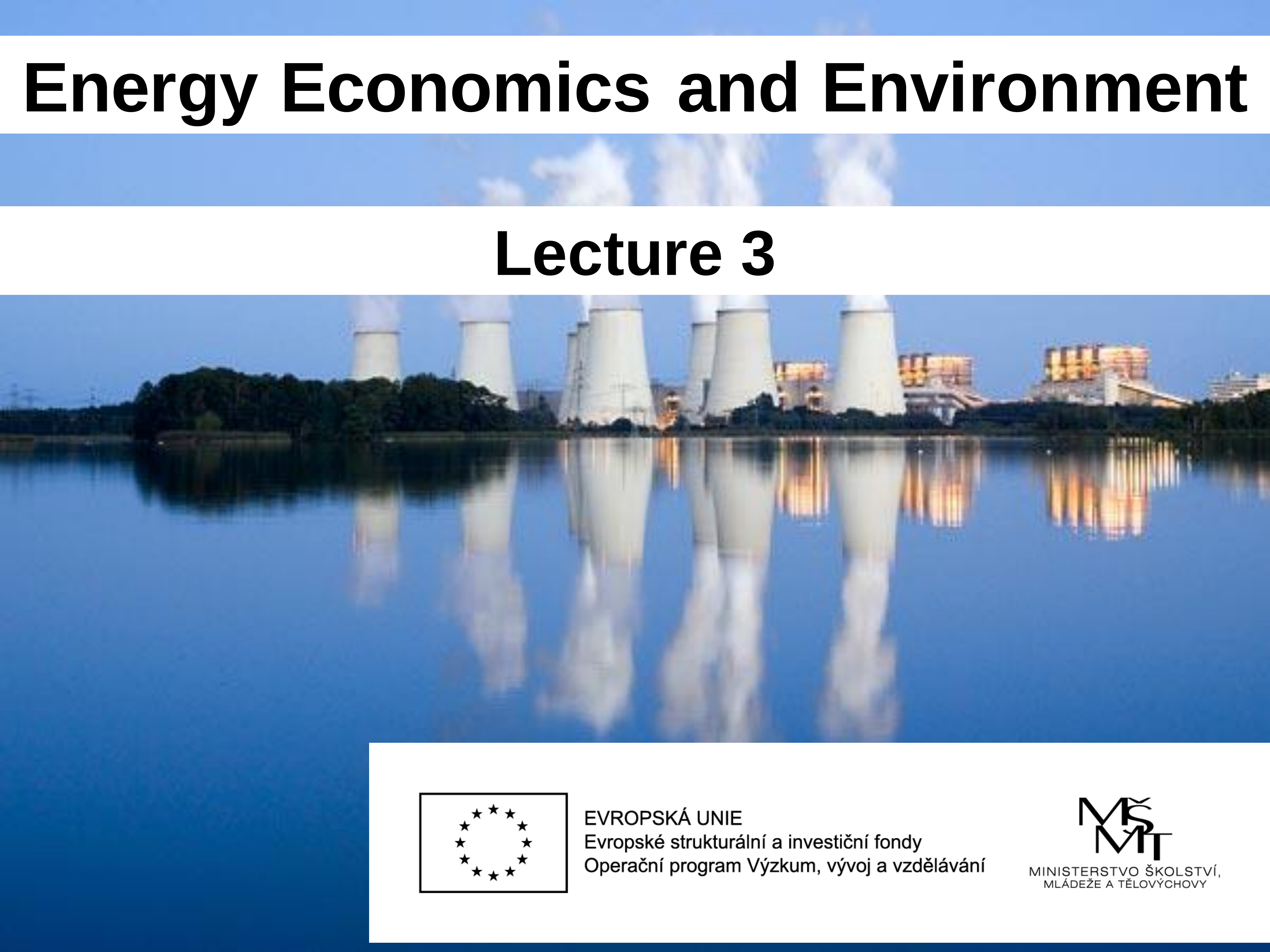
Energy Economics and Environment

Lecture 3



EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

- Transmission pricing (Nodal versus Zonal)
  - Peak-load pricing
  - Nodal and zonal dispatch in meshed networks

- Consumer:
- Maximum buying price

10 10 10

9 9 9

7 7 7

3 3 3

Producer:

Minimal selling price

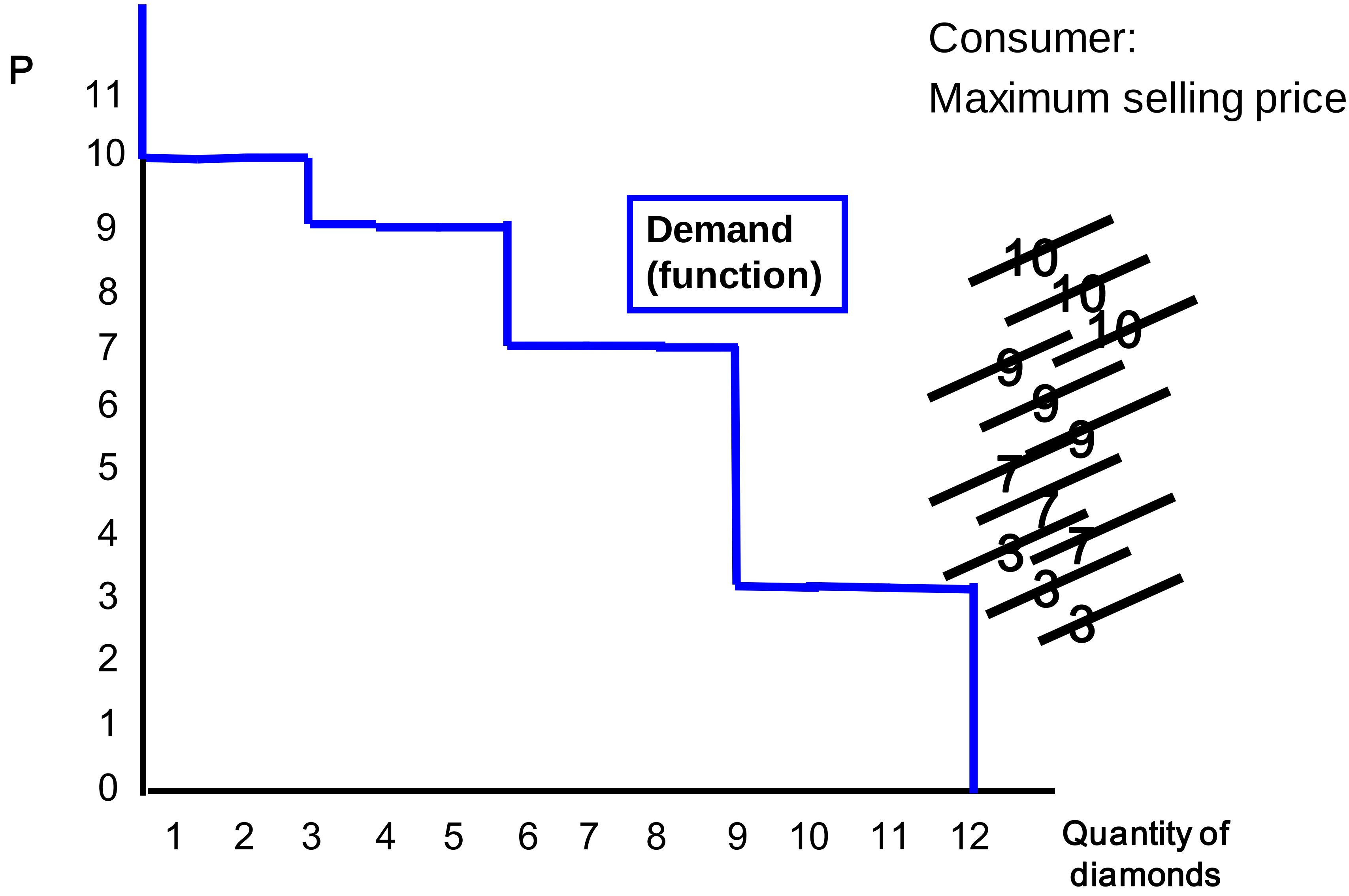
9 9 9

7 6 6

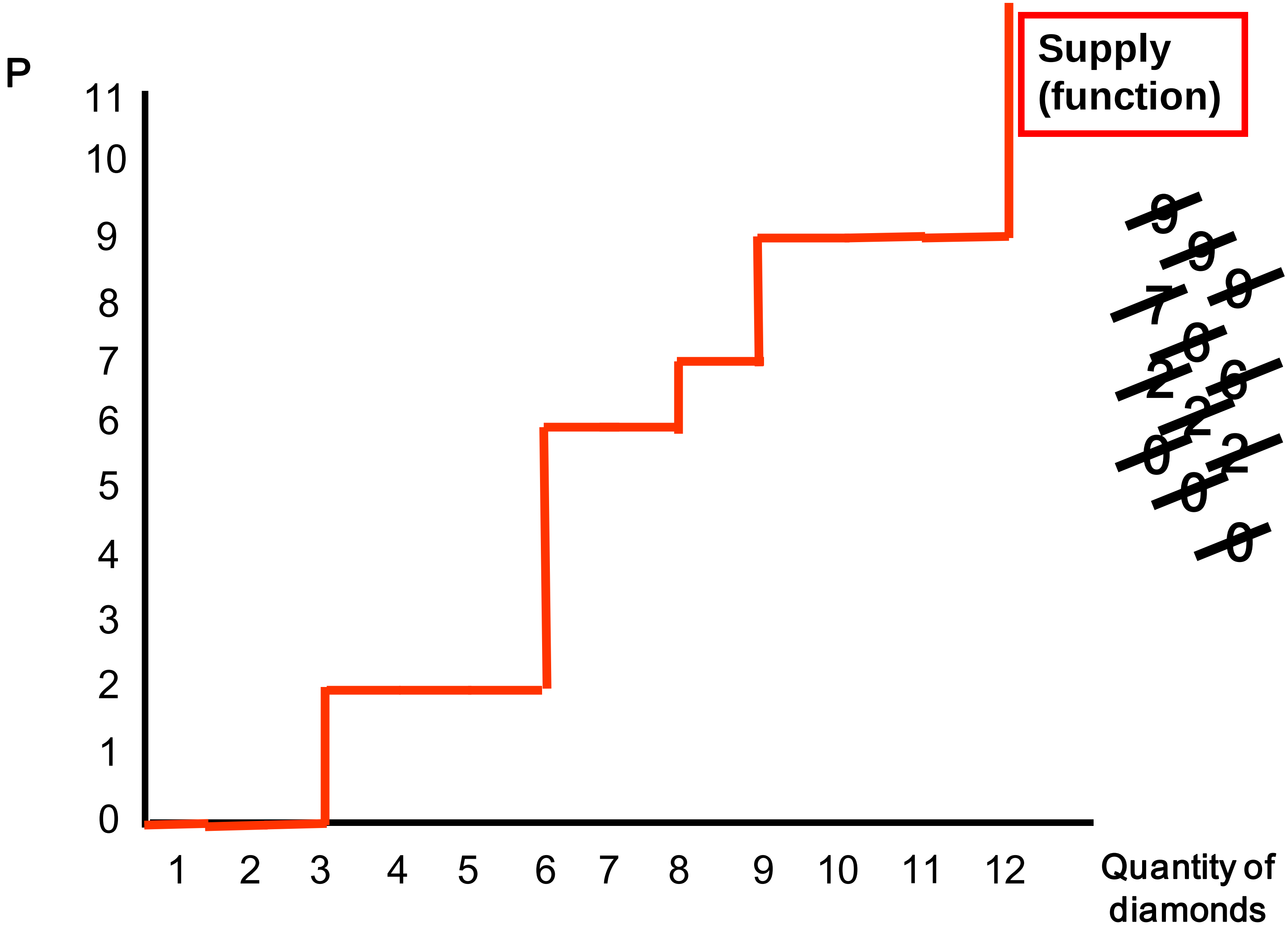
2 2 2

0 0 0

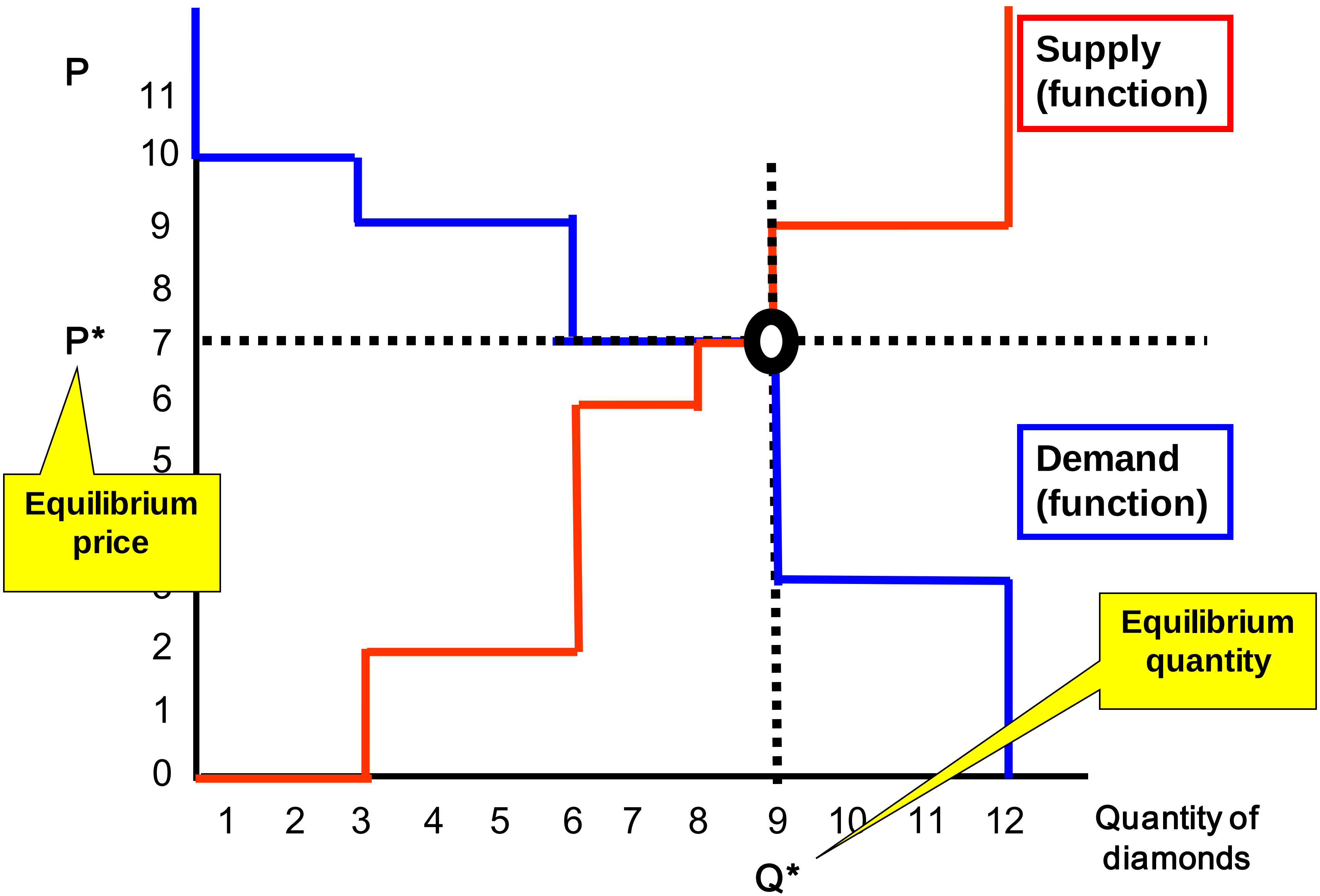
Deriving a the equilibrium price



Deriving a the equilibrium price



Deriving a the equilibrium price



- Consumer:
- Maximum buying price

10 10 10
9 9 9
7 7 7
3 3 3

Producer:

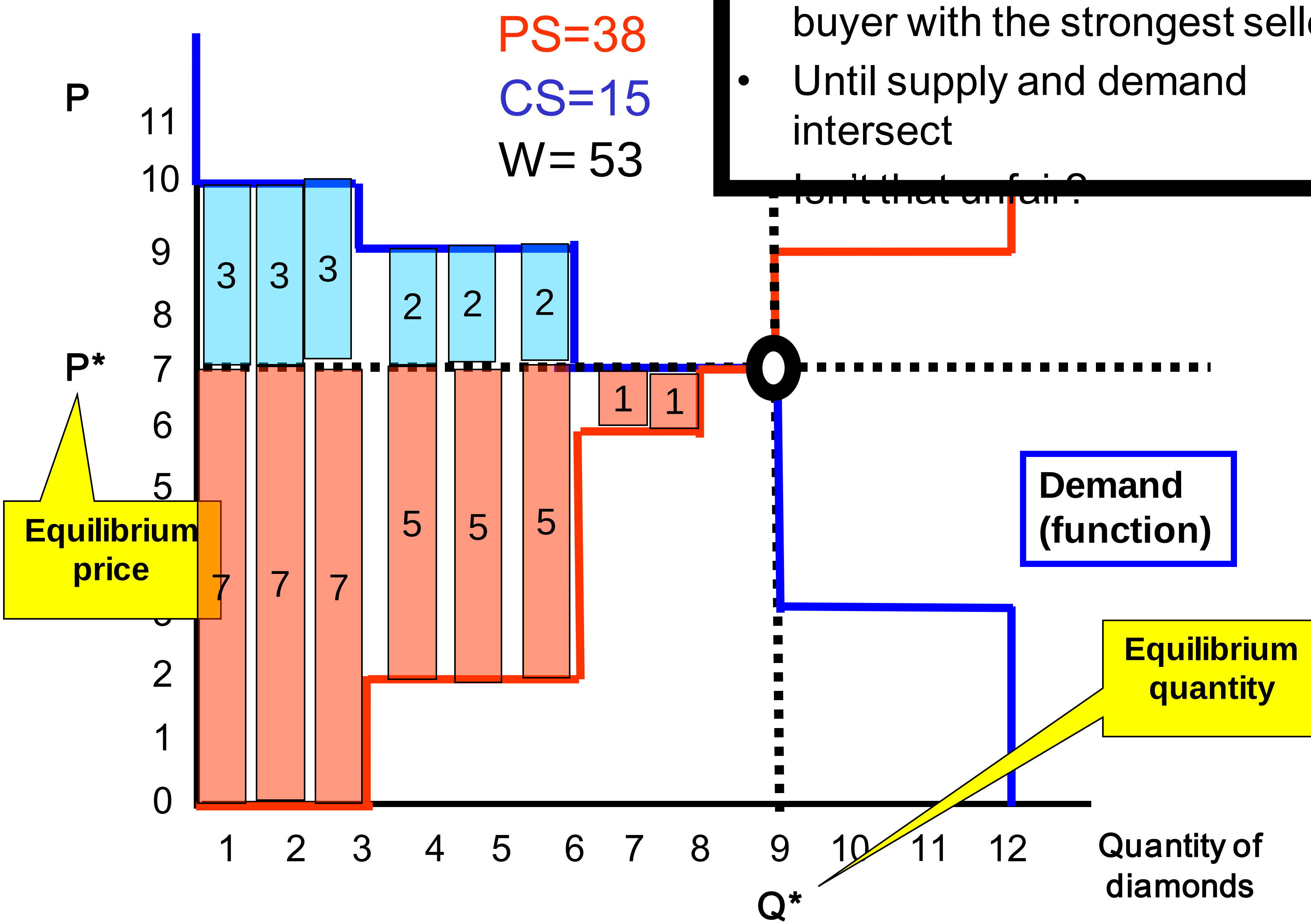
Minimal selling price

9 9 9
7 6 6
2 2 2
0 0 0

Free market mechanism imposes a rich structure

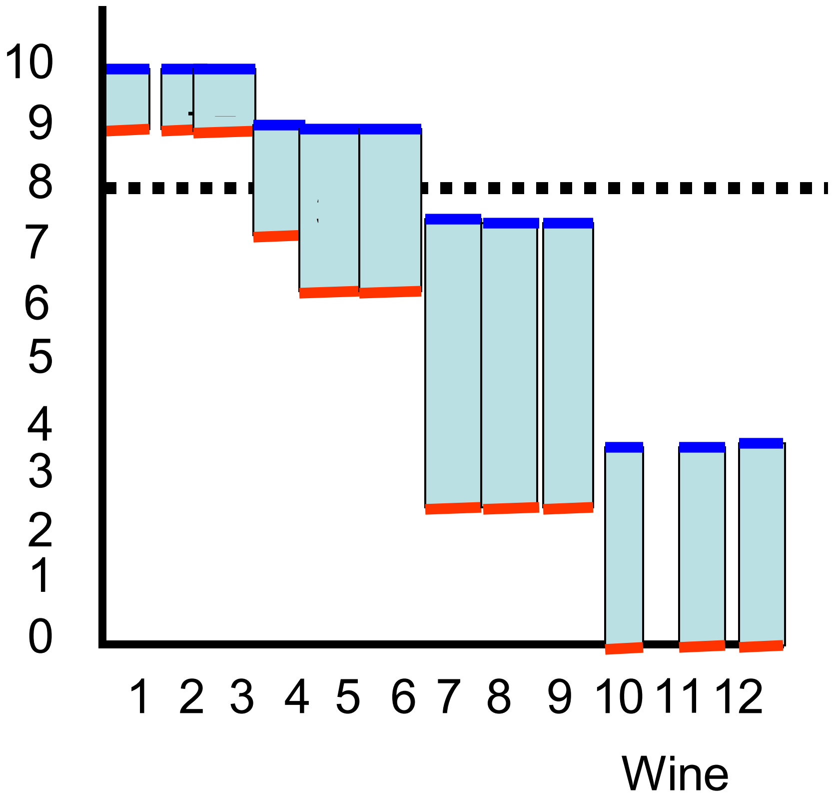
Looking at t

- You always pair the strongest buyer with the strongest seller.
- Until supply and demand intersect



Other possible arrangements: Communist *"fair"* dictator

Could this be more efficient?



Consumer

~~10~~ ~~10~~ ~~10~~ ~~9~~ ~~9~~ ~~9~~ ~~7~~ ~~7~~ ~~7~~ ~~3~~ ~~3~~ ~~3~~

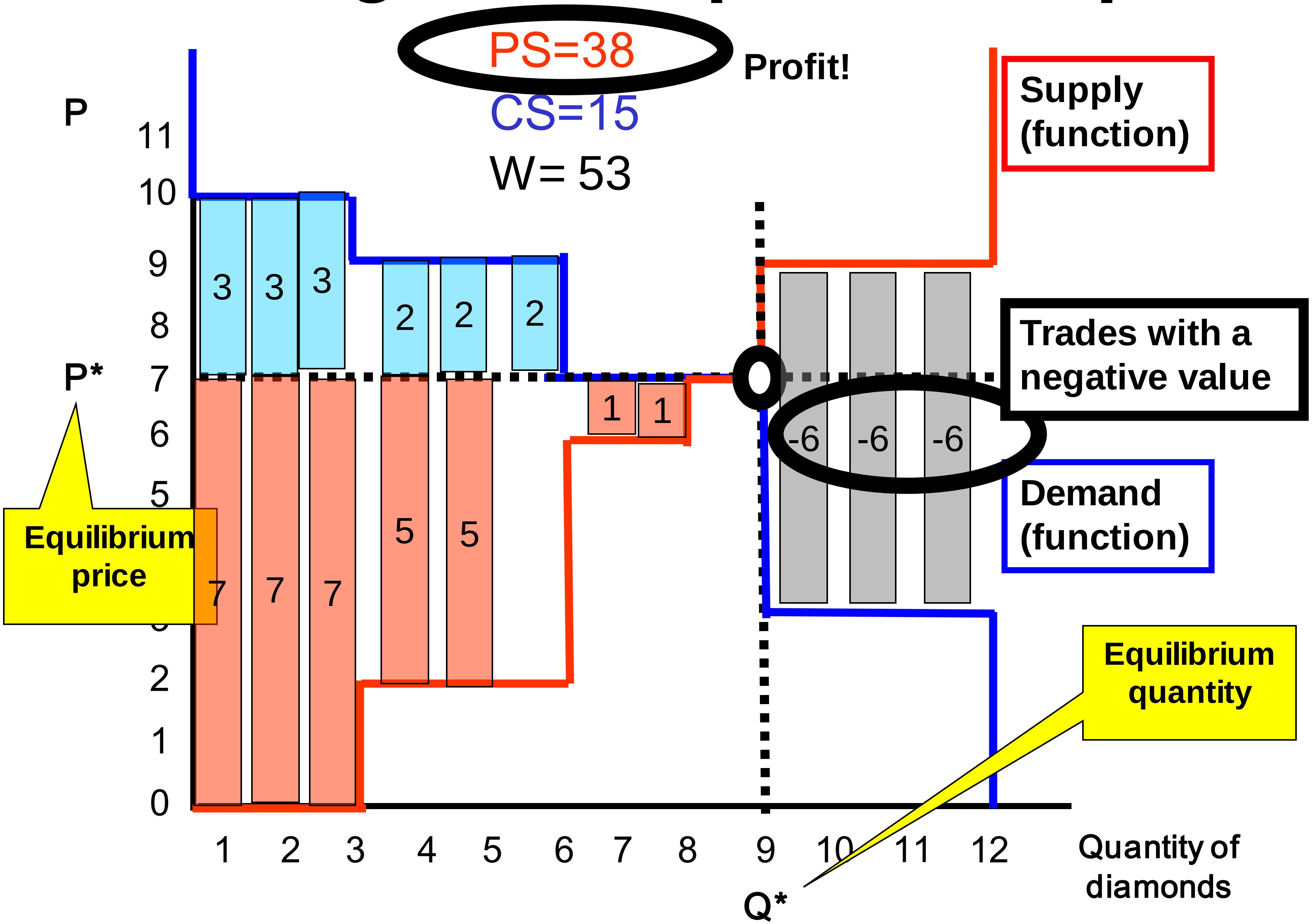
Producer

~~9~~ ~~9~~ ~~9~~ ~~7~~ ~~6~~ ~~6~~ ~~2~~ ~~2~~ ~~2~~ ~~0~~ ~~0~~ ~~0~~

W= 35
W(Free market)=53
(difference =18)

Free market maximizes
W=CS+PS

Deriving a the equilibrium price



- Peak-load pricing
- Nodal & Zonal

- A new transmission line has to be build. Once it will be running, demand per hour is given by:

$$P_L[Q_L] = 8 - Q_L$$
- for the hours 20:00-08:00

$$P_H[Q_H] = 20 - Q_H$$
- for the hours 08:00-20:00

- Amortized fixed cost per unit capacity per hour:
  - *f=1.5$*.
- The marginal cost of use
  - *mc=0$*.
- Imagine that it has already be decided that a line of size 10 will be built.
  - What is the welfare maximizing tariff in $/hour (given this size)?
  - What is the average earning per unit per hour for the line owner?
  - Is the capacity of 10 the welfare maximizing size of the line? Why (not)?
  - What is the average total surplus W (consumer surplus + producer surplus) in $/hour?

- What is the welfare maximizing tariff in $/hour (given this size)?

- What is the average net earning per unit per hour for the line owner?

- Is the capacity of 10 the welfare maximizing size of the line? Why (not)?

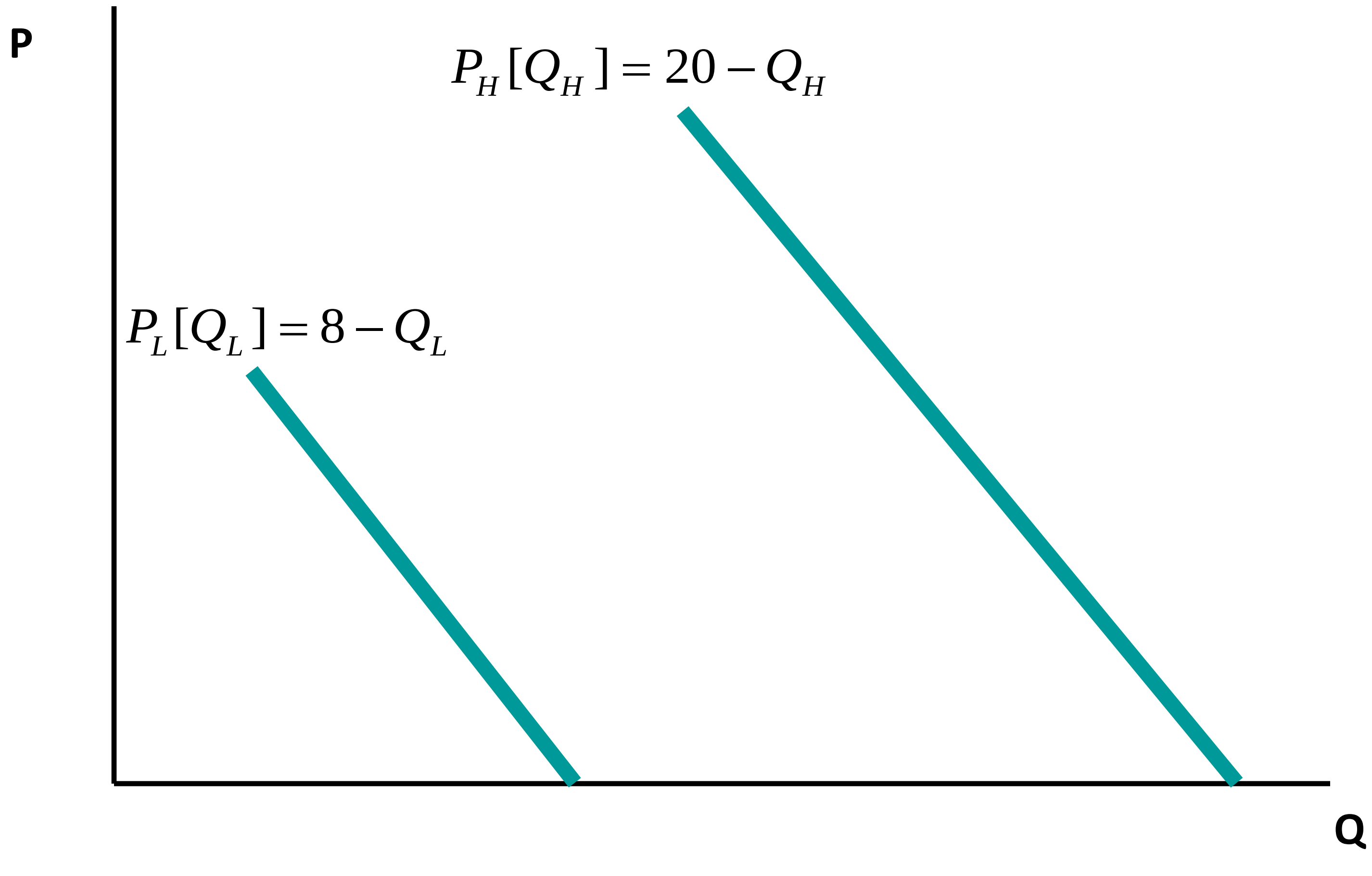
- What is the average total surplus W (consumer surplus + producer surplus) in $/hour?

