Game Theory: Auctions

Tomáš Miklánek

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

Jar with money

Jar with money

- You are going to place your bids for the jar full of coins.
- You will write your bid on the piece of paper together with your name.
- A bidder with the highest bid wins the jar a pays me his/her bid.
- Amount of money in the jar will not be announced publicly.
- By submitting the bid, you agree with these rules.

The winner's curse

- The estimates are correct, on average

What is an Auction?

1. A public sale in which property or merchandise are sold to the highest bidder.
2. A market institution with explicit rules determining resource allocation and prices on the basis of bids from participants.
3. Games: The bidding in bridge

[Latin: auctiō, auctiōn – from auctus, past participle of augēre, to increase]

Examples of auctions

- Internet
  - EBay.com, Amazon.com, airline companies
- Government
  - Treasury Bills, mineral rights (e.g. oil fields), assets (e.g. privatization)
  - Electromagnetic spectrum
- Stocks
  - IPO: Google, Repurchases
- Procurement auctions/Subcontracting
  - Automobiles: Valeo (GM, Daimler-Chrysler, etc..), Visteon (Ford)

Examples of auctions

- Internet
  - EBay.com, Amazon.com, airline companies
- Government
  - Treasury Bills, mineral rights (e.g. oil fields), assets (e.g. privatization)
  - Electromagnetic spectrum
- Stocks
  - IPO: Google, Repurchases
- Procurement auctions/Subcontracting
  - Automobiles: Valeo (GM, Daimler-Chrysler, etc..), Visteon (Ford)
- *Are auctions a competitive market?*
- *Are auction outcomes pareto-optimal?*

Spectrum auctions

- UK 2000: EUR 38.3 billions
- Germany 2008: EUR 50.8 billions
- CR 2016: EUR 100 millions

Ways to Categorize Auctions

- Number of objects
  - We focus on single object auctions, but ideas apply to multi-unit auctions
- Role of information / Type of valuation
  - Private value vs. Common value
- Rules of auction – Ascending (Ebay or Sotheby's), first-price (oil tracts), etc..

Auction Formats

Open outcry auction

- Ascending Auctions (English)
  - Auctioneer announces ever increasing prices to solicit bids. Continues until only one person left in.
- Descending Auctions (Dutch)
  - Auctioneer announces decreasing prices until someone puts up their hand.

Sealed bid (closed) auction

- Everyone puts bids in envelopes and gives to seller at the same time.
  - First price, Second price

Example: Art Auction

- 3 individuals with private values of \$60,000, \$70,000 and \$80,000 compete for a work of art
- What will happen in the ascending auction?
- What if we used a descending auction?

Ascending vs. Descending Auctions

- Do bidders bid their true valuation in ascending or descending auctions?
- Ascending: Yes - bidding up to your true valuation is the dominant strategy
- Descending: No - each bidder "shades down" his bid.
  - Risk averse bidders shade less

More Bidders Matters

- What if there were more bidders, say with private values of \$60,000, \$61,000, ... , \$79,000, \$80,000?
- More revenue in both ascending and descending
- Ascending: Second-highest value is now \$79,000
- Descending: Less incentive to shade bid since it's more likely someone else will jump in with more bidders

Bidding for an Oil Block

- Five People will bid, in one-time first-price auction. The top bid gets the block
- The true value of the field $= X^{True}$, but no bidder knows what X^{True} is. It will be revealed (drilled) after the bidding.
- Each bidder hires his/her own consultant to give an expert estimate of the value.

How Good Are the Experts?

- Oil experts can estimate reserves only with some error.
- The distribution of these types of oil estimates tends to be the following:

$$\text{Consultant's estimate} = \begin{cases} X^{True} + 40 & \textit{with prob.}\ 1/5 \\ X^{True} + 20 & \textit{with prob.}\ 1/5 \\ X^{True} & \textit{with prob.}\ 1/5 \\ X^{True} - 20 & \textit{with prob.}\ 1/5 \\ X^{True} - 40 & \textit{with prob.}\ 1/5 \end{cases}$$

- Given your consultant's estimate, how much will you bid?

Winner's Curse

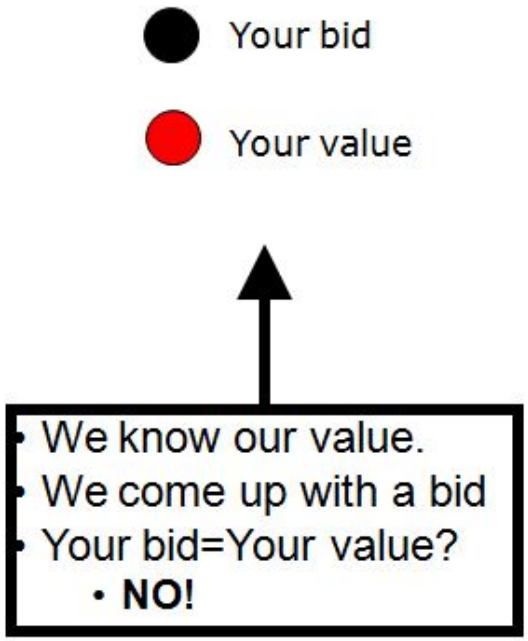
- Winner's Curse = Tendency to overbid due to the fact that bidder with highest estimate (or signal) will win
  - An issue in all common value auctions, worse with more bidders participating
  - To avoid the curse, simply assume your signal is the most overly optimistic when bidding

