players, one is the "row" player, and the other, the "column" player.
- Each rows or column represents a strategy and each box represents the payoffs to each player for every combination of strategies.

Example of a Two- Player Game

- The players are called A and B.
- Player A has two strategies, called “Up” and “Down”.
- Player B has two strategies, called “Left” and “Right”.
- The table showing the payoffs to both players for each of the four possible strategy combinations is the game’s payoff matrix.

Example of a Two- Player Game

- This is the game's payoff matrix.

	Player 2		
Player 1		Left	Right
	Up	(3, 9)	(1, 8)
	Down	(0, 0)	(2, 1)

- Player A's payoff is shown first.
- Player B's payoff is shown second.

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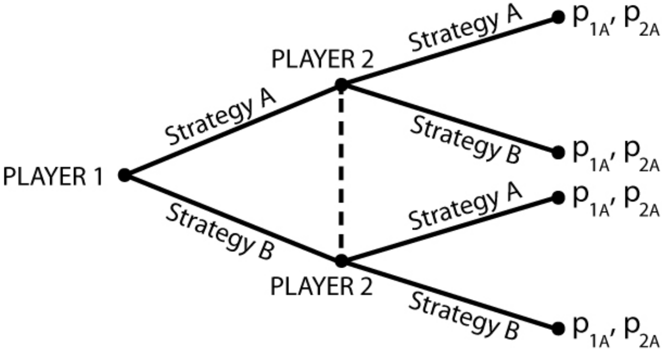
- *What plays are we likely to see for this game?*

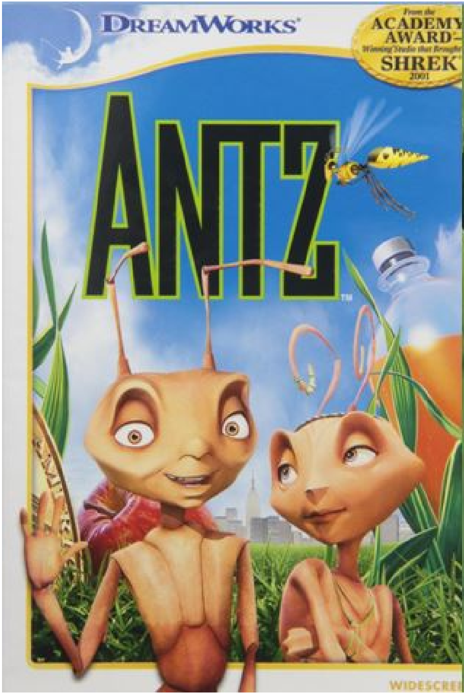
Normal form example



- *Can you express this game in the normal form?*

Solution

	Player 2		
Player 1		Strategy A	Strategy B
	Strategy A	(p_{1A}, p_{2A})	(p_{1A}, p_{2B})
	Strategy B	(p_{1B}, p_{2A})	(p_{1B}, p_{2B})


DREAMWORKS
ACADEMY
AWARD-
SHREK
2001
ANTZ


from the creators of "toy story"
Walt Disney Pictures
Presents
A PIXAR Film
a
bug's
life

Another problem

Represent the following game in an extensive form. Firm A decides whether to enter firm B's industry. Firm B observes this decision. If firm A enters, then the two firms simultaneously decide whether to advertise. Otherwise, firm B alone decides whether to advertise. With two firms in the market, the firms earn profits of \$3 million each if they both advertise and \$5 million if they both do not advertise. If only one firm advertises, then it earns \$6 million and the other earns \$1 million. When firm B is solely in the industry, it earns \$4 million if it advertises and \$3.5 million if it does not advertise. Firm A earns nothing if it does not enter.

Review: optimization

Maximizing function of one variable:

$$\max_x f(x) \quad \Rightarrow \quad \frac{\mathrm{d}f(x)}{\mathrm{d}x} = 0$$

- one equation in one variable
- intuition: at maximum, slope of the function must be zero

Maximizing function of several variables:

$$\max_{x_1,\ldots,x_n} f(x_1,\ldots,x_n) \quad \Rightarrow \quad \frac{\partial f(x_1,\ldots,x_n)}{\partial x_i} = 0, i = 1,\ldots,n$$

- n equations in n variables
- at maximum, slope in *each direction* must be zero

Review: optimization

- the above known as *first-order necessary conditions*
- *necessary*: each maximum satisfies them, but not every point satisfying them is a maximum
  - some may be minima
  - some may be neither (try $f(x) = x^3$)
- *second-order sufficient conditions* ensure that we have indeed found maximum or minimum
  - $\frown$ concave function, $f''(x) < 0$: maximum
  - $\smile$ convex function, $f''(x) > 0$: minimum
  - can be generalized for multiple variables
- these conditions find *local* maxima or minima; generally, a function can have several

Review: optimization

Maximization subject to constraint:

$$\max_{x_1,\ldots,x_n} f(x_1,\ldots,x_n) \text{ subject to } g(x_1,\ldots,x_n)=0$$

- form a Lagrange function

$$L(x_1,\ldots,x_n,\lambda)=f(x_1,\ldots,x_n)-\lambda\cdot g(x_1,\ldots,x_n)$$

  - λ is the Lagrange *multiplier*
- first order conditions

$$\frac{\partial L(x_1,\ldots,x_n,\lambda)}{\partial x_i}=0, i=1,\ldots,n$$

- together with the constraint, $n+1$ equations in $n+1$ variables

Review: optimization

- multiple constraints: everybody gets a multiplier

$$\max_{x_1,\ldots,x_n} f(x_1,\ldots,x_n) \text{ s. t. } g_j(x_1,\ldots,x_n) = 0, j = 1,\ldots,m$$

$$L(x_1,\ldots,x_n,\lambda_1,\ldots,\lambda_m) = f(x_1,\ldots,x_n) - \sum_{j=1}^{m} \lambda_j \cdot g_j(x_1,\ldots,x_n)$$

$$\frac{\partial L(x_1,\ldots,x_n,\lambda_1,\ldots,\lambda_m)}{\partial x_i} = 0, i = 1,\ldots,n$$

  - together with constraints, $n+m$ equations in $n+m$ variables
- second-order conditions
  - bit trickier with constraints
  - for our purposes: if constraints $g_j()$ are *linear*, the usual concave/convex rule for objective function still holds

Review examples 1

- *Suppose that the firm uses only one factor of production (input), x. Production function is given by $f(x) = \ln x$. The firm can sell each unit of its production for 5 EUR. Each unit of input costs 2 EUR. What will be the profit maximizing level of inputs and what will be the corresponding profit of the firm?*

Review examples 1

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- *We need to enclose a rectangular field with a fence. We have 500 feet of fencing material and a building is on one side of the field and so won't need any fencing. Determine the dimensions of the field that will enclose the largest area.*

Review examples 2

- *Imagine, that you are bidding simultaneously in two* ***first price auctions*** *auctions against one computer in each of them. You are endowed with 8 EUR which can be used for your bids (sum of your bids). Higher bid in each auction wins the prize and the winner needs to pay his/her own bid.*
- *Computerized bidder in the first auction chooses his bid from interval 0-10 with equal probability of each value (uniform distribution). Prize from this first auction is worth 12 EUR to you.*
- *Computerized bidder in the second auction chooses his bid from interval 0-5 with equal probability of each value (uniform distribution). Prize from this second auction is worth 7 EUR to you.*
- *How would you divide your 8 EUR between these two auctions in order to maximize your expected payoff?*

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- *How would you divide your 8 EUR between these two auctions in order to maximize your expected payoff?*

- *Can you come up with examples of situations resembling the described problem?*

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