Demand part

$$P_L[Q_L] = 8 - Q_L$$

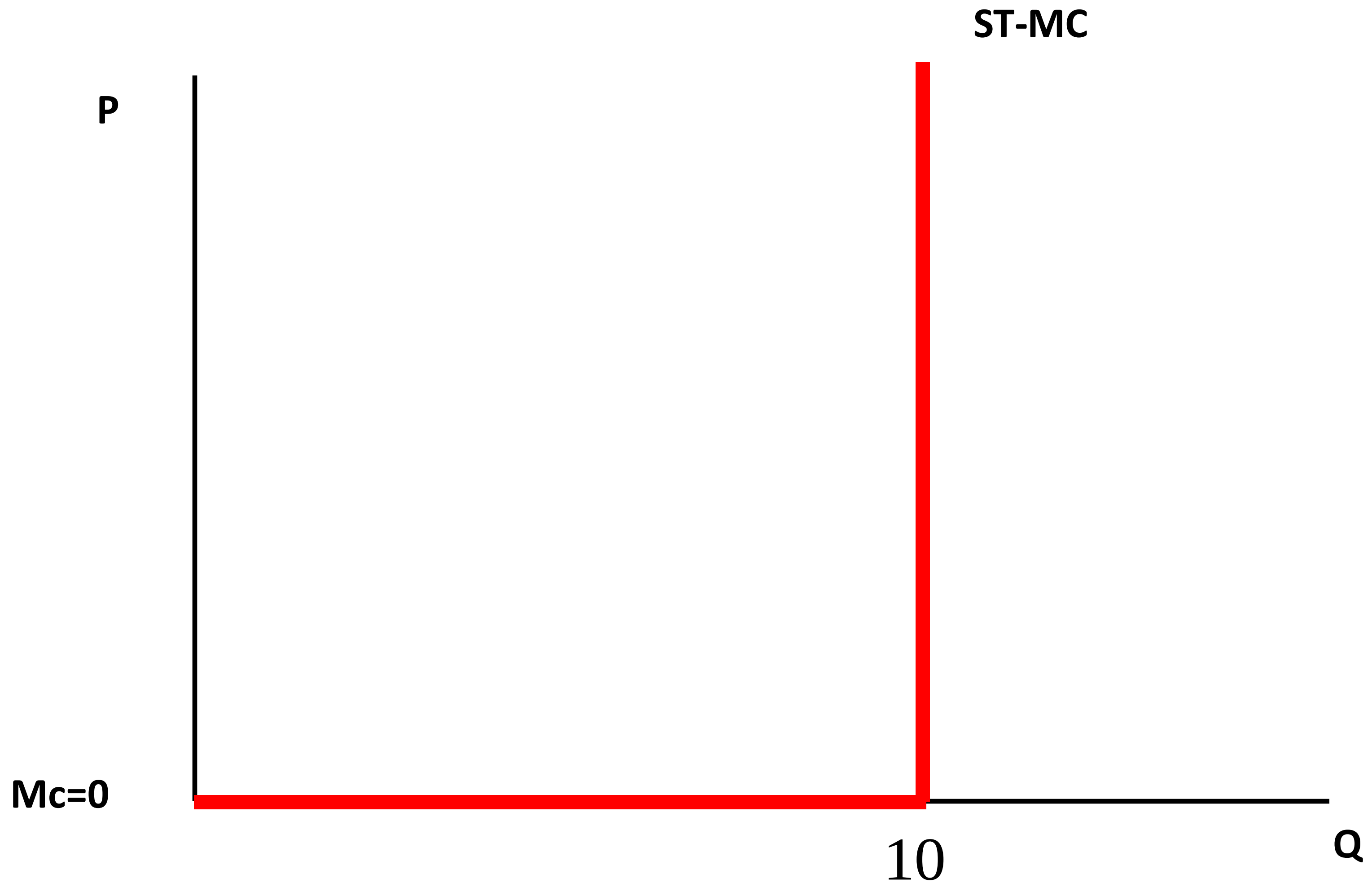
$$P_H[Q_H] = 20 - Q_H$$

- for the hours 20:00-08:00
- for the hours 08:00-20:00



Supply part

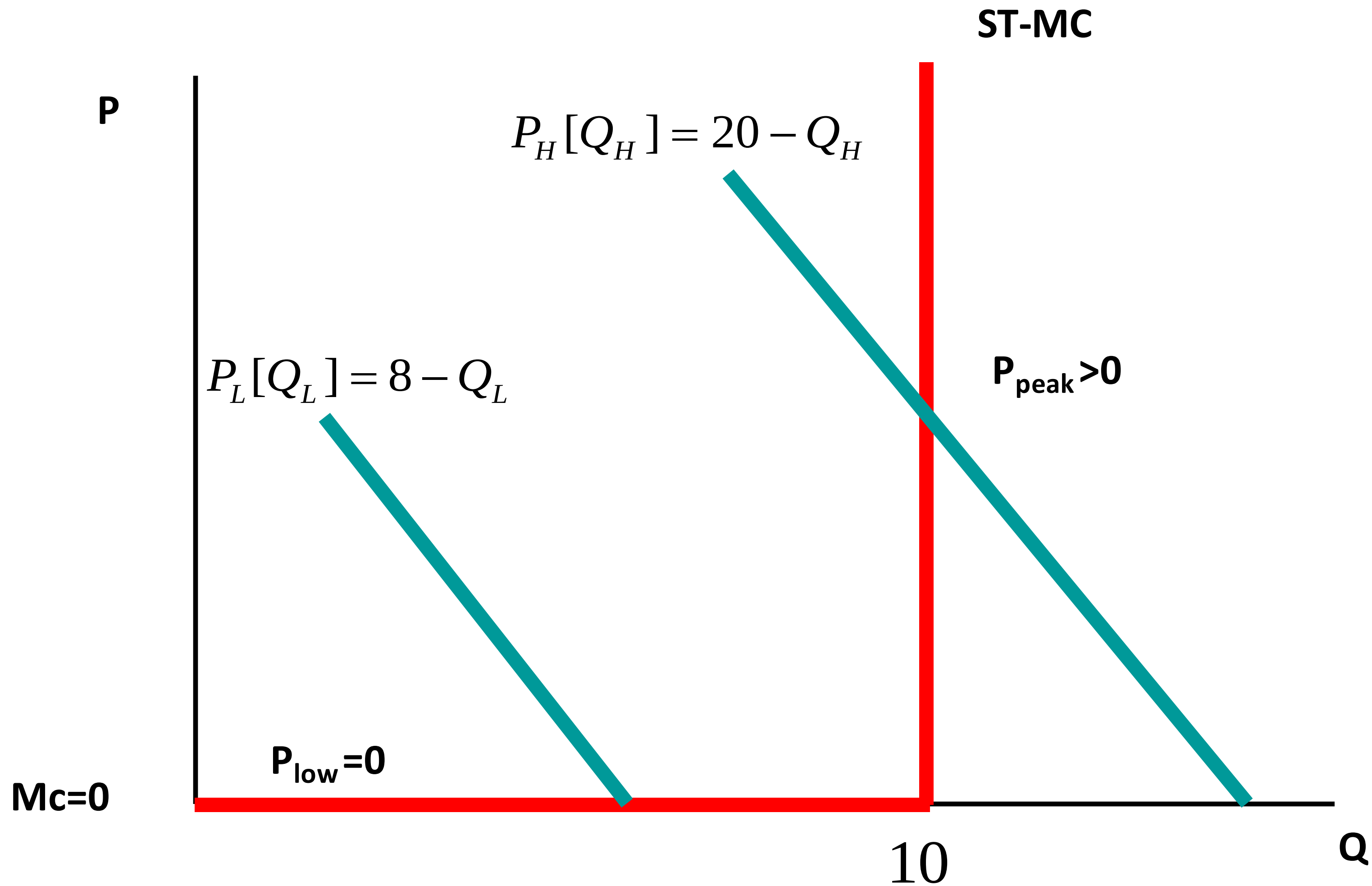
- Amortized fixed cost per unit capacity per hour: *f=1.5$*.
- The marginal cost of use: *mc=0$*.
- already been decided that size of line: 10



$p_L = mc = 0\$$

$D_L = 8$

$$\underbrace{P_H[Q_H] = 20 - Q_H \quad \& \quad D_H[p_H] = 10}_{p_H = 10\$}$$



$p_L = 0\$$ $p_H = 10\$$

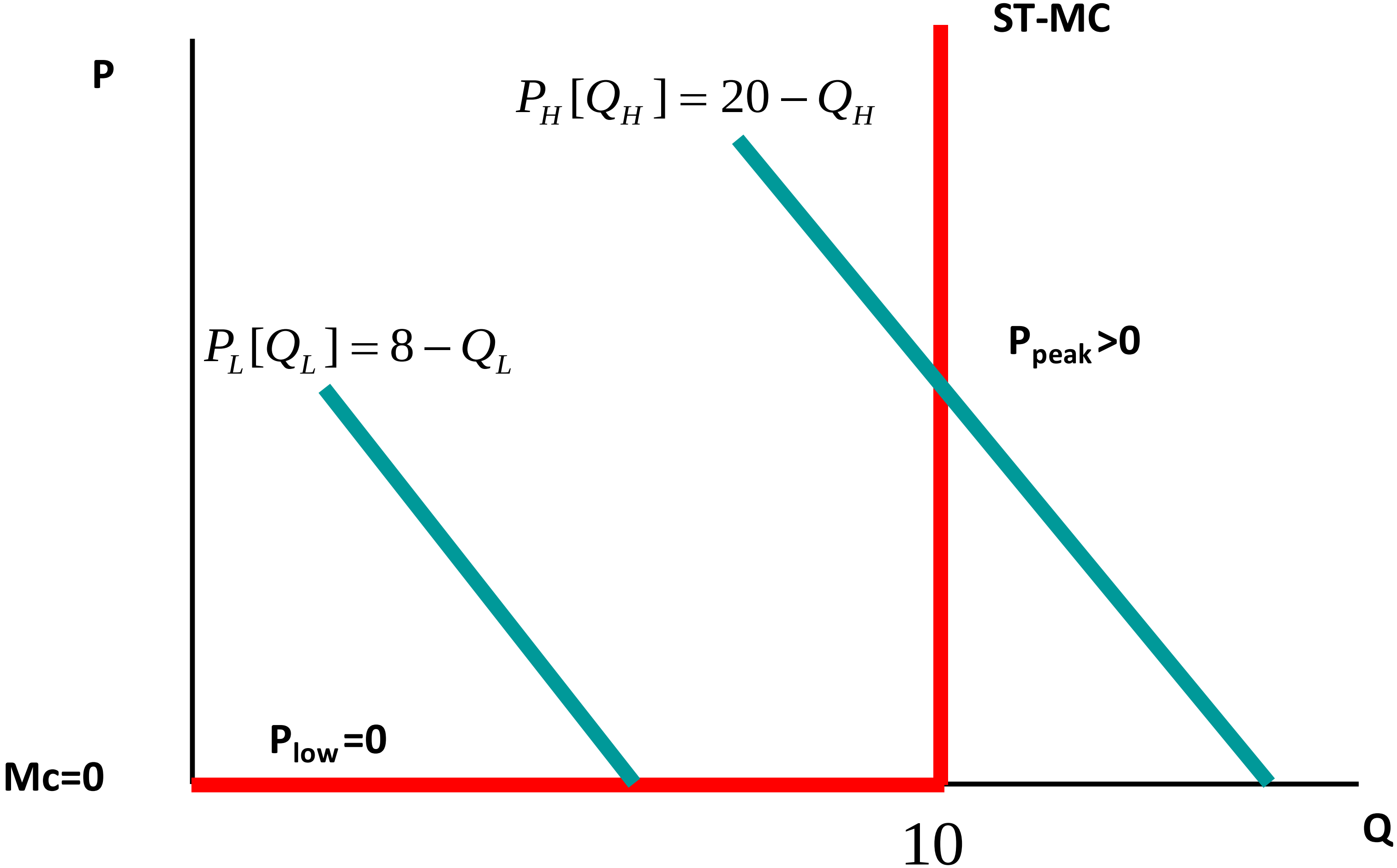
$D_L = 8$ $D_H = 10$

The average gross earnings per unit per hour:

$$0.5 \cdot p_L + 0.5 \cdot p_H = 0.5 \cdot 0\$ + 0.5 \cdot 10\$ = 5\$$$

The average net earnings (minus fixed costs) per unit per hour:

$$5\$ - 1.5\$ = 3.5\$ / h$$



- What is the welfare maximizing tariff in $/hour (given this size)?

  $$p_L = 0\$ \qquad p_H = 10\$$$

- What is the average net earning per unit per hour for the line owner?

  $$3.5\$ / h$$

- Is the capacity of 10 the welfare maximizing size of the line? Why (not)?
  - Under perfect competition, owner would earn on average zero net profits per hour!
  - positive net profits attracts competition
  - Capacity increases, price decreases until net profit =0
  - Thus the line is now too small.

- What is the average total surplus W (consumer surplus + producer surplus) in $/hour?

- What is the welfare maximizing tariff in $/hour (given this size)?

$$p_L = 0\$ \qquad p_H = 10\$$$

- What is the average net earning per unit per hour for the line owner?

$$3.5\$/h$$

- Is the capacity of 10 the welfare maximizing size of the line? Why (not)?
  - Under perfect competition, owner would earn on average zero net profits per hour!
  - positive net profits attracts competition
  - Capacity increases, price decreases until net profit =0
  - Thus the line is now too small.