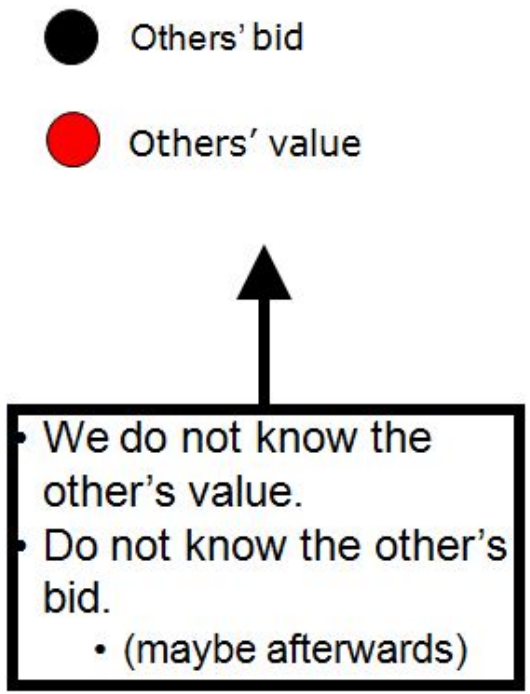
First Price Auction

First Price Auction

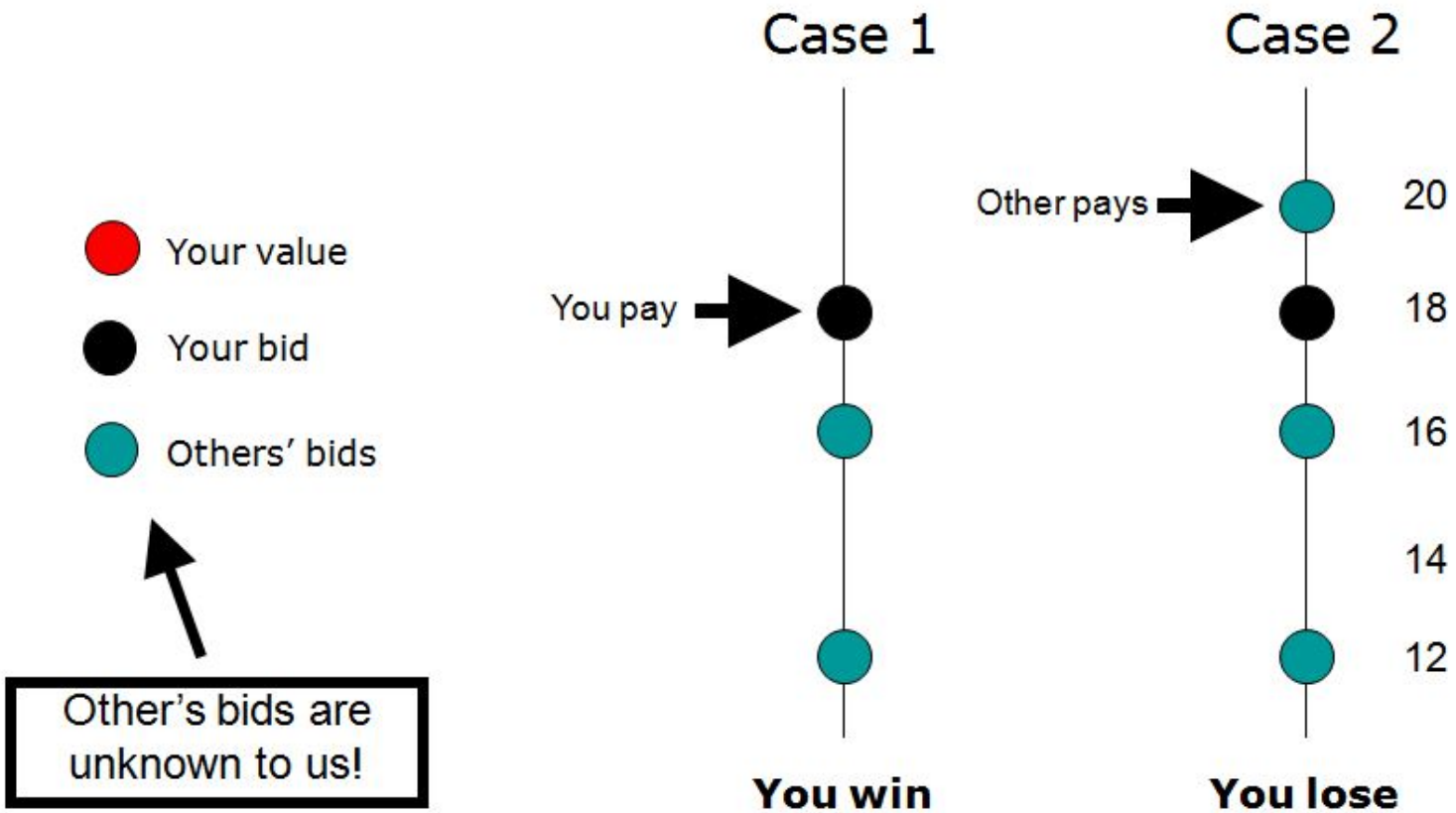
- Everybody submits her/his bid.
- A bidder with the highest bid wins the price.
- *What is the best strategy?*

Bidding your valuation?

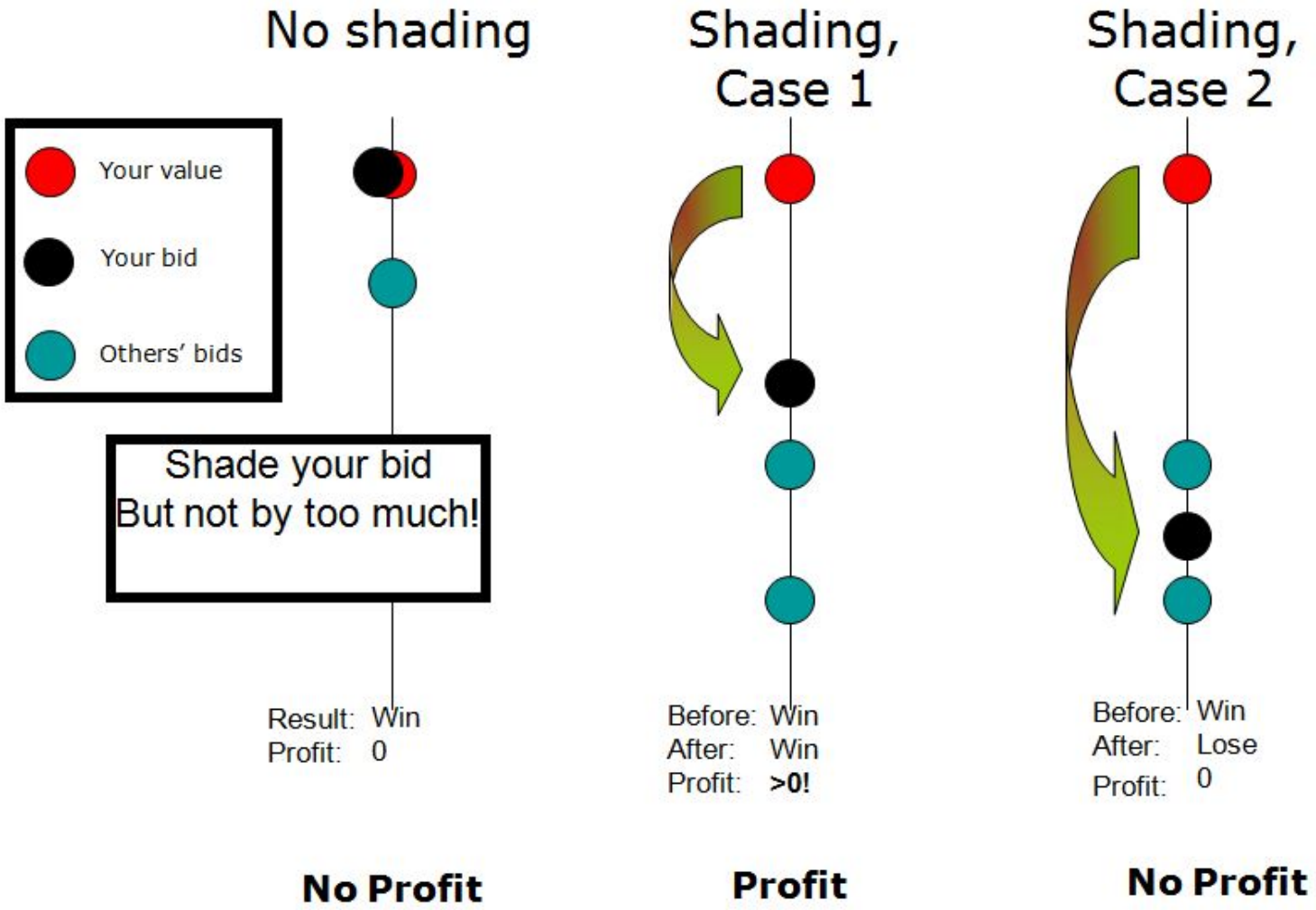




Outcomes


Your value
Your bid
Others' bids
Other's bids are unknown to us!
Case 1
You pay
You win
Case 2
Other pays
20
18
16
14
12
You lose

Bid shading


No shading
Your value
Your bid
Others' bids
Shade your bid
But not by too much!
Result: Win
Profit: 0
No Profit
Shading, Case 1
Before: Win
After: Win
Profit: >0!
Profit
Shading, Case 2
Before: Win
After: Lose
Profit: 0
No Profit

FPA

- First price auction presents tradeoffs
  - bidding your valuation: no surplus
- Lower your bid below your valuation
  - Smaller chance of winning, lower price
- Bid shading
  - Depends on the number of bidders
  - Depends on your information
  - Optimal bidding strategy is complicated!

Bidding in FPA

- Imagine that one good is sold in an FPA auction there are 2 bidders: bidder X and bidder Y.
- You are bidder X. You do not know the value or the bid of bidder Y for the good, but you know
  - the value of Y will be between 0 and 1, with all values having an equal probability (a uniform distribution).

Bidding in FPA

- Imagine that one good is sold in an FPA auction there are 2 bidders: bidder X and bidder Y.
- You are bidder X. You do not know the value or the bid of bidder Y for the good, but you know
  - the value of Y will be between 0 and 1, with all values having an equal probability (a uniform distribution).
- Y doesn't know the value or the bid of bidder X, but Y knows
  - the value of X will be between 0 and 1, with all values having an equal probability (a uniform distribution).

Bidding in FPA

- You also know that Y has a strategy to bid $\frac{1}{2}$ his value, thus: $b_y(v_y) = \frac{1}{2}v_y$.

Bidding in FPA

- You also know that Y has a strategy to bid $\frac{1}{2}$ his value, thus: $b_y(v_y) = \frac{1}{2}v_y$.
- So for example, if Y happens to have a value of 1 for the good, what will Y bid?
  - 0.5
- If Y happens to have a value of 0.8 for the good, what will Y bid?
  - 0.4

Bidding in FPA

- You also know that Y has a strategy to bid $\frac{1}{2}$ his value, thus: $b_y(v_y) = \frac{1}{2}v_y$.
- So for example, if Y happens to have a value of 1 for the good, what will Y bid?
  - 0.5
- If Y happens to have a value of 0.8 for the good, what will Y bid?
  - 0.4
- What is your optimal bidding strategy $b_x(v_x)$ (the bid you would make as a function of the value for the good for you)?

