Game Theory

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Bargaining

- Government policy is typically the outcome of negotiations among cabinet ministers.
- National governments are often engaged in a variety of international negotiations on matters ranging from economic issues to global security, and environmental and related issues..
- Mergers and acquisitions require negotiations over the price at which such transactions are to take place
- Couples negotiate over a variety of matters such as who will do which domestic chores
- Wages, prices...

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- Another individual, called Mohan, is willing to pay up to £ 70,000 for Aruna's house.
- If trade occurs, at a price that lies between £ 50,000 and £ 70,000, then both Aruna (the 'seller') and Mohan (the 'buyer') would become better of.
    - common interest to trade
    - they have conflicting interests over the price at which to trade

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    - common interest to trade
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- *What would you suggest?*

Nash bargaining solution

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- Two players, A and B , bargain over the partition of a cake (or surplus) of size π ($\pi > 0$).
- The set of possible agreements is $X = \{(x_a, x_b) : 0 \leq x_a \leq \pi \ and \ x_b = \pi - x_a\}$ where x_i is the share of the cake to player i.
- $U(x_i)$ is player i's from obtaining a share x_i of the cake.
- The utility pair (d_a, d_b) is disagreement point with utilities if players fail to reach agreement.

Nash bargaining solution

- The Nash bargaining solution (NBS) of the bargaining situation described above is the unique pair of utilities, denoted by (u_a^N, u_b^N), that solves the following maximization problem:

$$\max_{u_a, u_b}(u_a - d_a)(u_b - d_b)$$

- Solve for linear utilities.

Solution example 1

Split the difference rule

- if utilities are linear in payoffs: $u_i(x_i) = x_i$

$$u_A^N = d_A + \frac{1}{2}\Big(\pi - d_A - d_B\Big) \quad and \quad u_B^N = d_B + \frac{1}{2}\Big(\pi - d_A - d_B\Big)$$

- *How would you interpret this result?*

Solution example

Risk aversion present

- One of the agents is risk averse: $u_a(x_a) = x_a^\gamma$ where $0 < \gamma < 1$ and $u_b(x_b) = x_b$
- If $d_a = d_b = 0$ then division is as follows:

$$x_A^N = \frac{\gamma\pi}{1+\gamma} \quad and \quad x_B^N = \frac{\pi}{1+\gamma}$$

- *Intuition?*

Asymmetric Nash bargaining

- Different bargaining power captured by parameter τ :

$$\max_{u_a,\, u_b} (u_a - d_a)^{\tau} (u_b - d_b)^{1-\tau}$$

- If τ -> 1/2: symmetric equilibrium

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$$\max_{u_a,\, u_b}(u_a - d_a)^{\tau}(u_b - d_b)^{1-\tau}$$

- If τ -> 1/2: symmetric equilibrium
- *Examples of different bargaining power?*

Quick practice

- Suppose that a worker with alternative wage offer of 10 EUR per hour can generate profit of 30 EUR per hour for a firm.
- The firm would like to hire this worker. If this worker rejects an offer, there is another worker who will accept for sure offer of 5 EUR, but his productivity is only 15 EUR per hour.

Quick practice

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- The firm would like to hire this worker. If this worker rejects an offer, there is another worker who will accept for sure offer of 5 EUR, but his productivity is only 15 EUR per hour.

- Assume risk neutrality for both parties, $U(x_i) = x_i$
- What would be an outcome of symmetric Nash bargaining?
- What if bargaining power of the worker is two times greater than bargaining power of the firm (captured by τ)?

Main message from Nash bargaining

- If there is potential for trade, trade should happen.
- Outside options are not used in equilibrium but determine the outcome
- Reasons for non-equal split of the surplus:
  - risk aversion
  - bargaining power

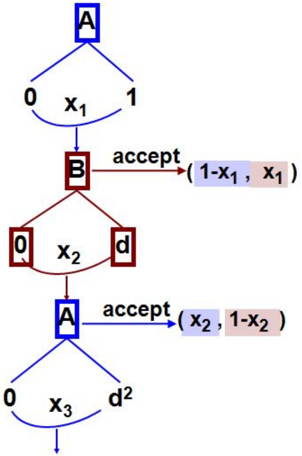
Rubinstein model

- Key feature of this model is that it specifies a rather attractive procedure of bargaining:
  - the players take turns to make offers to each other until agreement is secure
- This model has much intuitive appeal, since making offers and counteroffers lies at the heart of many real-life negotiations.
- One insight is that friction-less bargaining processes are indeterminate.
  - if the players do not incur any costs by haggling

Rubinstein model

- It seems intuitive that for the players to have some incentive to reach agreement they should find it costly to haggle.
- A player's bargaining power depends on the relative magnitude of the players' respective costs of haggling.
- This model provides a basic framework, which can be adapted, extended and modified for the purposes of application

Illustration

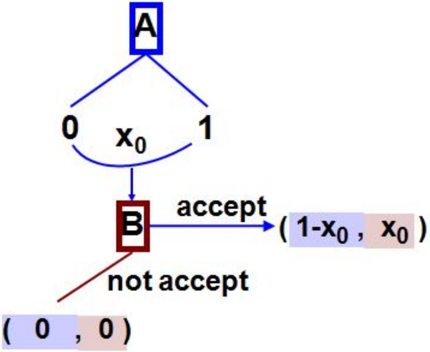


- *Notice, that the size of the pie is shrinking after each round due to discounting.*