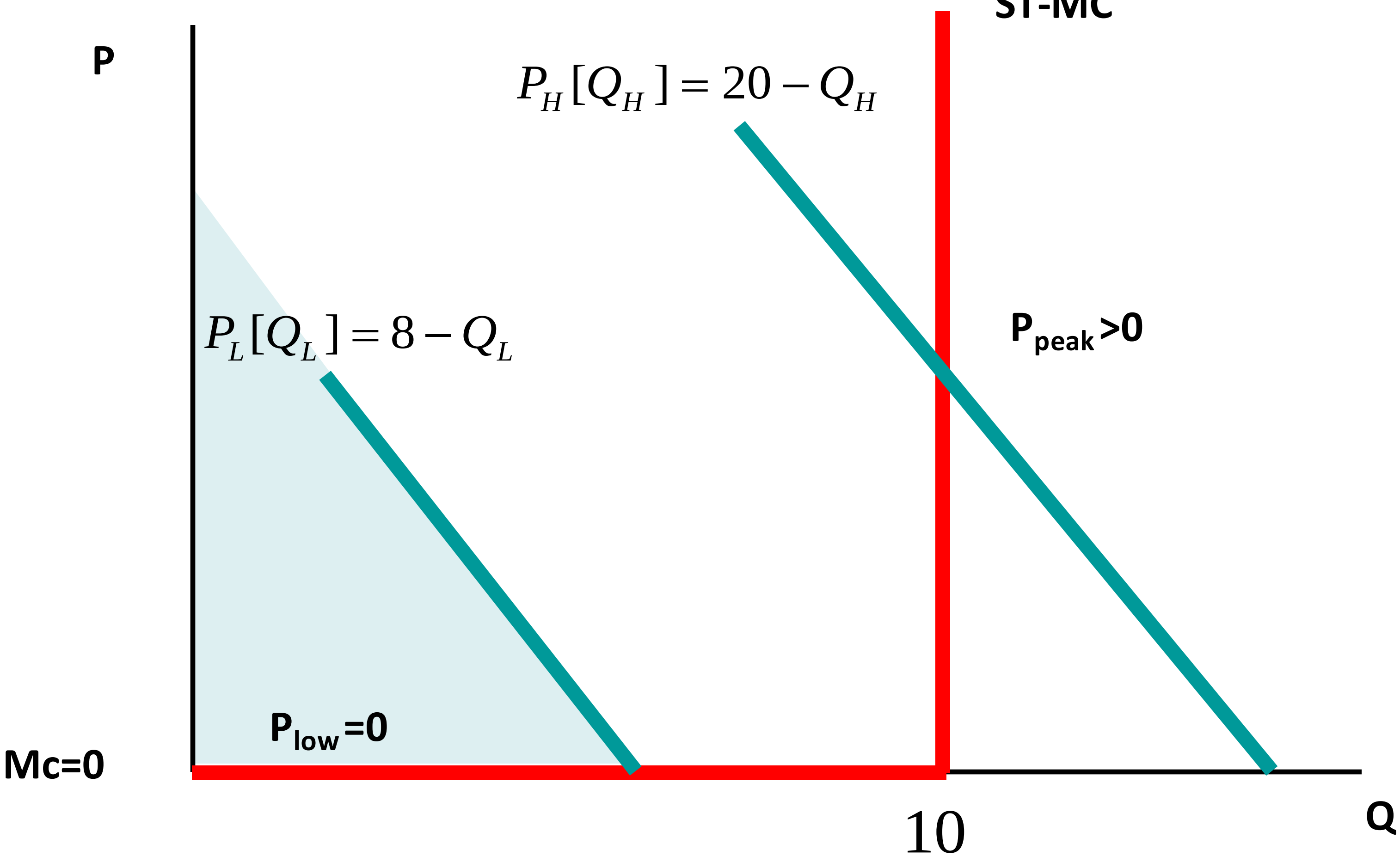
- What is the average total surplus W (consumer surplus + producer surplus) in $/hour?

$$PS_L / h = 0\$ \cdot D_L = 0\$$$

$$PS_H / h = 10\$ \cdot D_H = 10\$ \cdot 10 = 100\$/h$$

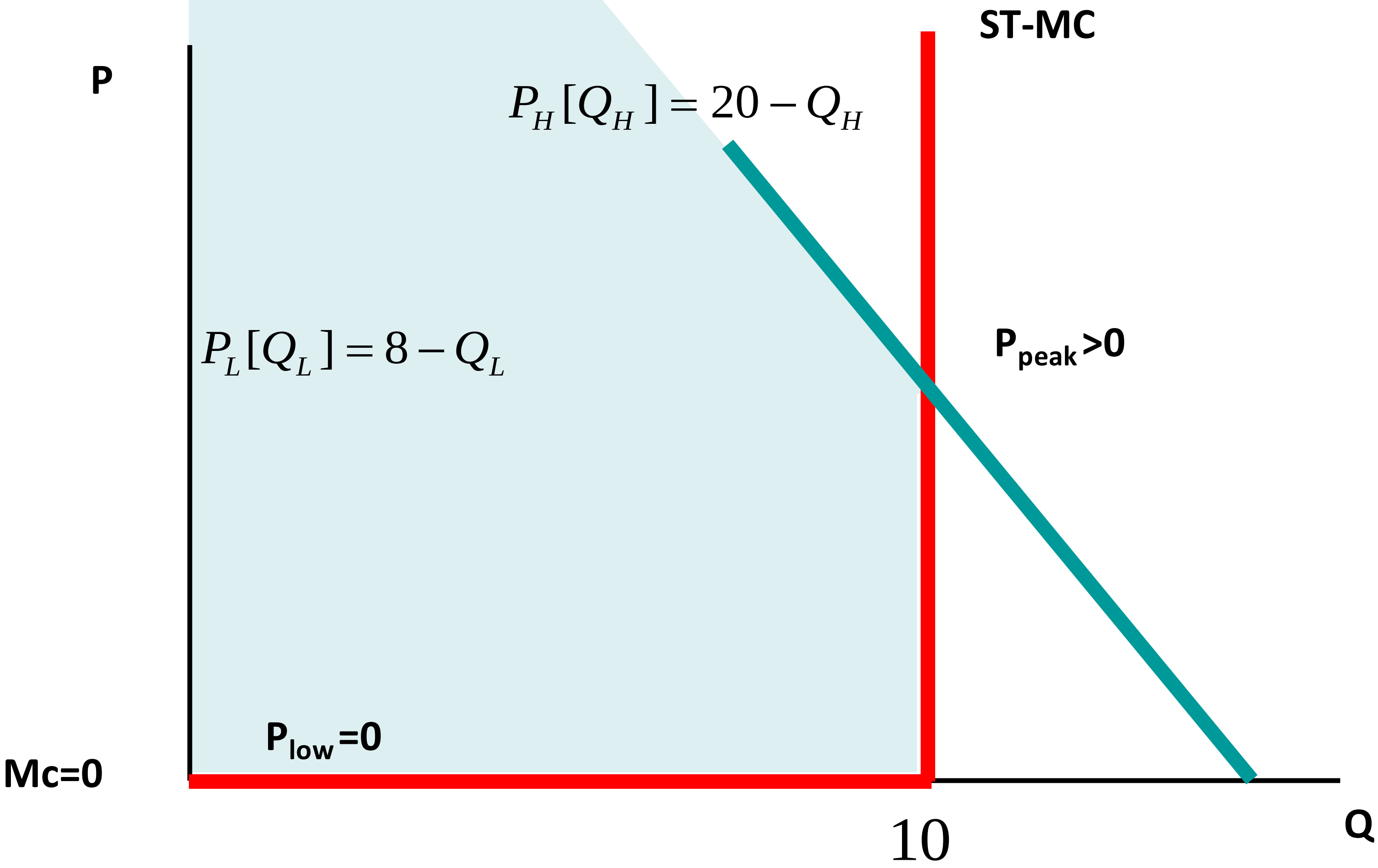
$$CS_L = 8 \cdot 8 \cdot 0.5 \;\; = 32\$ / h$$

$$\int_0^8 \left(8 - Q\right) dQ = [8Q - .5Q^2]_0^8 = 32$$



$$CS_H = 10 \cdot 10 \cdot 0.5 + 10 \cdot 10 = 50 + 100 = 150\$ / h$$

$$\int_0^{10} \left(20 - Q\right) dQ = [20Q - .5Q^2]_0^{10} = 200 - 50 = 150$$



- What is the welfare maximizing tariff in \$/hour (given this size)?

$$p_L = 0\$ \qquad p_H = 10\$$$

- What is the average net earning per unit per hour for the line owner?

$$3.5\$ / h$$

- Is the capacity of 10 the welfare maximizing size of the line? Why (not)?
  - Under perfect competition, owner would earn on average zero net profits per hour!
  - positive net profits attracts competition
  - Capacity increases, price decreases until net profit =0
  - Thus the line is now too small.

- What is the average total surplus W (consumer surplus + producer surplus) in \$/hour?

$$PS_L / h = 0\$ \qquad PS_H / h = 100\$ / h$$

$$CS_L / h = 32\$ / h \qquad CS_H / h = 150\$ / h$$

$$W / h = 0.5 \cdot (32) + 0.5 \cdot (250) = 0.5 \cdot (282) = 141$$

- A new transmission line has to be build. Once it will be running, demand per hour is given by:

$$P_L[Q_L] = 8 - Q_L$$

- for the hours 20:00-08:00

$$P_H[Q_H] = 20 - Q_H$$

- for the hours 08:00-20:00

- Amortized fixed cost per unit capacity per hour:
  - *f=1.5$*.
- The marginal cost of use
  - *mc=0$*.
- Imagine that it has already be decided that a line of size 18 will be built.
  - What is the welfare maximizing tariff in $/hour (given this size)?
  - What is the average earning per unit per hour for the line owner?
  - Is the capacity of 18 the welfare maximizing size of the line? Why (not)?
  - What is the average total surplus W (consumer surplus + producer surplus) in $/hour?

- What is the welfare maximizing tariff in $/hour (given this size)?

- What is the average net earning per unit per hour for the line owner?

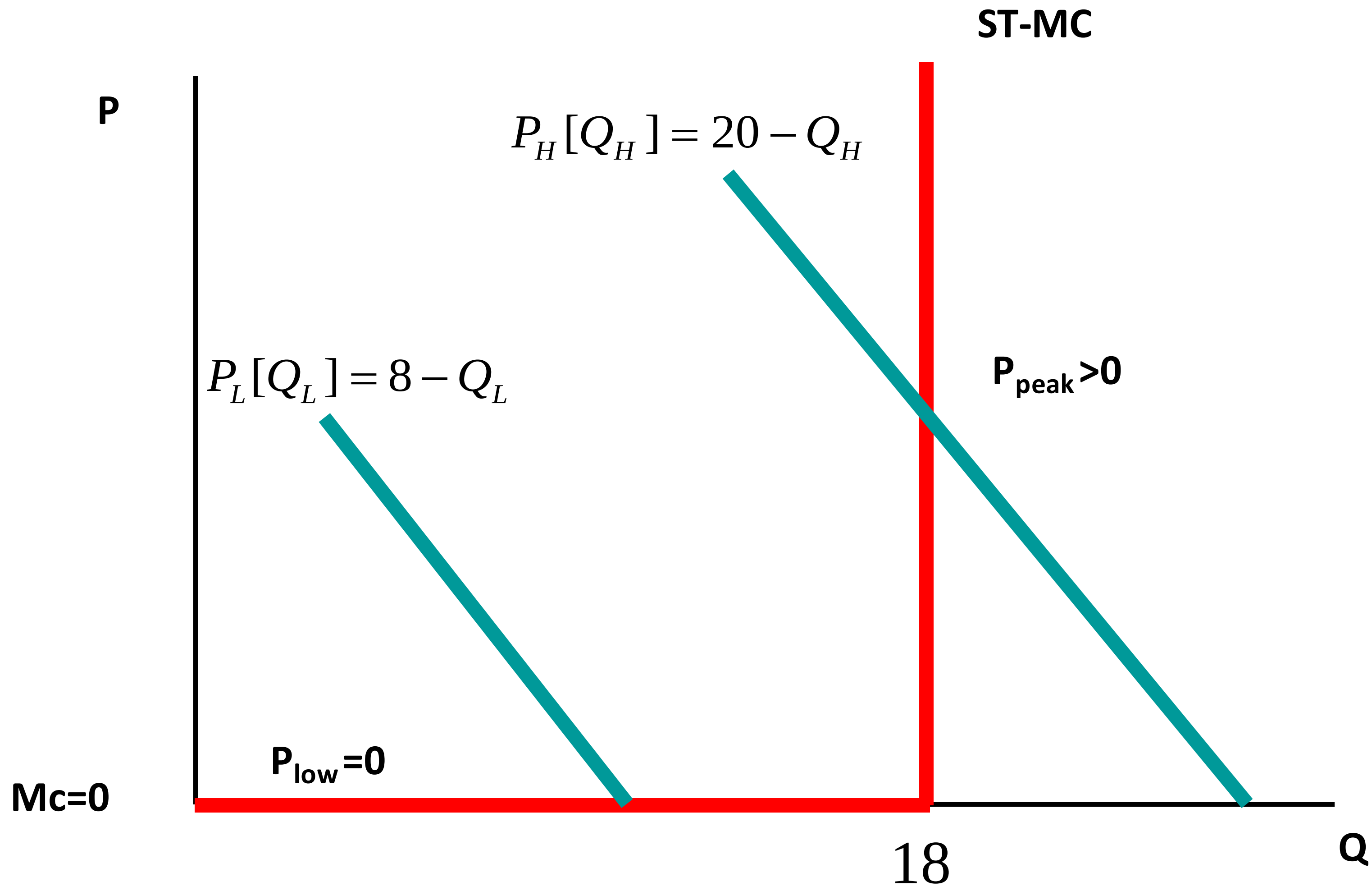
- Is the capacity of 18 the welfare maximizing size of the line? Why (not)?

- What is the average total surplus W (consumer surplus + producer surplus) in $/hour?

$p_L = mc = 0\$$

$D_L = 8$

$$\underbrace{P_H[Q_H] = 20 - Q_H \quad \& \quad D_H[p_H] = 18}_{p_H = 2\$}$$



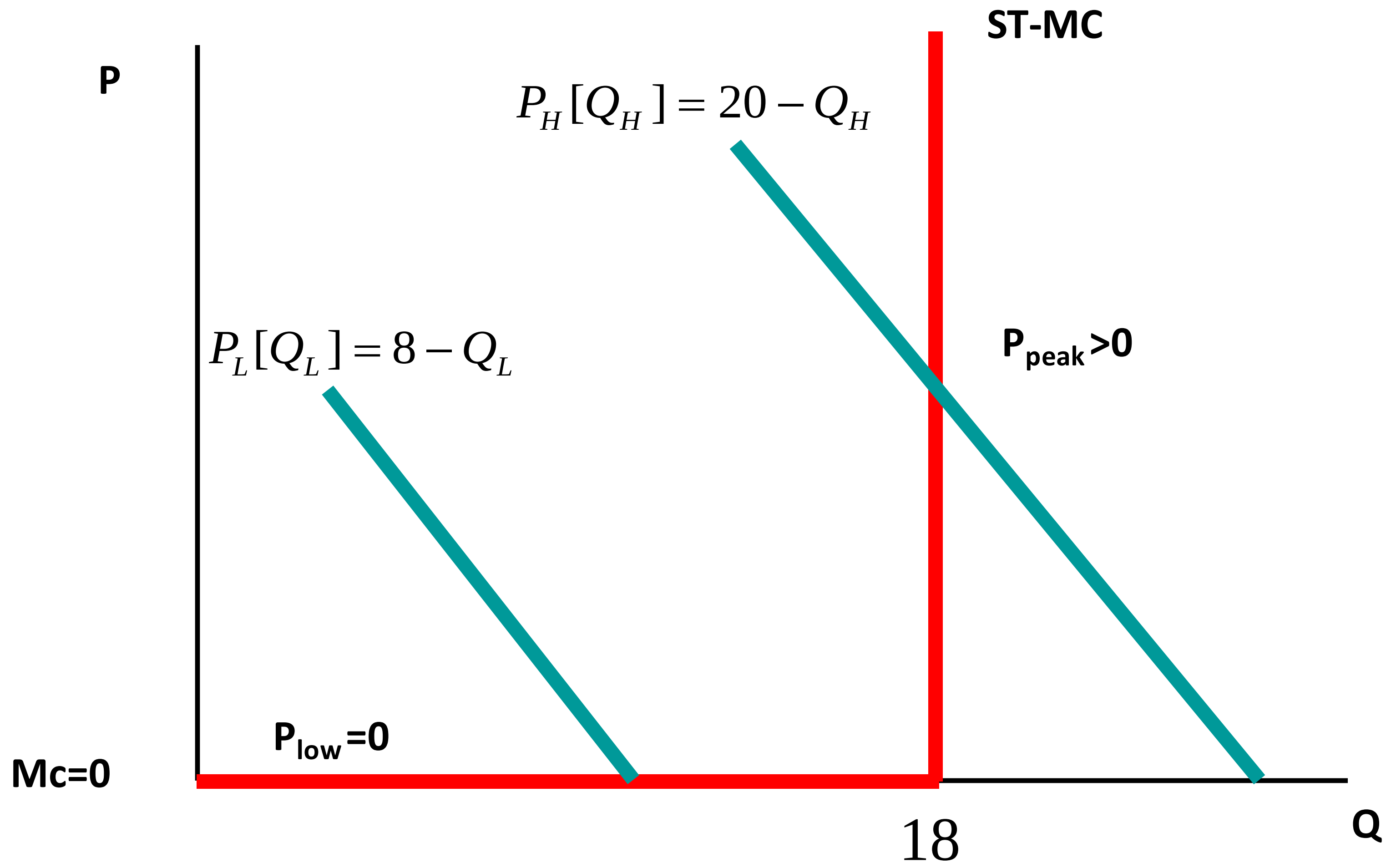
$p_L = 0\$$	$p_H = 2\$$
$D_L = 8$	$D_H = 18$

The average gross earnings per unit per hour:

$$0.5 \cdot p_L + 0.5 \cdot p_H = \quad 0.5 \cdot 0\$ + 0.5 \cdot 2\$ \quad = 1\$$$

The average net earnings (minus fixed costs) per unit per hour:

$$1\$ - 1.5\$ \quad = -0.5\$ / h$$



- What is the welfare maximizing tariff in $/hour (given this size)?

$$p_L = 0\$ \qquad p_H = 2\$$$

- What is the average net earning per unit per hour for the line owner?

$$-0.5\$ / h$$

- Is the capacity of 18 the welfare maximizing size of the line? Why (not)?
  - Under perfect competition, owner would earn on average zero net profits per hour!
  - negative net profits discourages investment
  - Capacity decreases, price increases until net profit =0
  - Thus the line is now too large.