Optimal bidding strategy

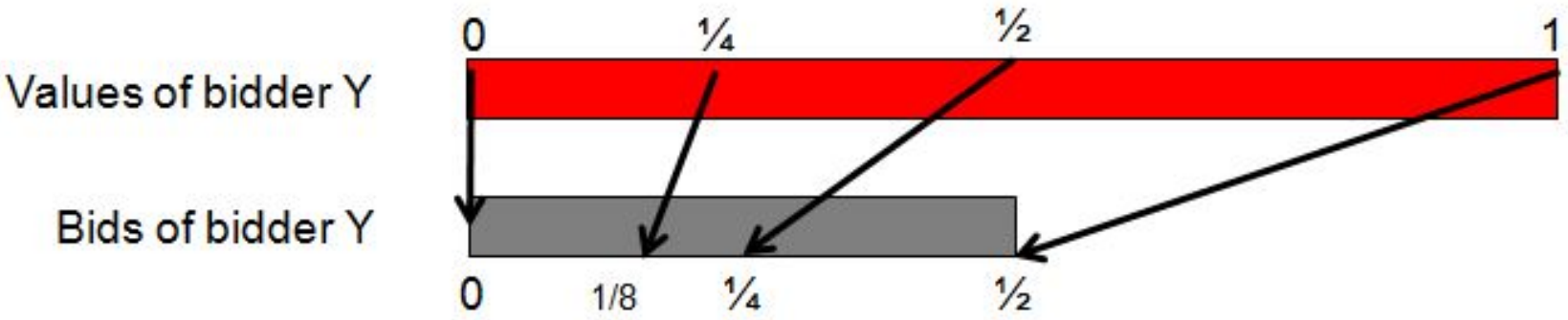
- My profit function is given by:
- PROFIT= (GAIN OF WINNING) * (PROBABILITY OF WINNING)

Optimal bidding strategy

- My profit function is given by:
- PROFIT= (GAIN OF WINNING) * (PROBABILITY OF WINNING)
- GAIN OF WINNING= $(v_x - b_x)$

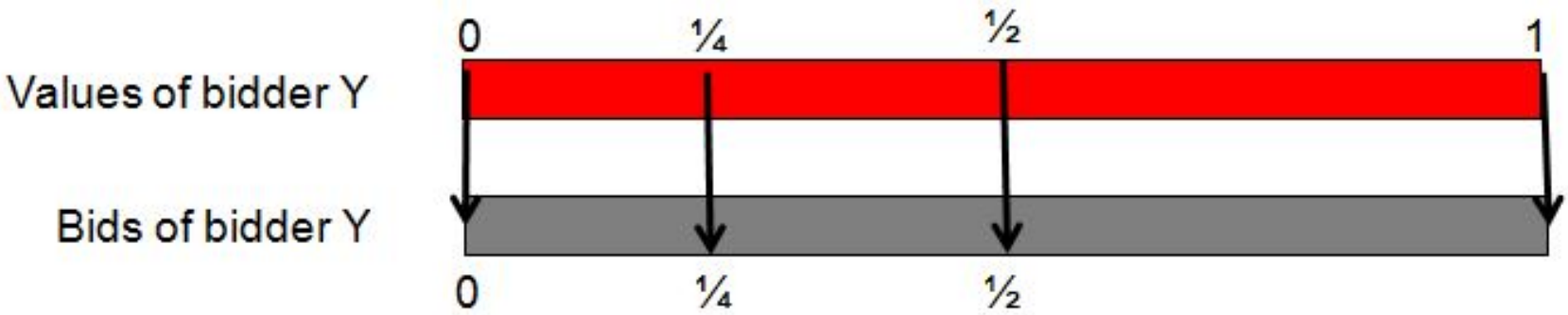
Probability?

- Y has a strategy to bid ½ his value, thus: $b_y(v_y) = ½\ v_y$.



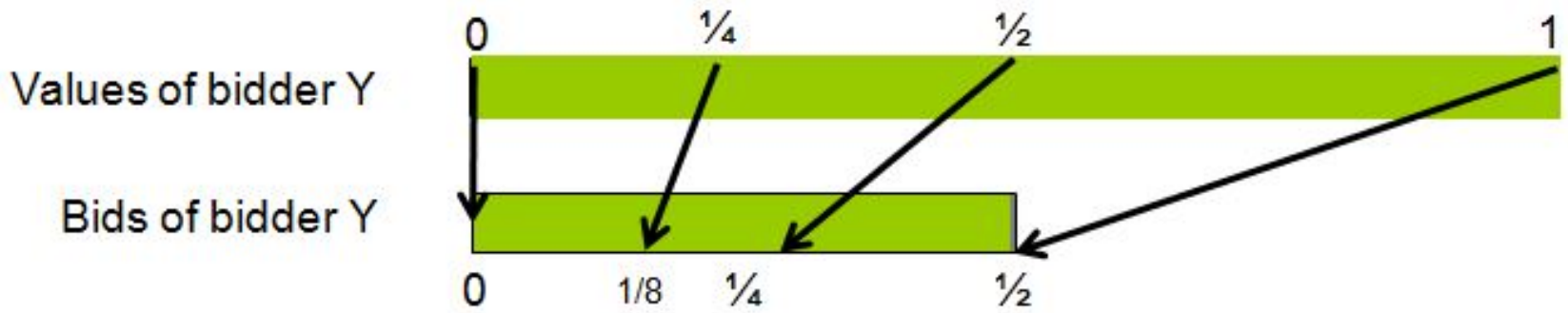
What if...

- Y has a strategy to bid his value, thus: $b_y(v_y) = v_y$.



If I bid b_x, my chance of winning is my bid: b_x

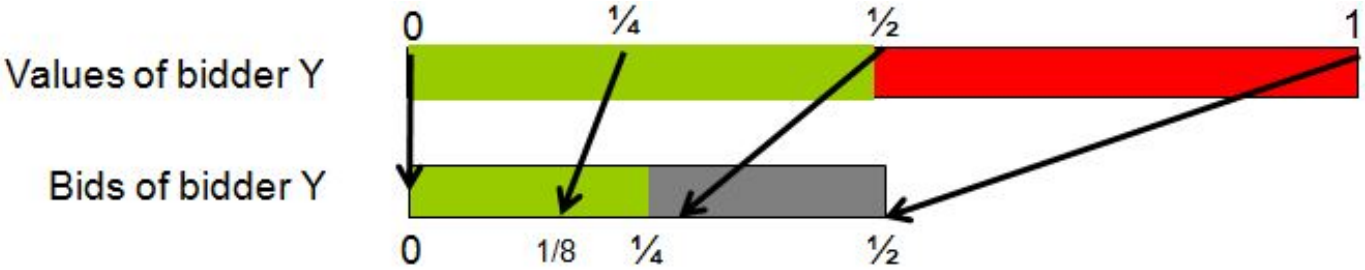
Y bids half of his value...



What is Pr(WINNING when my bid is bx)?

- What if I bid ½?
  - **pr(winning)=100%**
    - Y will never bid more than ½ -> I win for sure

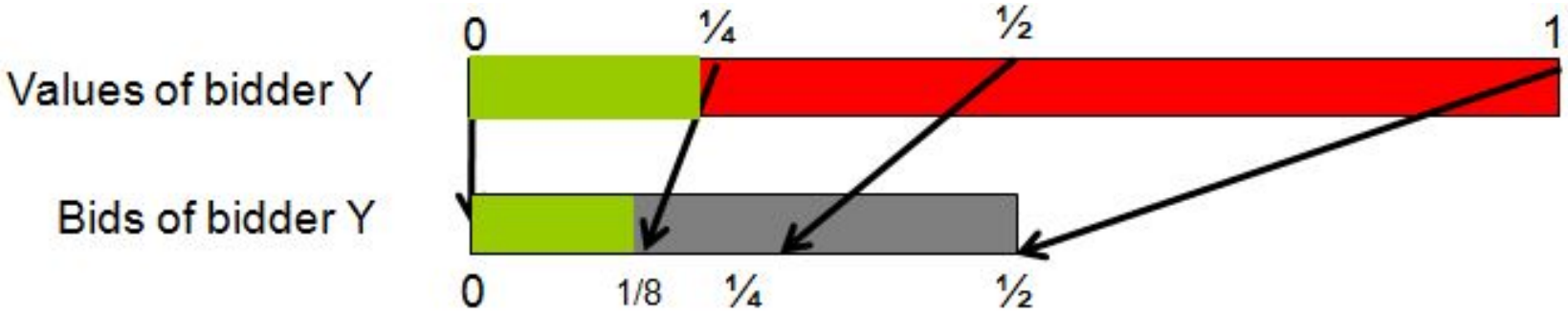
Y bids half of his value...