Different approach

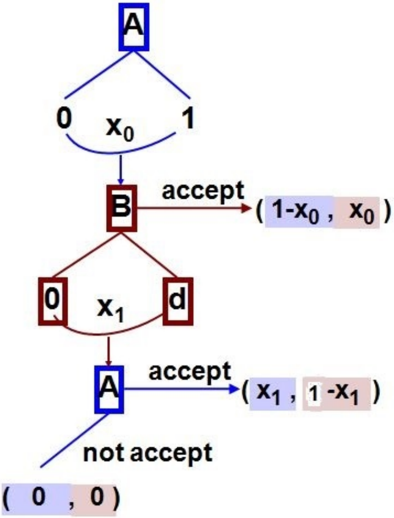
- Look for the sub-game perfect Nash equilibrium
- Variants of the game with different number of periods
  - $T=1$
  - $T=2$
  - ...
  - $T=+\infty$

T=0



- At time 0, what player B accepts?
  - 1-$x_0 \geq 0$, x_0=0
  - Or x_0= ε>0 with ε very small
- Thus: (1, 0)

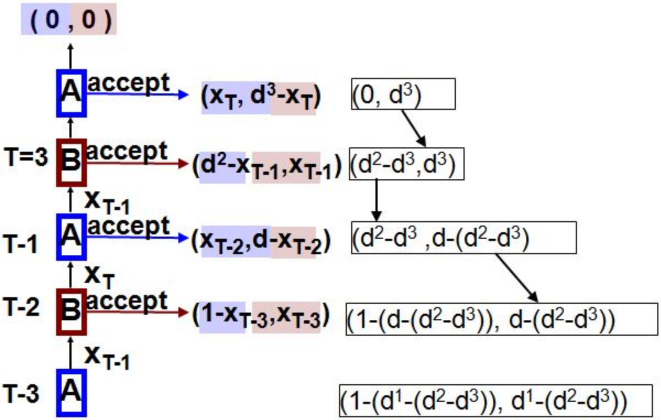
T=1



- At time 1, what Player A accepts?
- $1-x_1 \geq 0$
- $x_1=0$
- Thus: (0, d)

- At time 0, what player B accepts?
- $1-x_0 \geq d$
- $x_0=0$
- Thus: (1-d,d)

More periods



$(1-d^1+d^2-d^3+d^4-d^5+\ldots d^N,\ d^1-d^2+d^3-d^4+d^5-\ldots-d^N)$

Calculate it

- $(1 - d^1 + d^2 - d^3 + d^4 - d^5 + \ldots d^N, d^1 - d^2 + d^3 - d^4 + d^5 - \ldots - d^N) \qquad \text{for } N \to \infty$

Calculate it

- $(1 - d^1 + d^2 - d^3 + d^4 - d^5 + \dots d^N, d^1 - d^2 + d^3 - d^4 + d^5 - \dots - d^N)$ for $N \to \infty$
- Starting player (A) offers $1 - \frac{1}{1+d} = \frac{d}{1+d}$ and another other player (B) accepts
- $(\pi_a, \pi_b) = (\frac{1}{1+d}, \frac{d}{1+d})$

# of Offers	A's Equilibrium Utility	B's Equilibrium Utility
1	1	0
2	$1 - \delta$	δ
3	$1 - \delta + \delta^2$	$\delta - \delta^2$
4	$1 - \delta + \delta^2 - \delta^3$	$\delta - \delta^2 + \delta^3$
Infinite	$1/(1 + \delta)$	$\delta/(1 + \delta)$

Possible extensions

- Different discount factors
- Non-zero outside options

Unique subgame perfect equilibrium

- SPE that satisfies the following two properties

Property 1 (No Delay). *Whenever a player has to make an offer, her equilibrium offer is accepted by the other player.*

Property 2 (Stationarity). *In equilibrium, a player makes the same offer whenever she has to make an offer.*

Different discount rates

- In the limit, the shares η_a and η_b obtained by players A and B respectively in the unique SPE converge to $\eta_a\pi$ and $\eta_b\pi$ where

$$\eta_a = \frac{(1-\delta_b)}{1-\delta_a\delta_b} \textit{ and } \eta_b = \frac{\delta_b(1-\delta_a)}{1-\delta_a\delta_b}$$

- It seems reasonable to assume that the share of the cake obtained by a player in the unique SPE reflects her 'bargaining power'.
- Thus, a player's bargaining power is increasing in her discount rate, and decreasing in her opponent's discount rate.

Message

- Same discount factors
  - there exists a 'first-mover' advantage
- Different discount factors
  - relative magnitude of the players' discount rates critically influence the equilibrium partition of the cake

In a boxing match, the winner is the relatively stronger of the two boxers; the absolute strengths of the boxers are irrelevant to the outcome.

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In a boxing match, the winner is the relatively stronger of the two boxers; the absolute strengths of the boxers are irrelevant to the outcome.

- The basic alternating-offers game has a unique SPE, which is Pareto efficient.

Relationship to Nash's Bargaining solution

- It is straightforward to verify that the SPE payoff pair $(\eta_a\pi, \eta_b\pi)$ is identical to the asymmetric Nash bargaining solution of the bargaining problem with $\tau = \eta_a$.
- This remarkable result provides a strategic justification for Nash's bargaining solution.
- In particular, it provides answers to the questions of why, when and how to use Nash's bargaining solution.

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