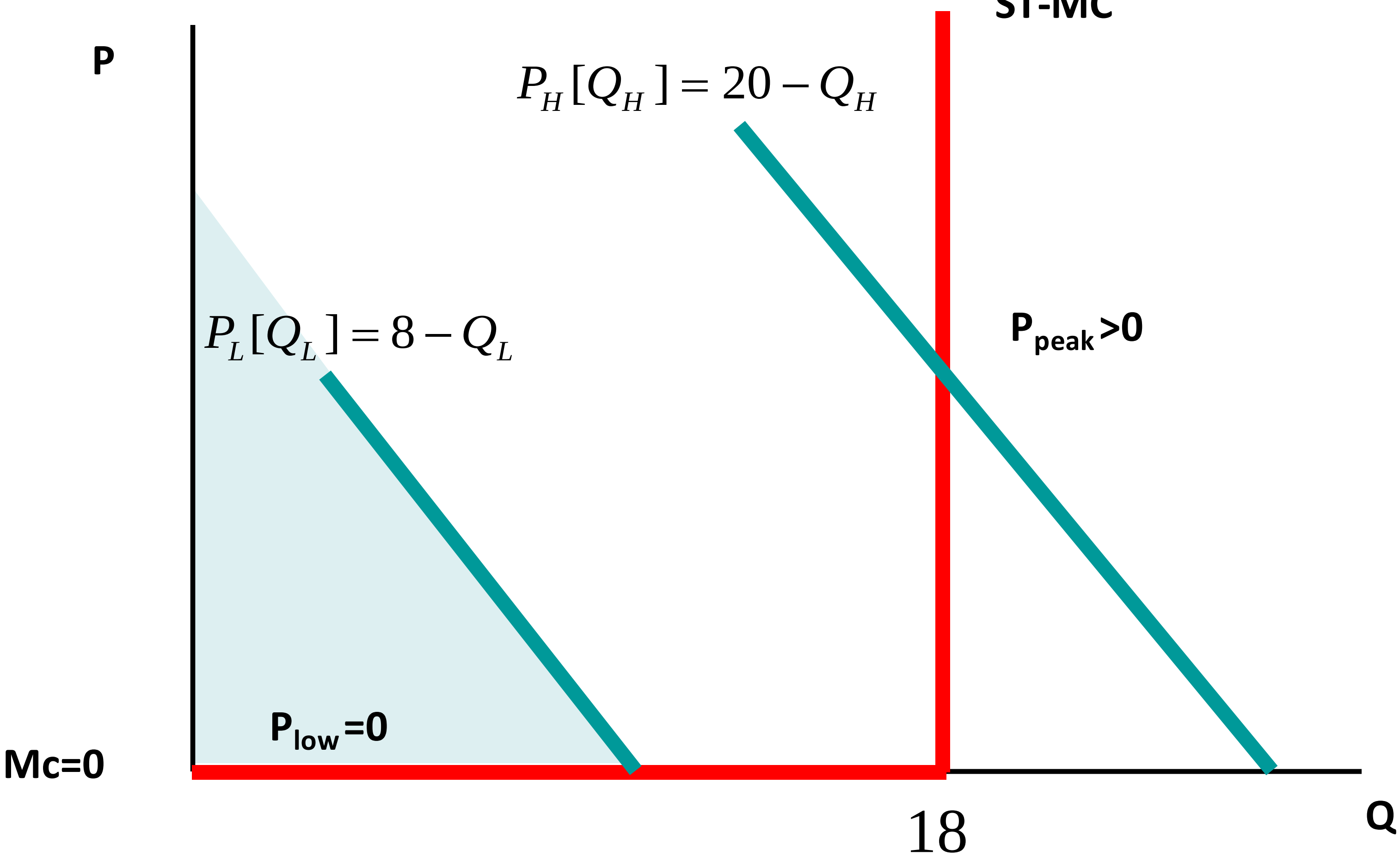
- What is the average total surplus W (consumer surplus + producer surplus) in $/hour?

$$PS_L / h = 0\$ \cdot D_L = 0\$$$

$$PS_H / h = 2\$ \cdot D_H = 2\$ \cdot 18 = 36\$ / h$$

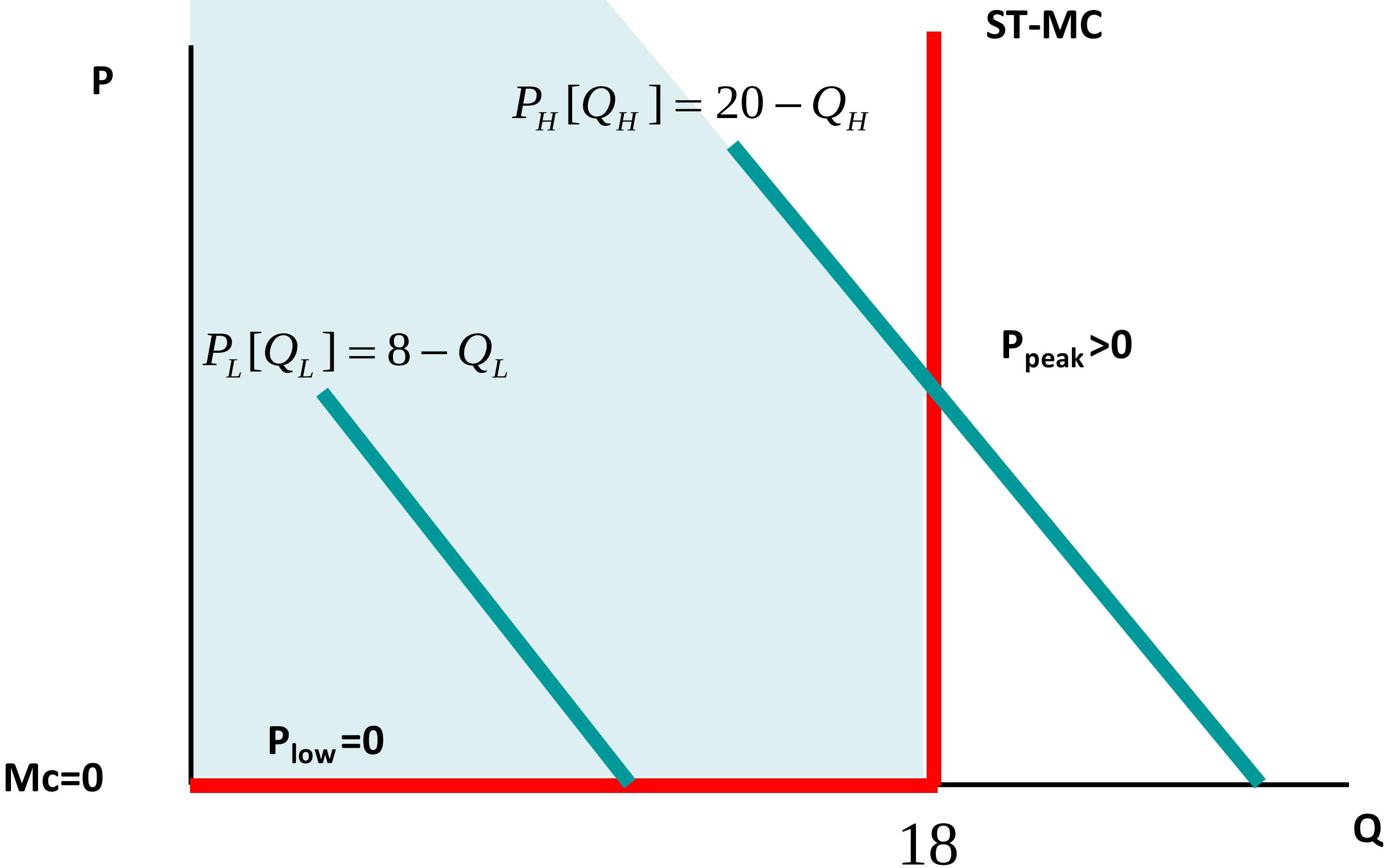
$$CS_L = 8 \cdot 8 \cdot 0.5 \;= 32\$/h$$

$$\int_0^8 (8-Q)\,dQ = [8Q - .5Q^2]_0^8 = 32$$



$$CS_H = 18 \cdot 18 \cdot 0.5 + 2 \cdot 18 = 162 + 36 = 198\$/h$$

$$\int_0^{18} (20 - Q)\, dQ = [20Q - .5Q^2]_0^{18} = 360 - 162 = 198$$



- What is the welfare maximizing tariff in $/hour (given this size)?

$$p_L = 0\$ \qquad p_H = 2\$$$

- What is the average net earning per unit per hour for the line owner?

$$-0.5\$/h$$

- Is the capacity of 18 the welfare maximizing size of the line? Why (not)?
  - Under perfect competition, owner would earn on average zero net profits per hour!
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  - Capacity decreases, price increases until net profit =0
  - Thus the line is now too large.

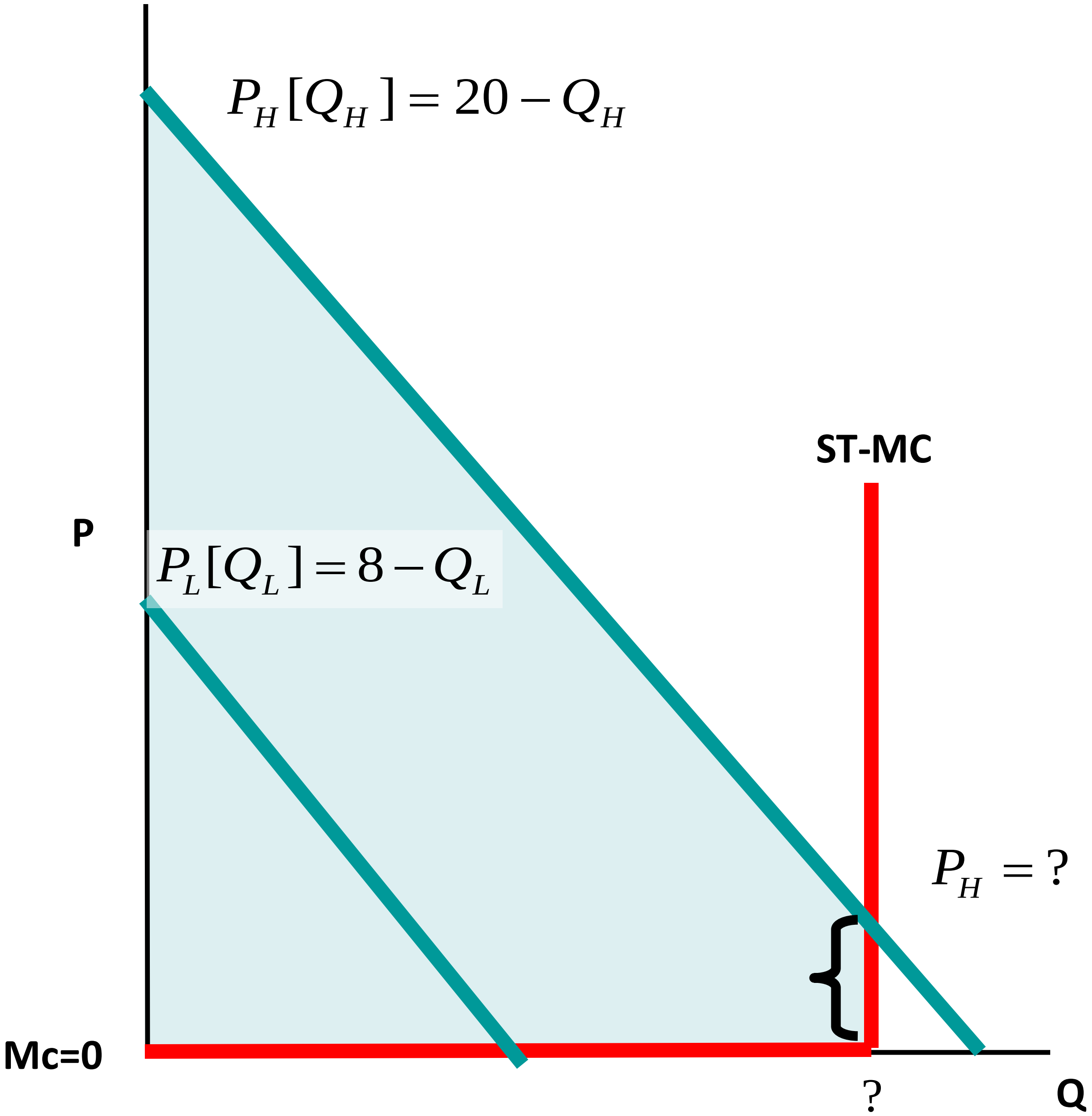
- What is the average total surplus W (consumer surplus + producer surplus) in $/hour?

$$PS_L / h = 0\$ \qquad CS_L / h = 32\$/h$$

$$PS_H / h = 36\$/h \qquad CS_H / h = 198\$/h$$

$$W/h = 0.5\cdot(32) + 0.5\cdot(234) = 0.5\cdot(266) = 133$$

So what is the optimal size of the line?