What is Pr(WINNING when my bid is bx)?

- What if I bid ½?
  - **pr(winning)=100%**
    - Y will never bid more than ½ -> I win for sure
- What if I bid ¼?
  - **pr(winning)=50%**
    - ½ chance that the bid of Y will be higher and ½ that it will be lower

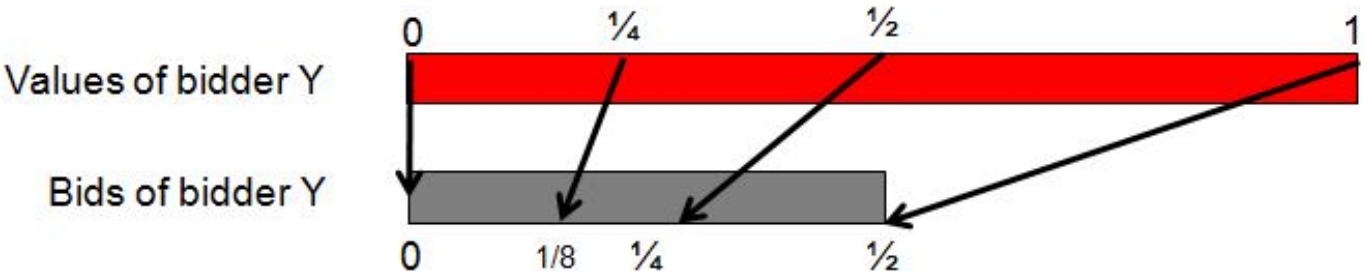
Y bids half of his value...



- What if I bid 1/8?
  - **pr(winning)=25%**
    - 75% chance that the bid of Y will be higher and 25% that it will be lower
- What is the general rule?
  - My chance of winning is two times my bid:

    2 bx

Optimal bidding



- Thus my profit function is:
- **PROF= (GAIN OF WINNING) * (PROBABILITY OF WINNING)**
- **PROF= (vx - bx) * 2 bx**
- Differentiate towards **bx**, gives the First Order Condition (FOC):
- **FOC: 2 (vx - bx) - 2 bx =0**
- **<-> 2 vx - 4 bx =0**
- **<-> bx = ½ vx**

Nash

- Is this a Nash equilibrium?
  - $b_y = \frac{1}{2} v_y$
  - $b_x = \frac{1}{2} v_x$
- Given that me (X) and Y have a value uniformly distributed on [0,1]

Nash

- Is this a Nash equilibrium?
  - $b_y = \frac{1}{2} v_y$
  - $b_x = \frac{1}{2} v_x$
- Given that me (X) and Y have a value uniformly distributed on [0,1]
- I maximize my profit given that Y has the strategy of bidding half its value Y maximizes its profit given that I have the strategy of bidding half its value
- YES! Is a Nash equilibrium.

If Y bids differently...

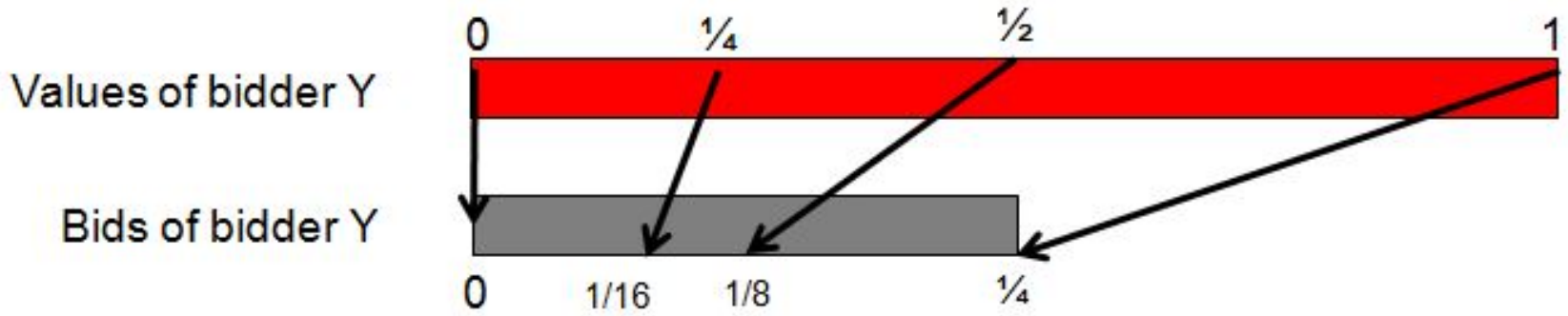
- You also know that Y has a strategy to bid $\frac{1}{4}$ his value, thus: $b_y(v_y) = \frac{1}{4}v_y$.
- So for example, if Y happens to have a value of 1 for the good, what will Y bid?
  - 0.25,
- If Y happens to have a value of 0.8 for the good, what will Y bid?
  - 0.2
- What is your optimal bidding strategy $b_x(v_x)$ (the bid you would make as a function of the value for the good for you)?

Optimal bidding

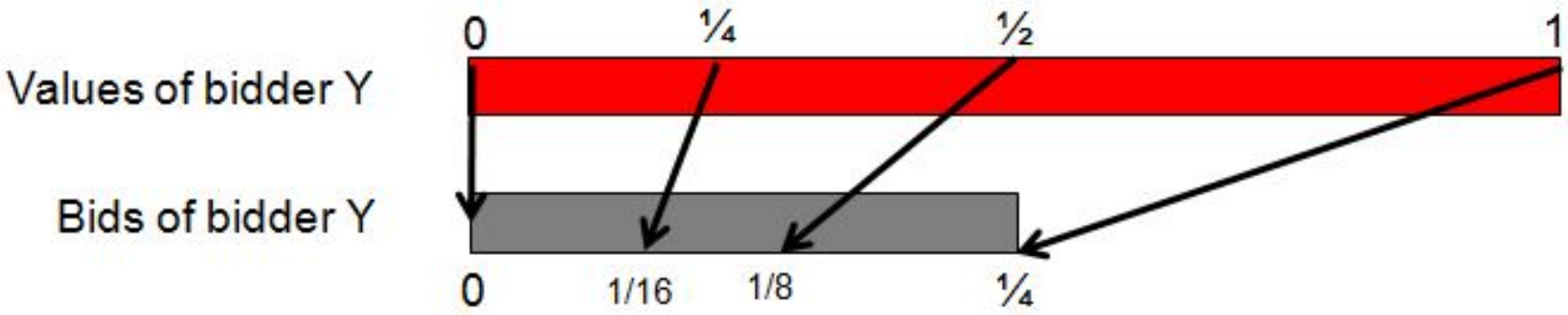
- My profit function is given by: PROFIT= (GAIN OF WINNING) * (PROBABILITY OF WINNING)
- GAIN OF WINNING= $(v_x - b_x)$

Probability of winning

- Y has a strategy to bid ¼ his value, thus: $b_y(v_y) = ¼\ v_y$.



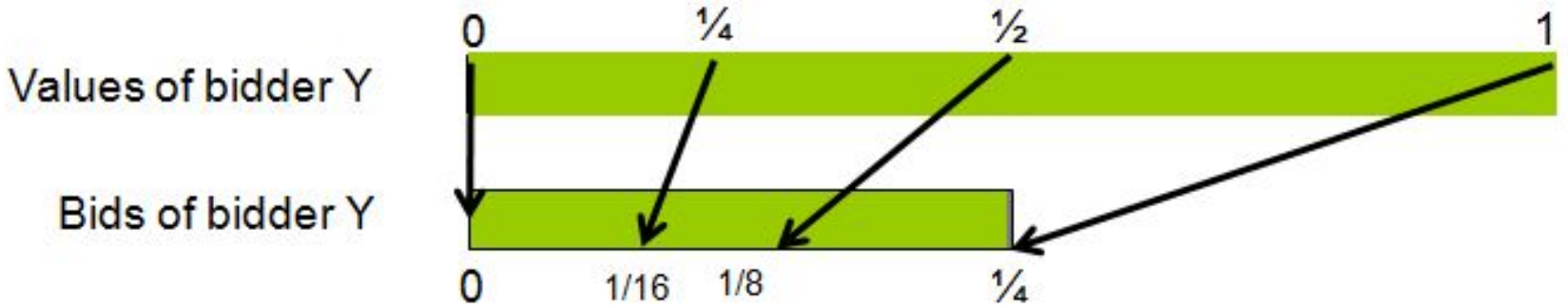
Probability of winning



What is Pr(WINNING when my bid is bx)?

- What if I bid ½?
  - **pr(winning)=100%**
    - Y will never bid more than ¼ -> I win for sure

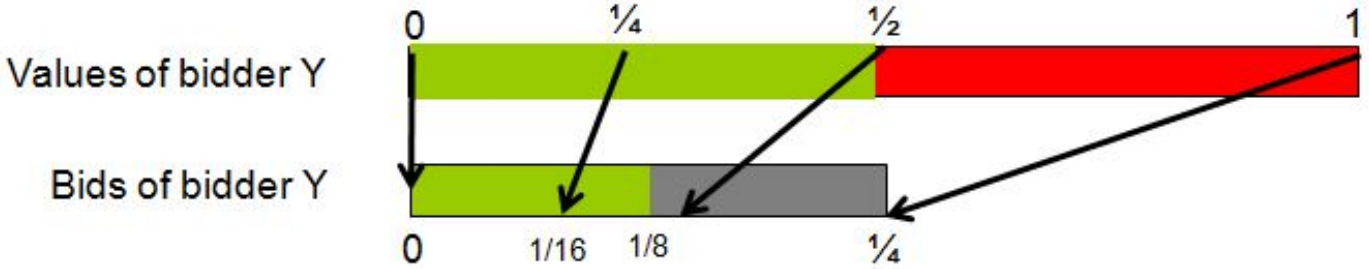
Probability of winning



What is Pr(WINNING when my bid is bx)?

- What if I bid ¼?
  - **pr(winning)=100%**
    - Y will never bid more than ¼ -> I win for sure

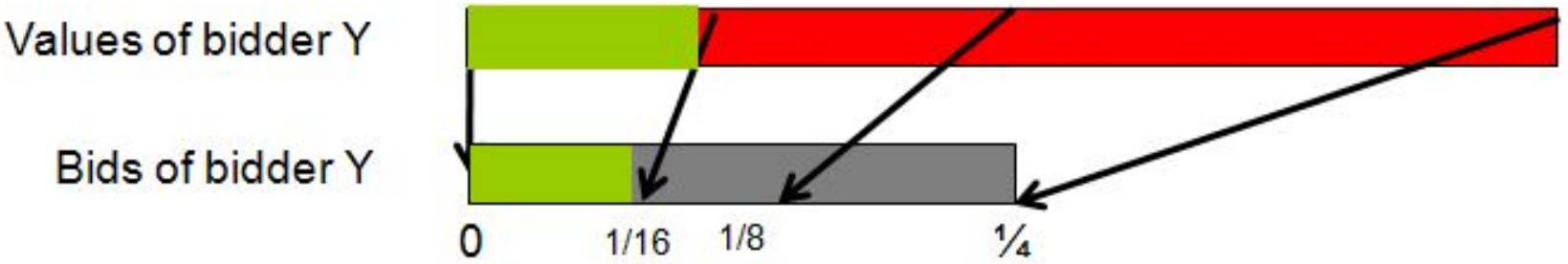
Probability of winning



What is Pr(WINNING when my bid is bx)?

- What if I bid ¼?
  - **pr(winning)=100%**
    - Y will never bid more than ¼ -> I win for sure
- What if I bid 1/8?
  - **pr(winning)=50%**
    - ½ chance that the bid of Y will be higher and ½ that it will be lower

Probability of winning



- What if I bid 1/16?
  - **pr(winning)=25%**
    - 75% chance that the bid of Y will be higher and 25% that it will be lower
- What is the general rule?
  - My chance of winning is four times my bid: 4 bx

Optimal bidding

- Thus my profit function is: PROF= (GAIN OF WINNING) * (PROBABILITY OF WINNING)
- PROFIT= $(v_x - b_x) * 4b_x$
- Differentiate wrt b_x, gives the First Order Condition (FOC):

Optimal bidding

- Thus my profit function is: PROF= (GAIN OF WINNING) * (PROBABILITY OF WINNING)
- PROFIT= $(v_x - b_x) * 4b_x$
- Differentiate wrt b_x, gives the First Order Condition (FOC):
- FOC: $4(v_x - b_x) - 4b_x = 0$
- $4v_x - 8b_x = 0 \implies b_x = \frac{1}{2}v_x$

Is this Nash Equilibrium?

- $b_y = \frac{1}{4} v_y$ and $b_x = \frac{1}{2} v_x$
- Given that me (X) and Y have a value uniformly distributed on [0,1]

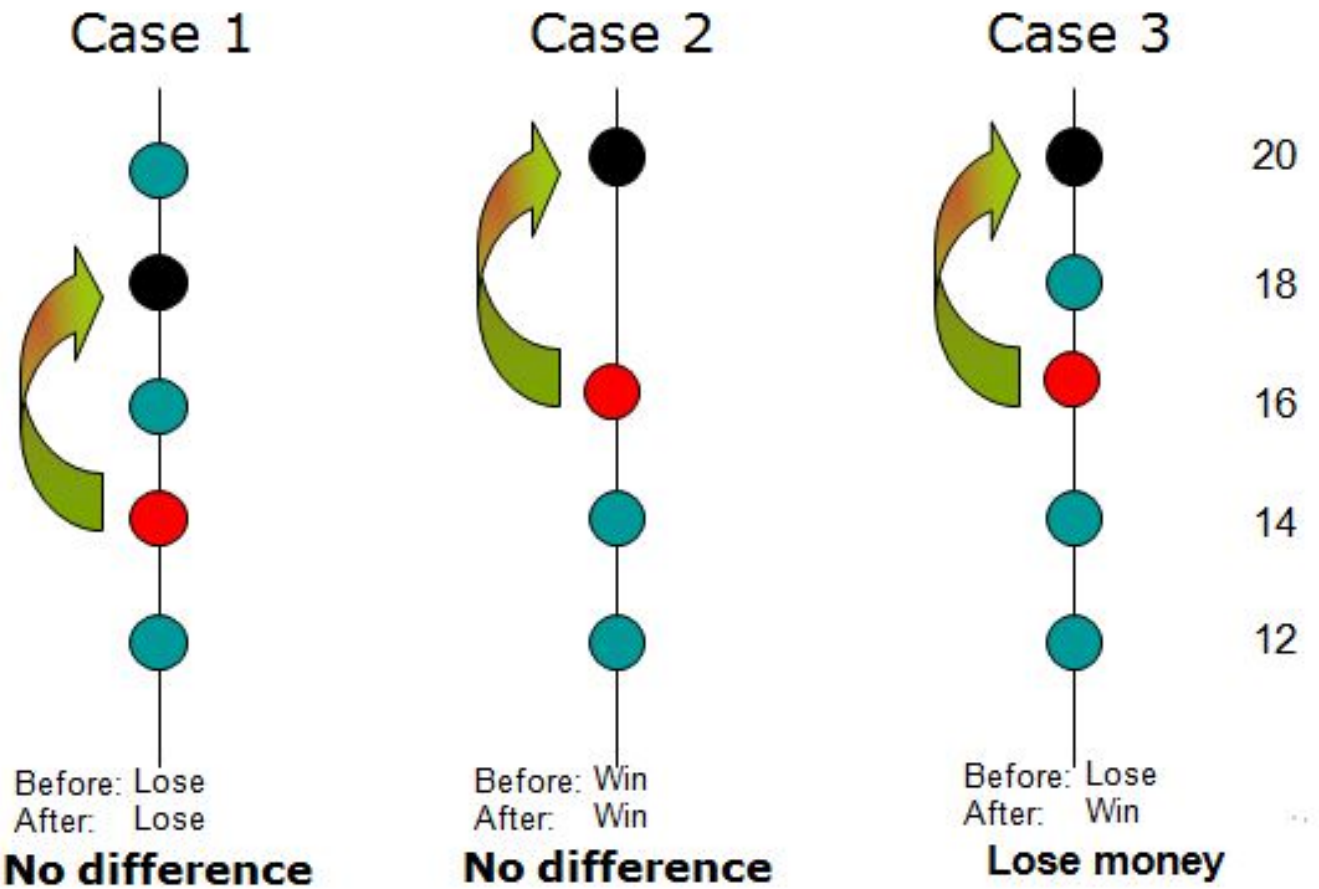
Is this Nash Equilibrium?

- $b_y = \frac{1}{4} v_y$ and $b_x = \frac{1}{2} v_x$
- Given that me (X) and Y have a value uniformly distributed on [0,1]
- I maximize my profit given that Y has the strategy of bidding half its value (for values lower than 1/2).
- Y maximizes its profit given that I have the strategy of bidding half its value?
  - No! This is not a Nash equilibrium.