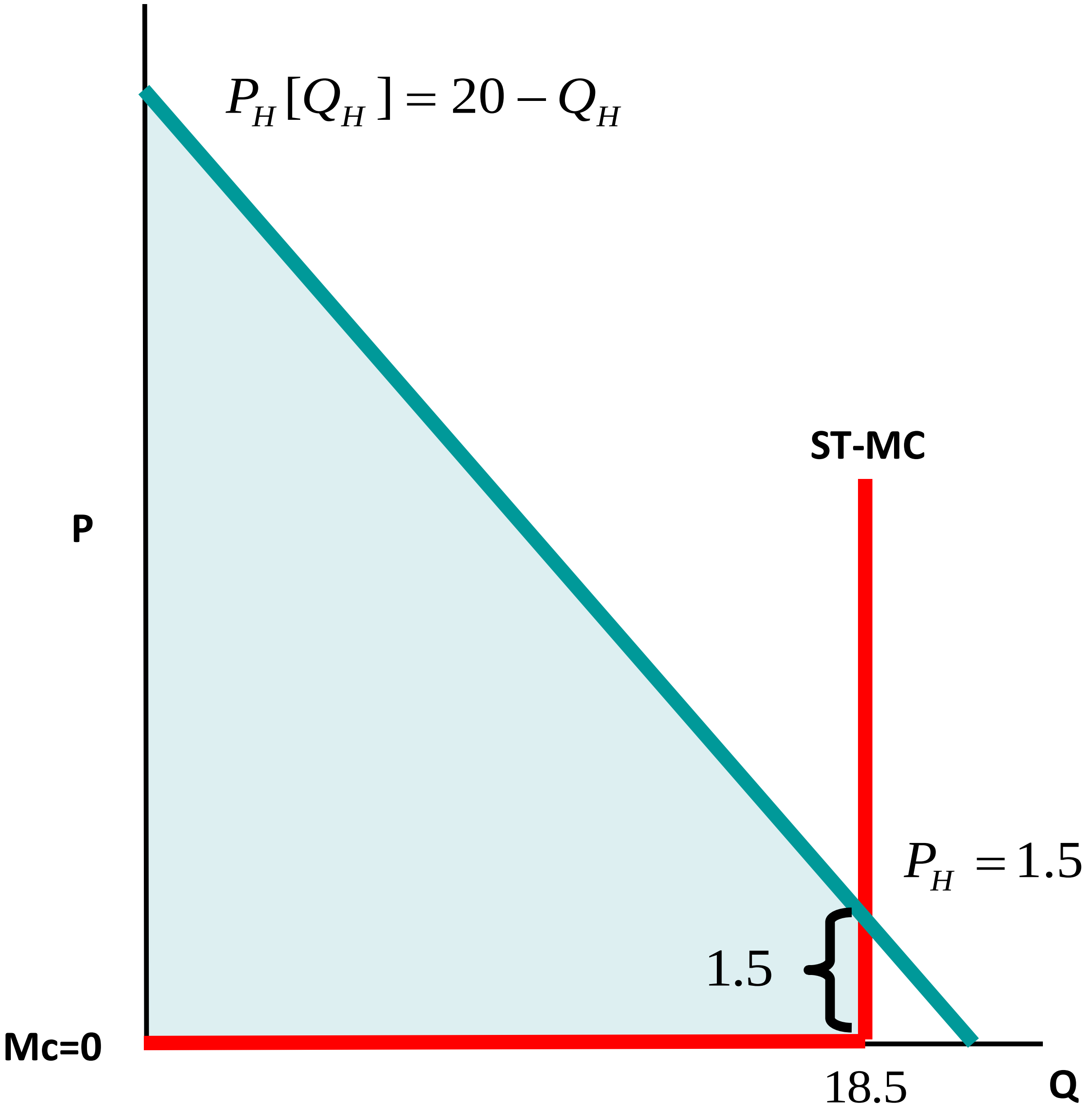


Supply side (production):

$mc = 0$

$f = 1.5$

- One demand curve – with 1 probability

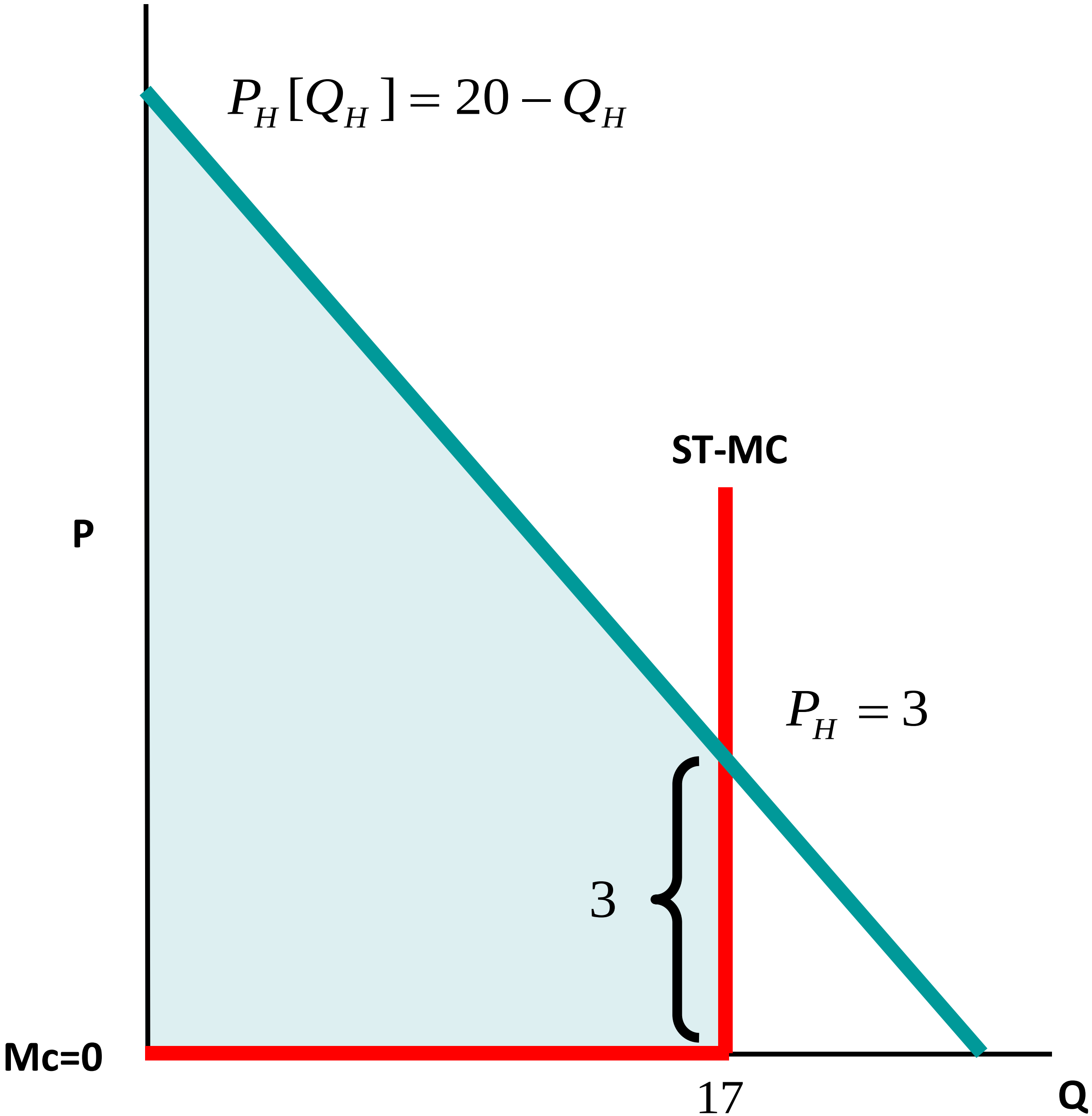


Supply side (production):

$mc = 0$

$f = 1.5$

- One demand curve – but only ½ probability
  - Otherwise no demand

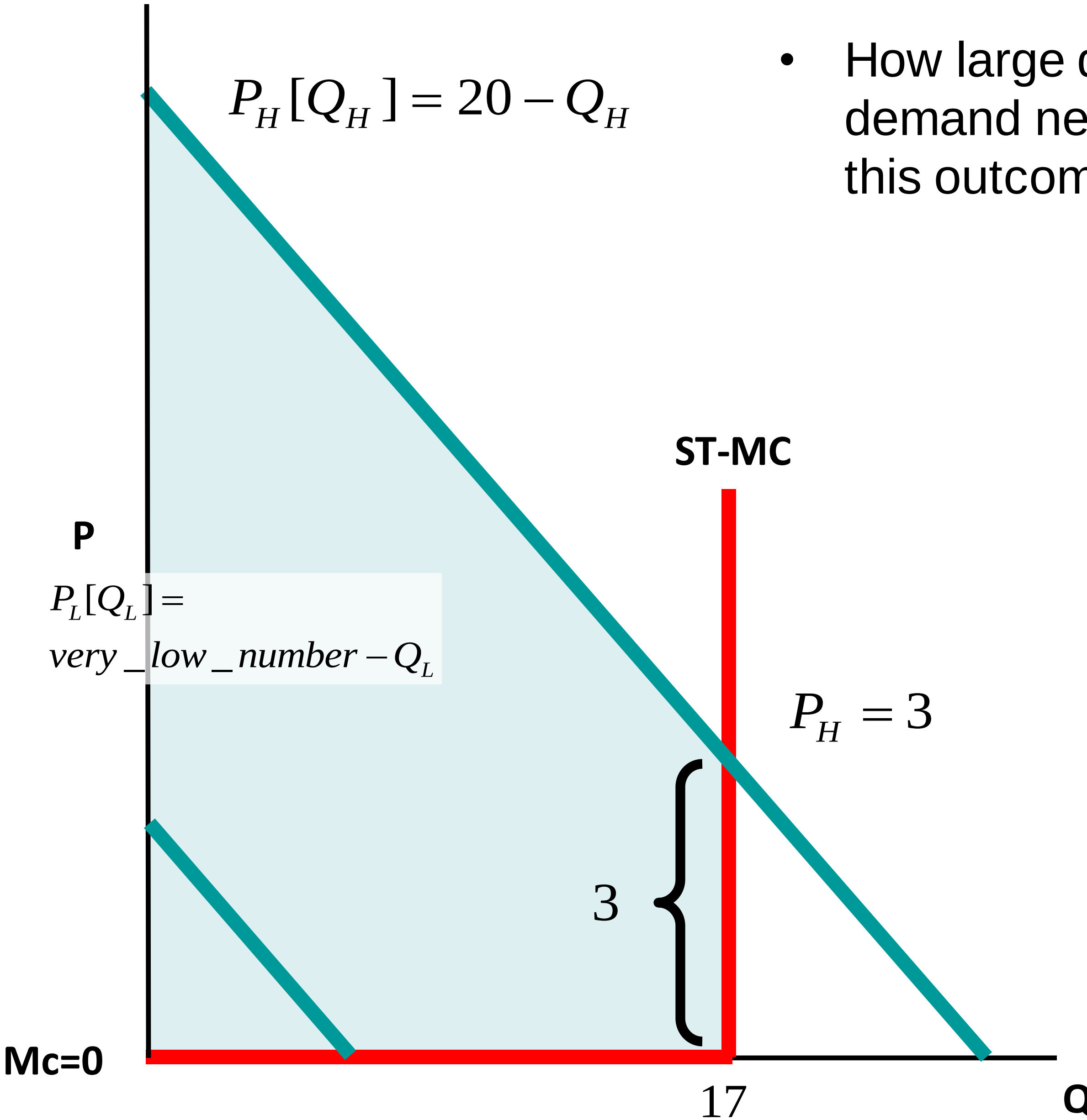


Supply side (production):

$mc = 0$

$f = 1.5$

- Two demand curve – each ½ probability
  - Second demand curve too low to contribute to fix cost recovery



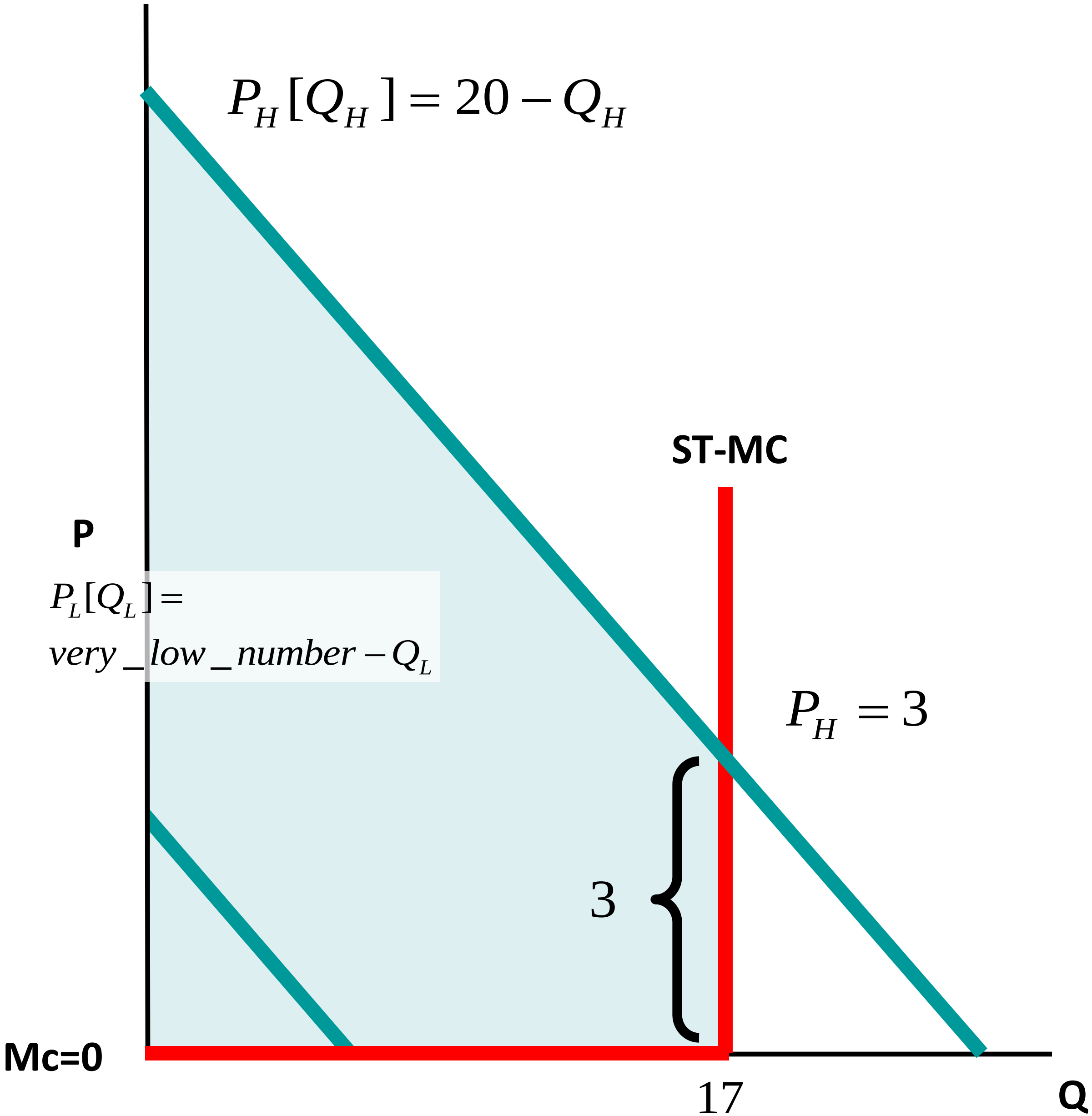
- How large does the lower demand need to be to affect this outcome?