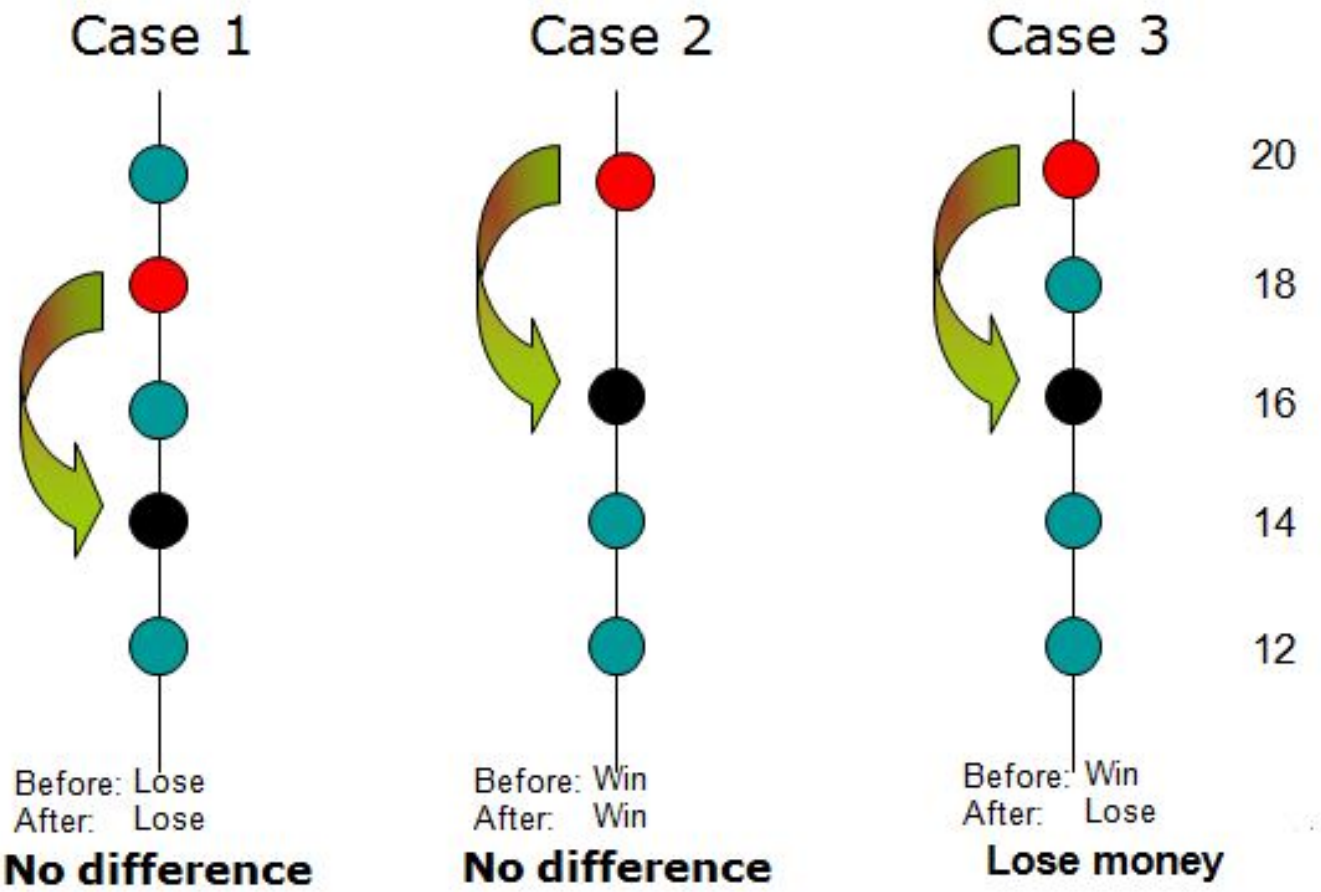
Second Price Auction

- An auction in which the bidder who submitted the highest bid is awarded the object being sold and pays a price equal to the second highest amount bid.
- Alternately, in a procurement auction, the winner is the bidder who submits the lowest bid, and is paid an amount equal to the next lowest submitted bid.
- The theoretical nicety of second price auctions, first pointed out by William Vickrey, is that bidding one's true value is a dominant strategy.

Bidding higher than my valuation



Bidding lower than my valuation



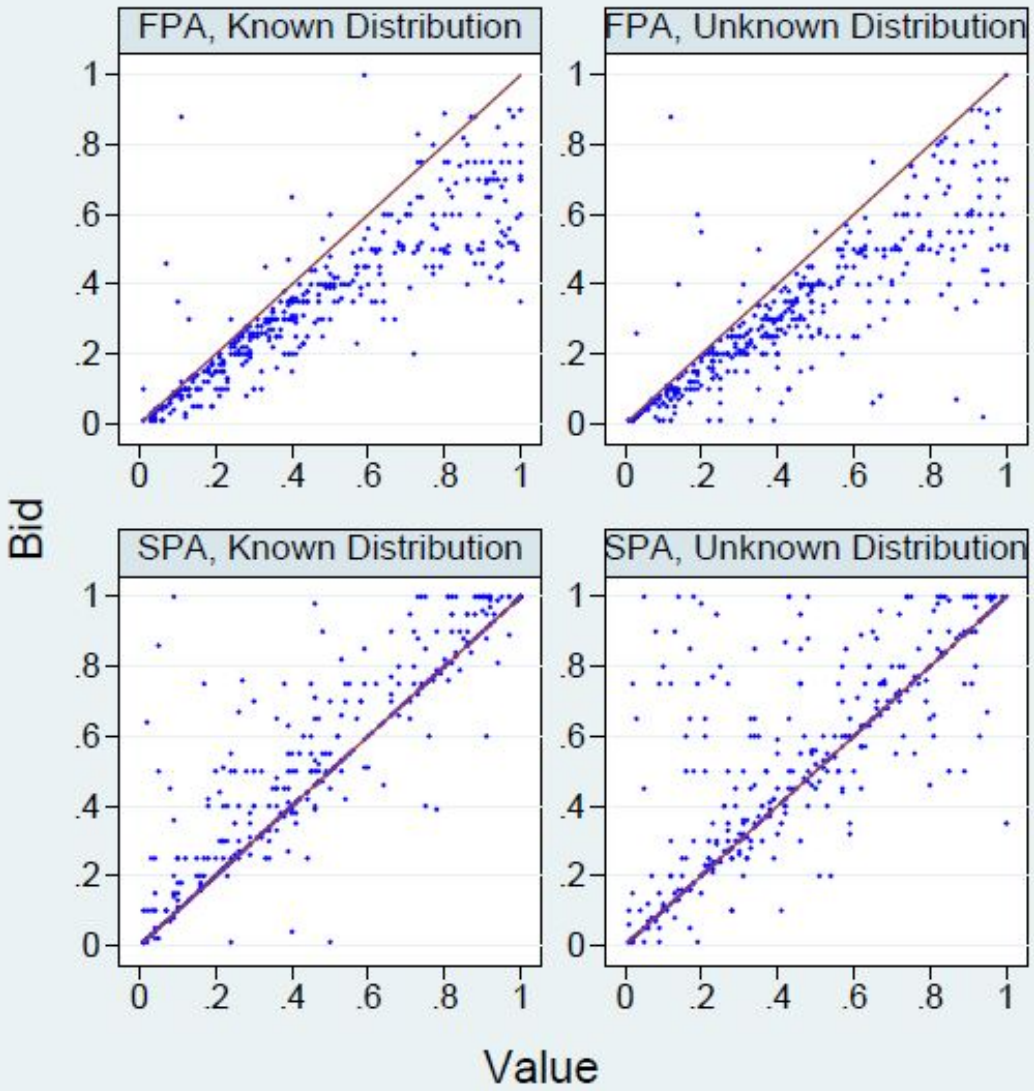
Best strategy

- In a second price auction, always bid your true valuation
- Winning bidder's surplus
  - Difference between the winner's valuation and the second highest valuation
- Surplus decreases with more bidders