Supply side (production):

$mc = 0$

$f = 1.5$

- Two demand curve – each ½ probability
  - Second demand curve too low to contribute to fix cost recovery

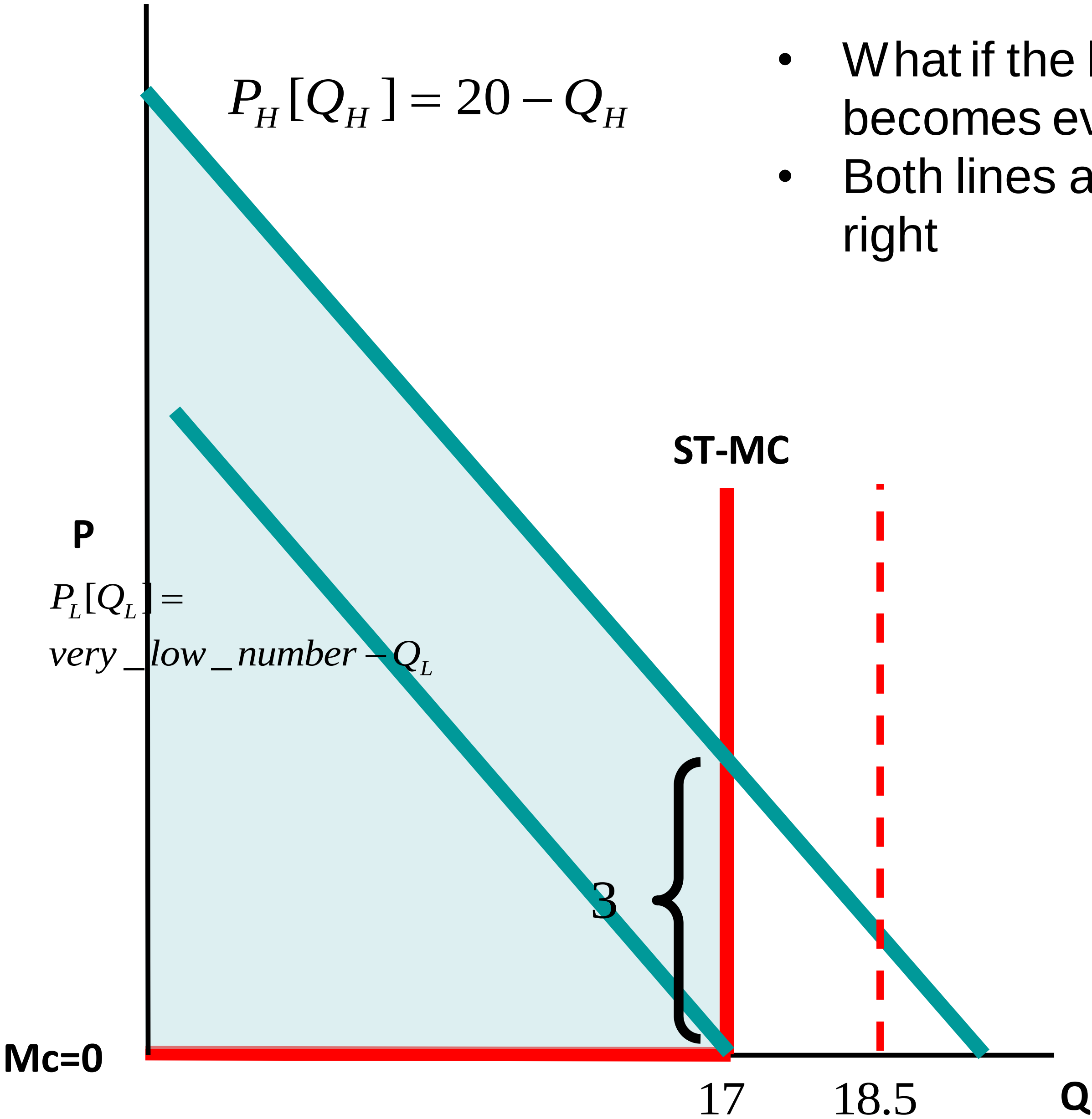


Supply side (production):

$mc = 0$

$f = 1.5$

- Two demand curve – each ½ probability
  - Second demand curve too low to contribute to fix cost recovery



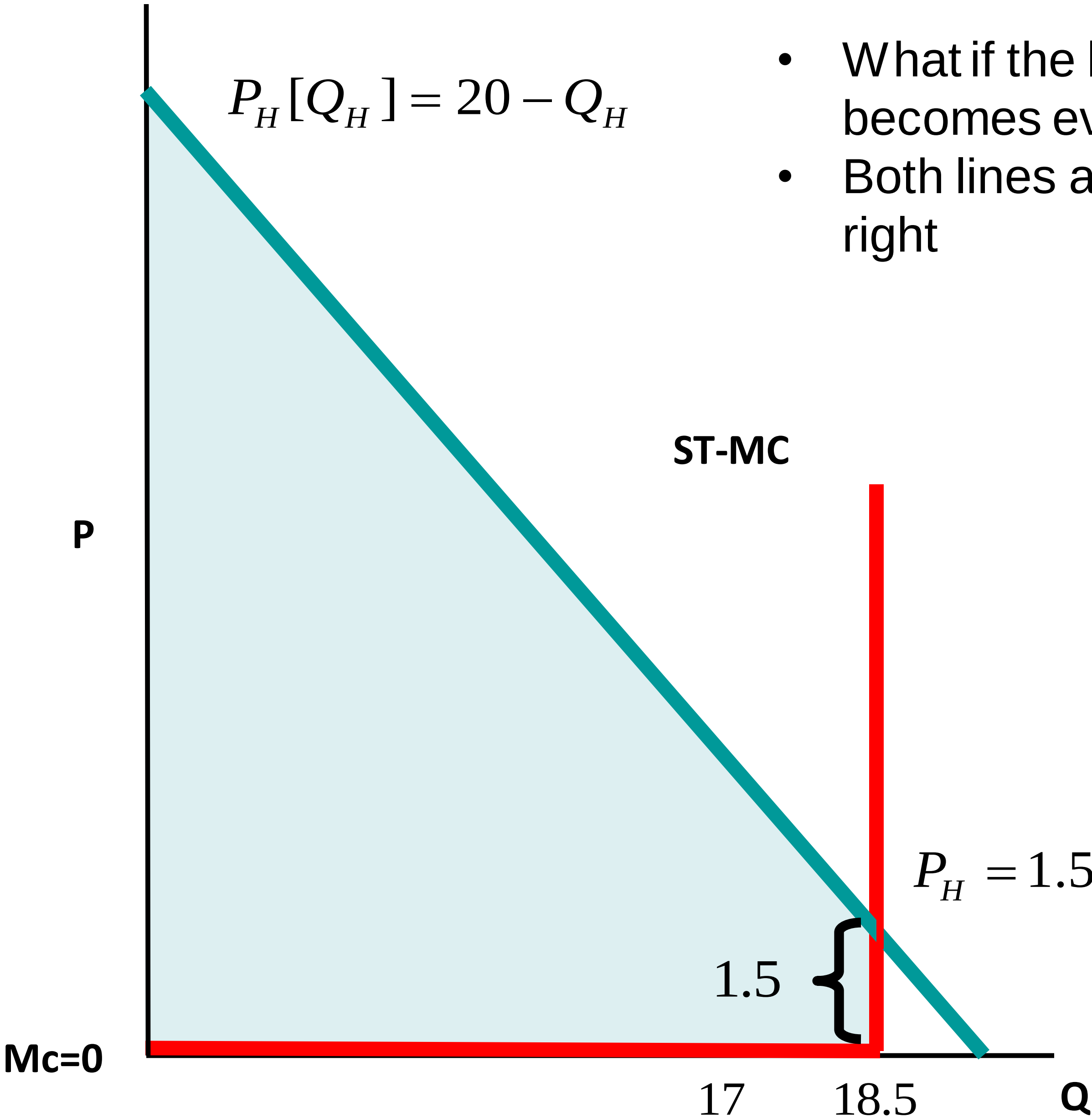
- What if the lower demand becomes even larger?
- Both lines are moving to the right

Supply side (production):

$$mc = 0$$

$$f = 1.5$$

- Two demand curve – each ½ probability
  - Second demand curve too low to contribute to fix cost recovery



- What if the lower demand becomes even larger?
- Both lines are moving to the right

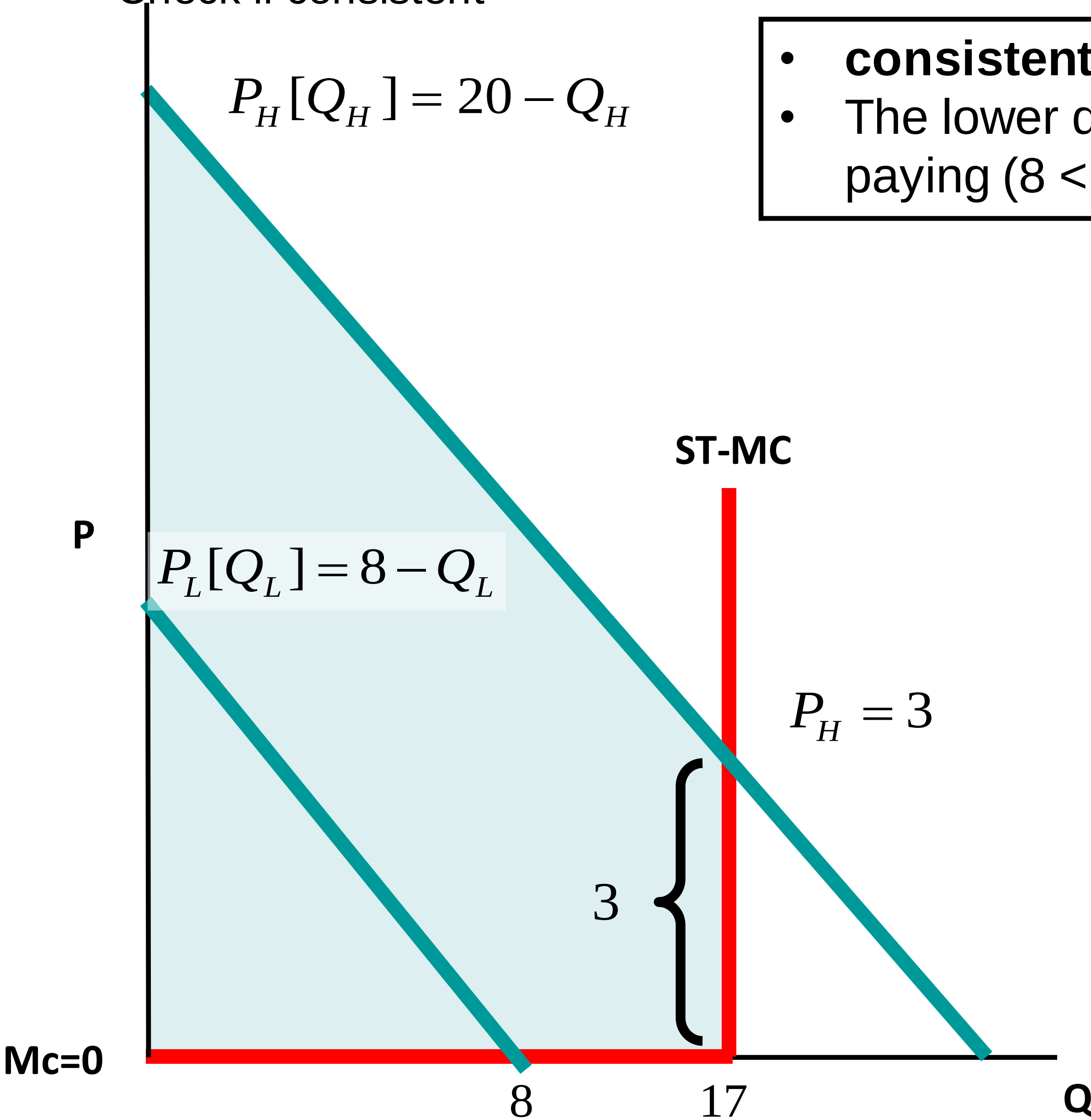
Supply side (production):

$mc = 0$

$f = 1.5$

- How to solve it

- Two demand curve – each ½ probability
  - **Assume** 2nd demand curve too low to contribute to fix cost recovery
  - Check if consistent



- **consistent!**
- The lower demand is not paying (8 < 17)

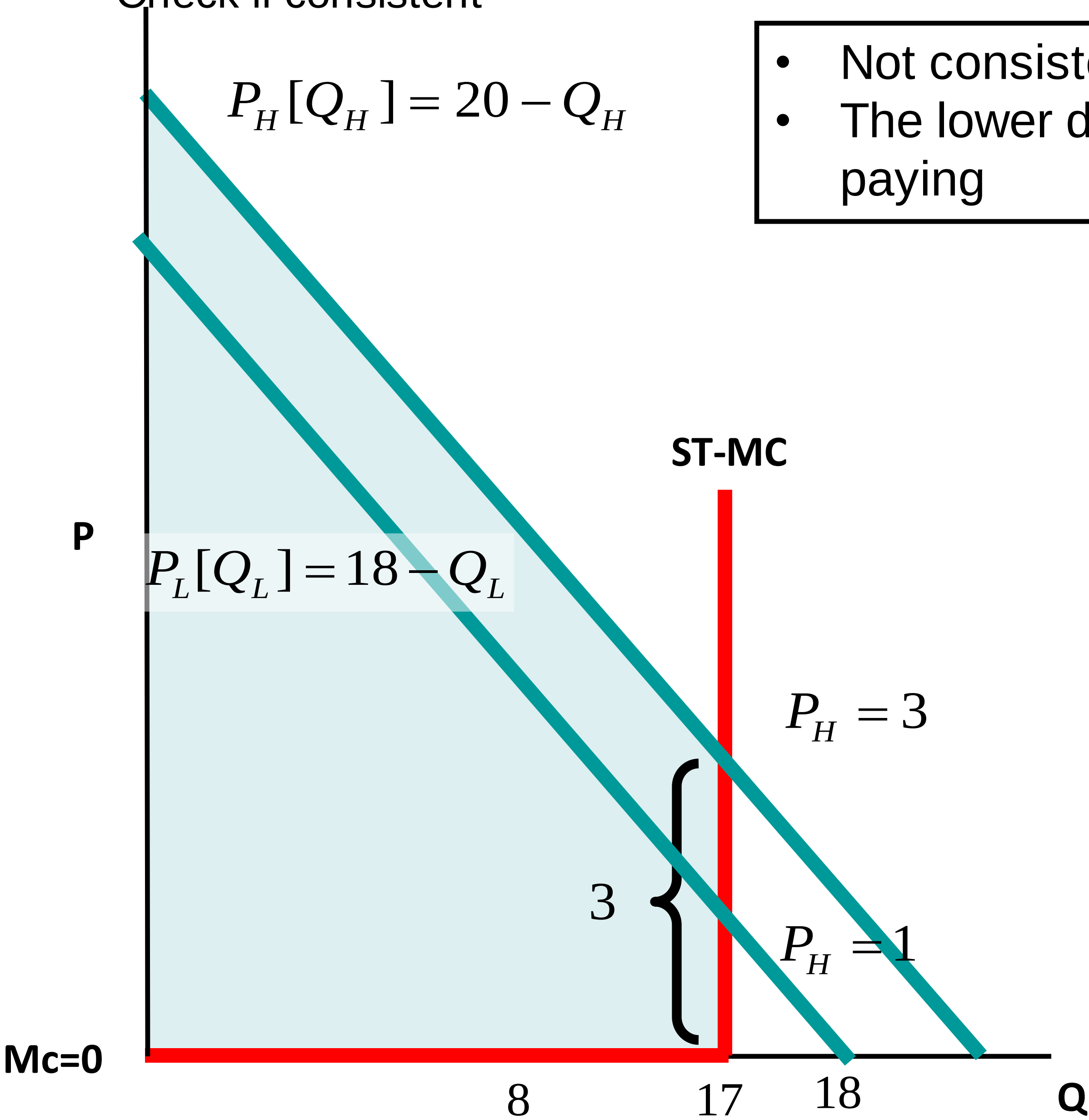
Supply side (production):

$$mc = 0$$

$$f = 1.5$$

- Another example

- Two demand curve – each ½ probability
  - **Assume** 2nd demand curve too low to contribute to fix cost recovery
  - Check if consistent