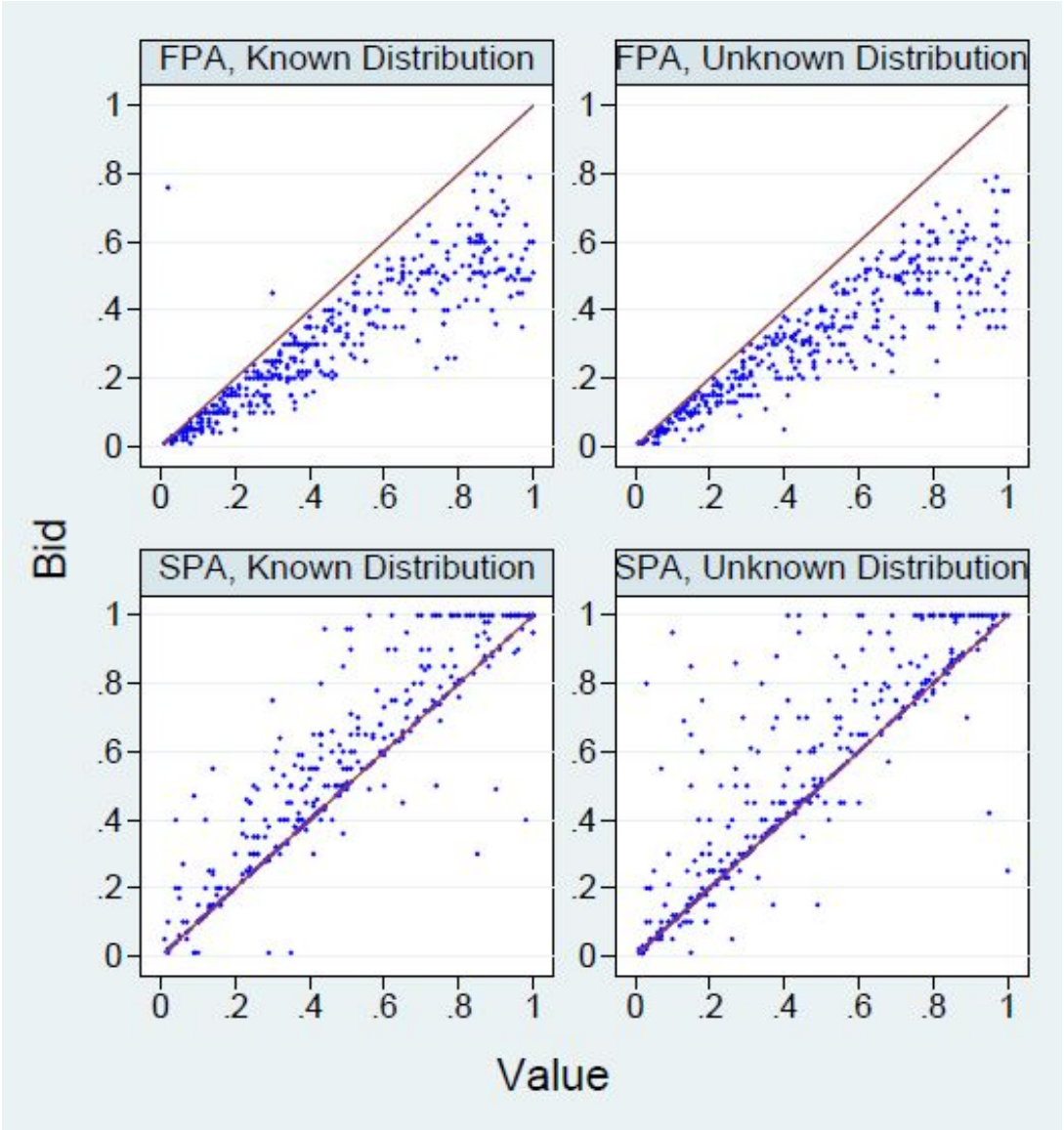
Which auction to use?

- In a second price auction bidders bid their true value
  - auctioneer receives the second highest bid
- In a first price auction bidders bid below their true value
  - auctioneer receives the highest bid

In practice- inexperienced subjects



In practice- experienced subjects



English Auction

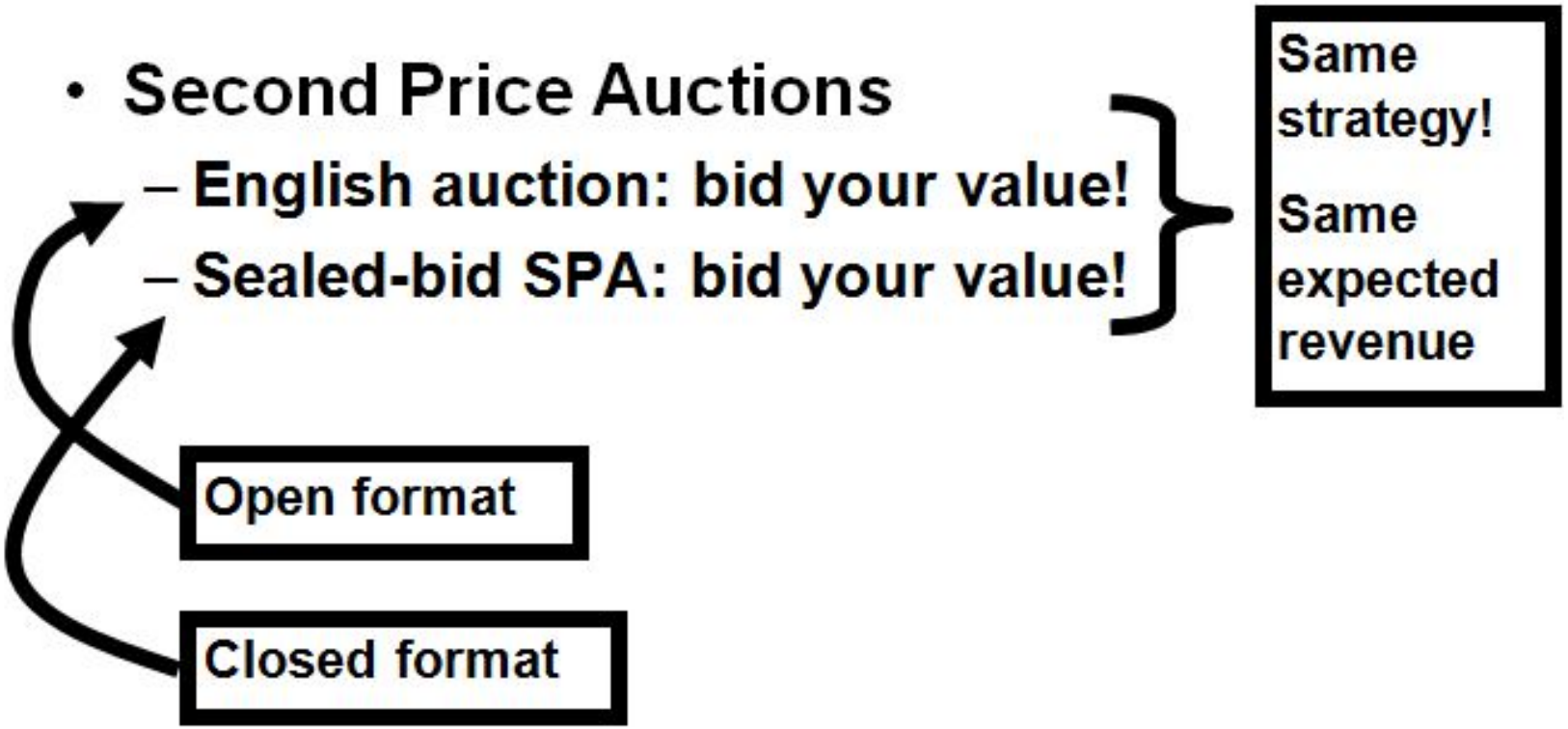
Auction procedure:

- I start with a price of zero
- Everybody who keeps his hand in the air is bidding.
- I increase the price slowly If the price becomes too high, you can withdraw from the auction by drawing back your hand
- The auction is over when only one bidder is still in the auction

The bidder left in the auction wins

- And pays the price level the auction stopped at.