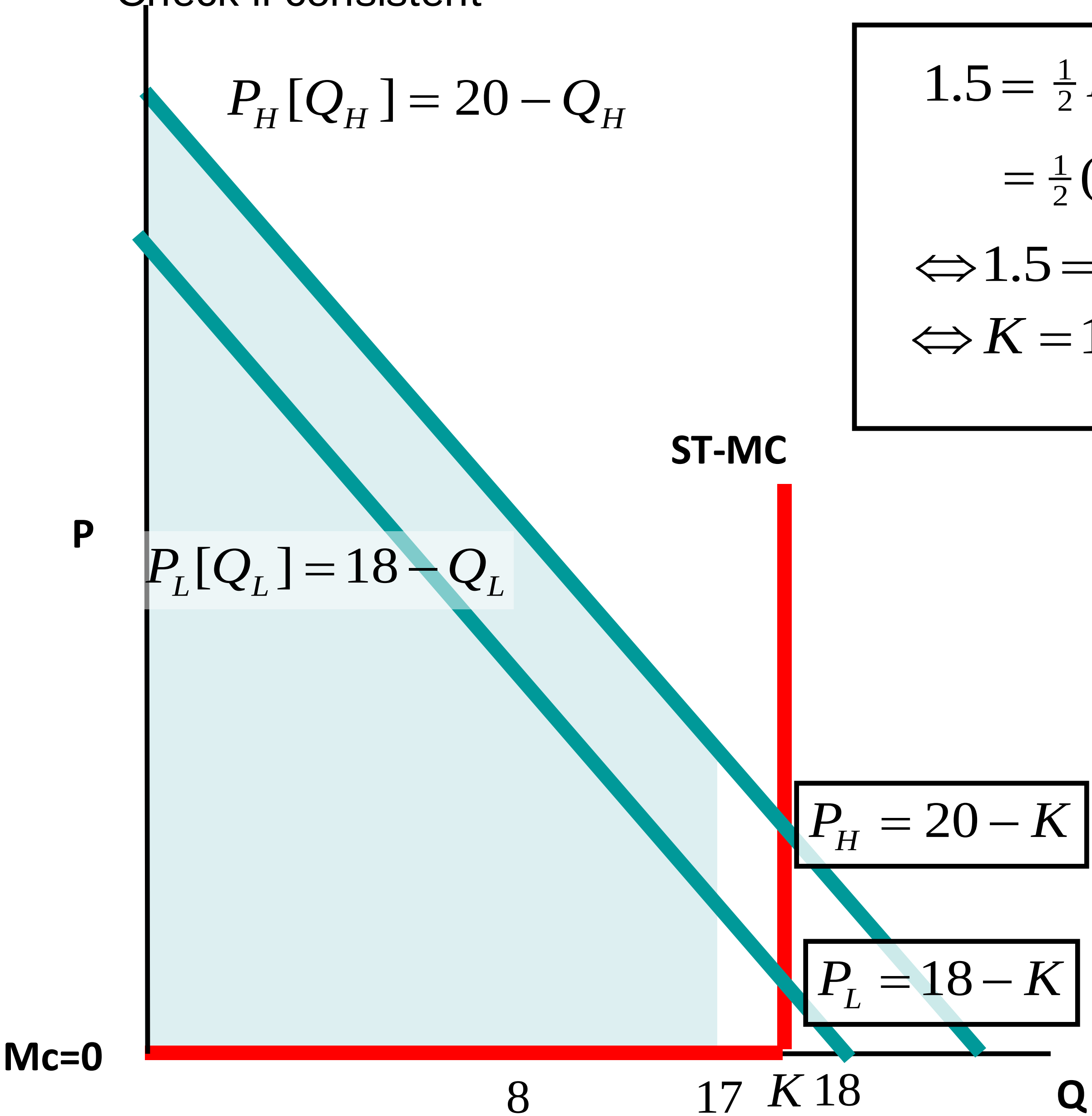
- Not consistent!
- The lower demand is also paying

Supply side (production):

$$mc = 0$$

$$f = 1.5$$

- Two demand curve – each ½ probability
  - **Assume** both demands contribute to fix cost recovery
  - Check if consistent



$$1.5 = \tfrac{1}{2} P_H + \tfrac{1}{2} P_L$$
$$= \tfrac{1}{2}(20 - K) + \tfrac{1}{2}(18 - K)$$
$$\Leftrightarrow 1.5 = 19 - K$$
$$\Leftrightarrow K = 17.5$$

Supply side (production):

$$mc = 0$$
$$f = 1.5$$

Peak-load pricing

Biggar, p.21

$$C[Q,K] = C_{ST}[Q,K] + C_{LT}[Q,K]$$

Cost function: $C_{ST}[Q,K] = cQ, \text{ s.t } Q \leq K \quad C_{LT}[Q,K] = f \cdot K$

$$C[Q,K] = cQ + f \cdot K, \text{ where } Q \leq K$$

- ***f* is the cost of capacity**
  - For a unit of capacity (eg a MW):
  - Amortized over the useful life-time
  - EG: lifetime of 30 year ≈300.000 hours
    - Important practical problems:
      - **what is the right interest rate?**
      - what is the right life-time? (30 versus 40 years)
        - » Not that important
      - **What is the REALIZED time of construction (2 versus 10 years)**
      - **What is the REALIZED cost of building?**

Peak-load pricing

Biggar, p.21

Cost function: $$C[Q,K] = cQ + f \cdot K, \text{ where } Q \le K$$

$$\text{when } c = 0$$

$$C[Q,K] = f \cdot K, \text{ where } Q \le K$$

State-dependent Utility functions $U_S[Q_S]$ with probability α_S

Welfare maximization

$$Max_{Q_S,K} \sum_s \left(\alpha_S \cdot U_S[Q_S]\right) - f \cdot K \qquad \text{such that } Q \le K$$

Write out Lagrangian:

$$L[\vec{Q}_S, K, \vec{\mu}_S] = \sum_s \alpha_S \cdot \left(U_S[Q_S] - cQ_S\right) - f \cdot K + \sum_s \alpha_S \cdot \mu_S (K - Q_S)$$

Peak-load pricing

$$L[\vec{Q}_S, K, \vec{\mu}_S] = \sum_s \alpha_S \cdot \left(U_S[Q_S] - cQ_S\right) - f \cdot K + \sum_s \alpha_S \cdot \mu_S (K - Q_S)$$

$$Q_S: \quad U_S{}'[Q_S] = c + \mu_S \qquad \#S \text{ equations}$$

$$K: \quad f = \sum_s \alpha_S \mu_S \qquad 1 \text{ equation}$$

$$\mu_S: \quad \mu_S(K - Q_S) = 0, \mu_S \geq 0, (K - Q_S) \geq 0 \qquad \#S \text{ equations}$$

- If using up to capacity $Q_S = K$
  - Then $\mu \geq 0$ $\quad \Rightarrow U_S{}'[Q_S] = c + \mu_S \geq c$
- If using less than capacity $Q_S < K$
  - *Then* $\mu = 0$ $\quad \Rightarrow U_S{}'[Q_S] = c$
- The fixed costs are recovered by the charges μ
  - Thus only the ones that use the full capacity contribute to cost-recovery!
    - Is this what is applied?
    - Yes, but often not: costs are “socialized” (average tariffs over all consumers)
    - Thus not optimal (trade-off of efficiency, complexity and “fairness”)

Peak-load pricing

$$L[\vec{Q}_S, K, \vec{\mu}_S] = \sum_s \alpha_S \cdot \left(U_S[Q_S] - cQ_S\right) - f \cdot K + \sum_s \alpha_S \cdot \mu_S (K - Q_S)$$

$$Q_S: \quad U_S'[Q_S] = c + \mu_S \qquad \# S \text{ equations}$$

$$K: \quad f = \sum_s \alpha_S \mu_S$$

$$\mu_S: \quad \mu_S (K - Q_S) = 0, \mu_S \geq 0, (K - Q_S) \geq 0 \qquad \# S \text{ equations}$$

$$\mu_S: \quad \mu_S(K - Q_S) = 0, \mu_S \geq 0, (K - Q_S) \geq 0 \qquad \#S \text{ equations}$$

Example:

$$P_H[Q_H] = 20 - Q_H \quad P_L[Q_L] = 8 - Q_L \quad \alpha_H = \alpha_L = \tfrac{1}{2} \quad f = 1.5 \quad c = 0$$

$$Q_L: \quad U_L'[Q_L] = c + \mu_L \quad \Leftrightarrow 8 - Q_L = c + \mu_L \quad \Leftrightarrow 8 - Q_L = \mu_L$$

$$Q_H: \quad U_H'[Q_H] = c + \mu_H \Leftrightarrow 20 - Q_H = c + \mu_H \Leftrightarrow 20 - Q_H = \mu_H$$

$$\left(\text{as } U[Q] = \int_0^Q P[j]dj, \text{ thus } U'[Q] = P[Q]\right)$$

$$K: \quad f = \sum_S \alpha_S \mu_S = \alpha_L \mu_L + \alpha_H \mu_H \Leftrightarrow 1.5 = 0.5\mu_L + 0.5\mu_H$$

Inspect the case $Q_L < K$ and $Q_H = K$ (formally all 4 combinations should be checked)

$$\mu_L = 0 \qquad \mu_H \geq 0$$

$$\mu_H \geq 0 \Leftrightarrow K: 1.5 = 0.5\mu_L + 0.5\mu_H = 0.5\mu_H \Leftrightarrow \mu_H = 3$$

$$\mu_H = 3 \Leftrightarrow Q_H: 20 - Q_H = 3 \quad \boxed{\Leftrightarrow Q_H = 17} \quad \boxed{\Leftrightarrow K = 17}$$

$$\mu_L = 0 \Leftrightarrow Q_L: 8 - Q_L = 0 \quad \boxed{\Leftrightarrow Q_L = 8} \quad \boxed{P_L = 0, P_H = 3}$$

$$\mu_S: \quad \mu_S(K-Q_S)=0, \mu_S \geq 0, (K-Q_S) \geq 0 \qquad \# S \text{ equations}$$

Example:

$$P_H[Q_H]=20-Q_H \quad P_L[Q_L]=18-Q_L \quad \alpha_H=\alpha_L=\tfrac{1}{2} \quad f=1.5 \quad c=0$$

$$Q_L: \quad U_L'[Q_L]=c+\mu_L \iff 18-Q_L=c+\mu_L \iff 18-Q_L=\mu_L$$

$$Q_H: \quad U_H'[Q_H]=c+\mu_H \iff 20-Q_H=c+\mu_H \iff 20-Q_H=\mu_H$$

$$\left(\text{as } U[Q]=\int_0^Q P[j]dj, \text{ thus } U'[Q]=P[Q]\right)$$

$$K: \quad f=\sum_S \alpha_S\mu_S = \alpha_L\mu_L+\alpha_H\mu_H \iff 1.5=0.5\mu_L+0.5\mu_H$$

Inspect the case $Q_L < K$ and $Q_H = K$ (formally all 4 combinations should be checked)

$$\mu_L=0 \qquad \mu_H \geq 0$$

$$\mu_H \geq 0 \iff K: 1.5=0.5\mu_L+0.5\mu_H = 0.5\mu_H \iff \mu_H=3$$

$$\mu_H=3 \iff Q_H: 20-Q_H=3 \quad \boxed{\iff Q_H=17} \quad \boxed{\iff K=17}$$

$$\mu_L=0 \iff Q_L: 18-Q_L=0 \quad \boxed{\iff Q_L=18} \quad \boxed{\iff Q_L=18>17=K!!!}$$

$$\mu_S: \quad \mu_S(K - Q_S) = 0, \mu_S \geq 0, (K - Q_S) \geq 0 \qquad \# S \text{ equations}$$

Example:

$$P_H[Q_H] = 20 - Q_H \quad P_L[Q_L] = 18 - Q_L \quad \alpha_H = \alpha_L = \tfrac{1}{2} \quad f = 1.5 \quad c = 0$$

$$Q_L: \quad U_L{}'[Q_L] = c + \mu_L \quad \Leftrightarrow 18 - Q_L = c + \mu_L \quad \Leftrightarrow 18 - Q_L = \mu_L$$

$$Q_H: \quad U_H{}'[Q_H] = c + \mu_H \quad \Leftrightarrow 20 - Q_H = c + \mu_H \quad \Leftrightarrow 20 - Q_H = \mu_H$$

$$\left(\text{as } U[Q] = \int_0^Q P[j]dj, \text{ thus } U'[Q] = P[Q] \right)$$

$$K: \quad f = \sum_s \alpha_S \mu_S = \alpha_L \mu_L + \alpha_H \mu_H \quad \Leftrightarrow 1.5 = 0.5\mu_L + 0.5\mu_H$$

Inspect the case $Q_L = K$ and $Q_H = K$ (formally all 4 combinations should be checked)

$$Q_L: 18 - Q_L = \mu_L \quad \Leftrightarrow \mu_L = 18 - K \qquad \mu_L = 0.5$$

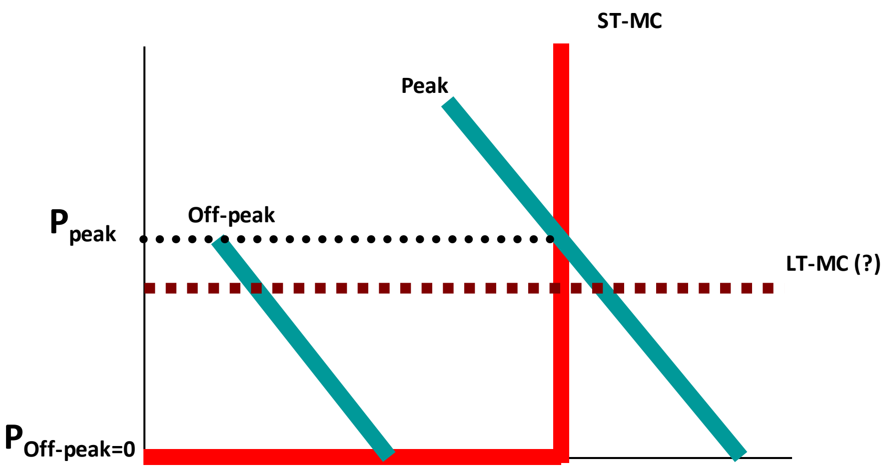
$$Q_H: 20 - Q_H = \mu_H \quad \Leftrightarrow \mu_H = 20 - K \qquad \mu_H = 2.5$$

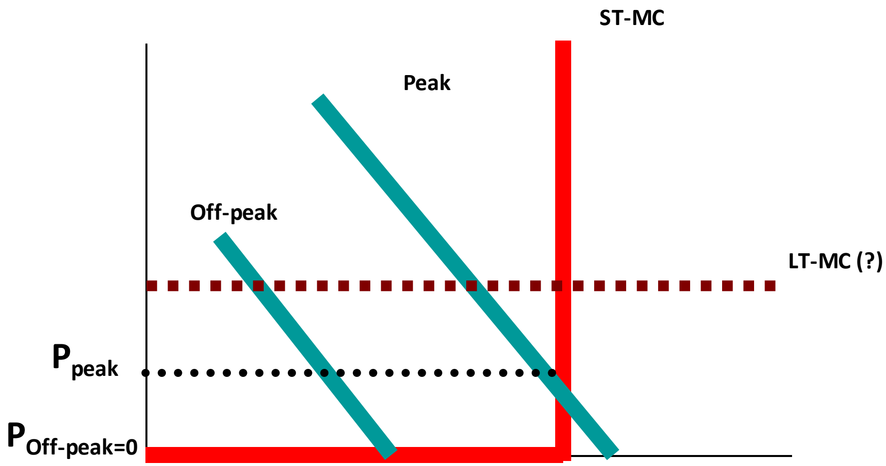
$$K: 1.5 = 0.5\mu_L + 0.5\mu_H$$
$$= 0.5(18 - K) + 0.5(20 - K) = 19 - K$$
$$\Leftrightarrow K = 19 - 1.5 = 17.5$$

$$\boxed{K = 17.5 \quad P_L = 0.5 \quad P_H = 2.5}$$

Peak-load pricing

Gives clear economic signals!





Invest in expansion of transmission capacity on the line

Do not invest in expansion

(wait or even remove a line)

- Nodal pricing is a generalization of peak-load pricing

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