Optimal bidding strategy in English Auction



Dutch Auction

Auction procedure:

- I start with a price of ...
- I slowly decrease the price
- The auction is over when one bidder raises his/her hand

The first bidder to raise the hand wins the auction

- And pays the price level the auction stopped at

Dutch Auction

Auction procedure:

- I start with a price of ...
- I slowly decrease the price
- The auction is over when one bidder raises his/her hand

The first bidder to raise the hand wins the auction

- And pays the price level the auction stopped at
- *What will be your strategy in such auction?*

Overview

- Closed auctions
  - FPA
  - SPA (Vickrey auction)
- Open auctions
  - English (ascending price auction)
  - Dutch (descending price auction)
- Outcome equivalence between
  - FPA & Dutch
  - SPA & English

All pay auction

Auction procedure:

- Write down a bid on a paper
- The person with the highest bid wins
  - And pays the price (s)he bid
  - All the others also pay the price they bid

All pay auction

Auction procedure:

- Write down a bid on a paper
- The person with the highest bid wins
  - And pays the price (s)he bid
  - All the others also pay the price they bid
- *Examples?*

All pay auction: examples

- Tender competitions
  - Time and effort to prepare a prospectus
- Elections
  - Advertising & promotion costs
- Lobbying effort
- Legal action
  - Lawyers
  - Expert reports from forensic specialists, statisticians, economists, psychologists

Revenue equivalence theorem

- In any auction where:
  - The bidder with the highest bid wins the auction
  - Values are distributed independently and identically
  - All bidders are risk neutral
  - The expected payment of a bidder with value zero is zero

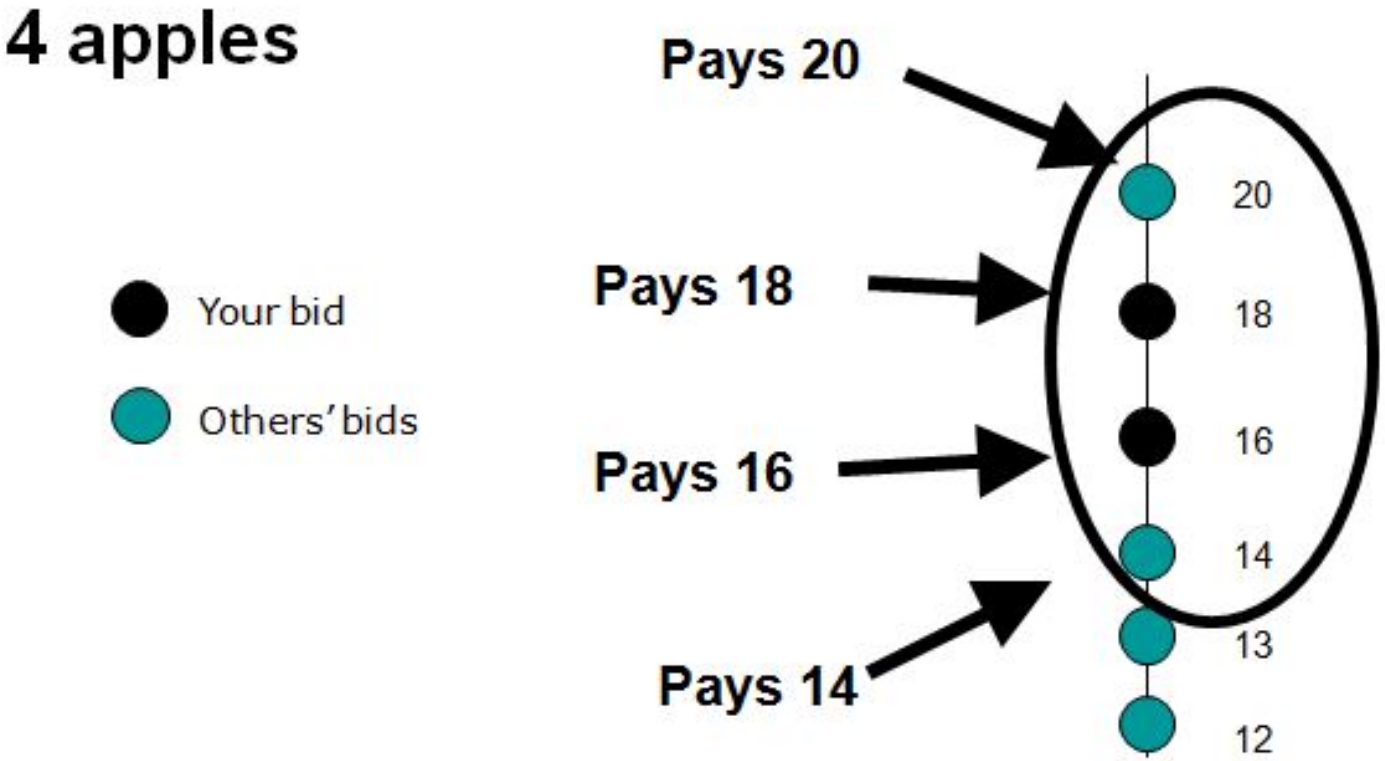
Revenue equivalence theorem

- In any auction where:
  - The bidder with the highest bid wins the auction
  - Values are distributed independently and identically
  - All bidders are risk neutral
  - The expected payment of a bidder with value zero is zero
- The seller yields the same expected revenue

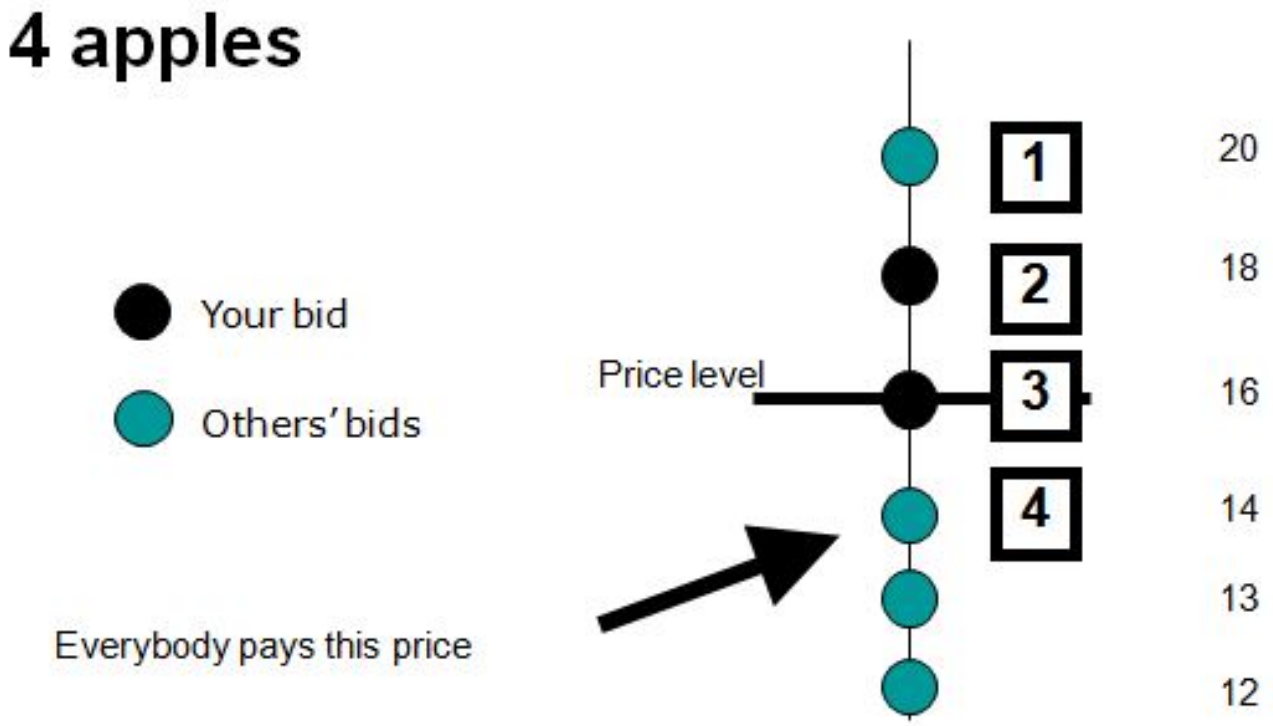
Multiple units

- More than 1 unit of the good is sold at the same time.
- You can also post (different or the same) bids for more than 1 unit.
- Types:
  - Discriminating auction (Pay-Your-Bid)
  - Modified Dutch auction (uniform price auction)

Discriminating auction (Pay-Your-Bid)



Modified Dutch auction (uniform price auction)



Uniform price (example)

Figure 10: Auction Bids for Treasury Bills

Discount Rate (%)	*Face Value* ($ billions)	*Cumulative Face Value* ($ billions)	
0.0000	5	5	
0.1081	3	3	8
0.1090	12	15	20
0.1098	8	23	28
0.1104	5	28	33
0.1117	8	36	41
0.1124	7	36	

- **Seller:**
  - Suppose the Treasury wants to sell $35b in bills
- **Buyer:**
  - Total competitive bids: $36b
    - specify the amount of bills (in face value) and price (yield=discount rate)
  - non-competitive bids: $5b
    - specify the amount of bills (in face value), but not price
    - Will accept any resulting price
  - Imagine simply that non-competitive bidders bid a price of zero
    - then determine the price at which the total quantity supplied of competitive + non-competitive bids is equal to $35b

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

Národohospodářská fakulta VŠE v Praze