Microeconomics 3

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

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MLÁDEŽE A TĚLOVÝCHOVY

Lecture 1

Silvester van Koten

- The presentation is based on the following sources:

- Snyder Christopher M Walter Nicholson and Robert Stewart. 2015. Microeconomic Theory : Basic Principles and Extensions EMEA ed. Andover: Cengage Learning. +
- Varian Hal R. 2020. Intermediate Microeconomics : A Modern Approach Ninth ed. New York: W.W. Norton & Company. +
- Excerpts from Gravelle H O Hart and R Rees. 2004. Advanced Microeconomics.

3 elasticities

$$\varepsilon = \frac{\Delta Q / Q}{\Delta P / P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

Price elasticity of demand

$$\xi = \frac{\Delta Q / Q}{\Delta Y / Y} = \frac{\Delta Q}{\Delta Y} \cdot \frac{Y}{Q}$$

Income elasticity of demand

$$\xi^{Q_{pork}}_{P_{beef}} = \frac{\Delta Q_{pork} / Q_{pork}}{\Delta P_{beef} / P_{beef}} = \frac{\Delta Q_{pork}}{\Delta P_{beef}} \cdot \frac{P_{beef}}{Q_{pork}}$$

Cross-price elasticity of demand

$$\xi^{Q_{pork}}_{P_{beef}} = \frac{\Delta Q_{pork} / Q_{pork}}{\Delta P_{beef} / P_{beef}} = \frac{\Delta Q_{pork}}{\Delta P_{beef}} \cdot \frac{P_{beef}}{Q_{pork}}$$

Cross-price elasticity of demand

$\boxed{\xi^{Q_{pork}}_{P_{beef}}}$ > or < 0? >0 **Substitutes**

$\boxed{\xi^{Q_{\text{Coca Cola}}}_{P_{\text{Pepsi Cola}}}}$

> or < 0? >0 **Substitutes**

$\boxed{\xi^{Q_{\text{Computers}}}_{P_{\text{El. Chips}}}}$

> or < 0? <0 **Complements!**

$$\varepsilon = \frac{\Delta Q / Q}{\Delta P / P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

Price elasticity of demand

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Cross-price elasticity of demand

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Income elasticity of demand

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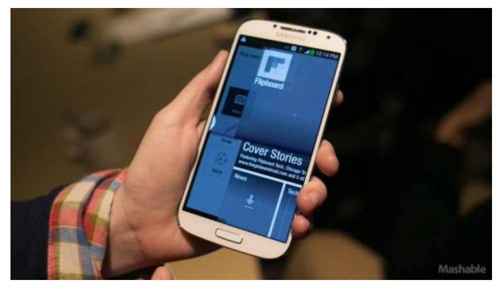
Income elasticity of demand

- Examples:
  - Beer: 0.88
  - Wine: 1.38

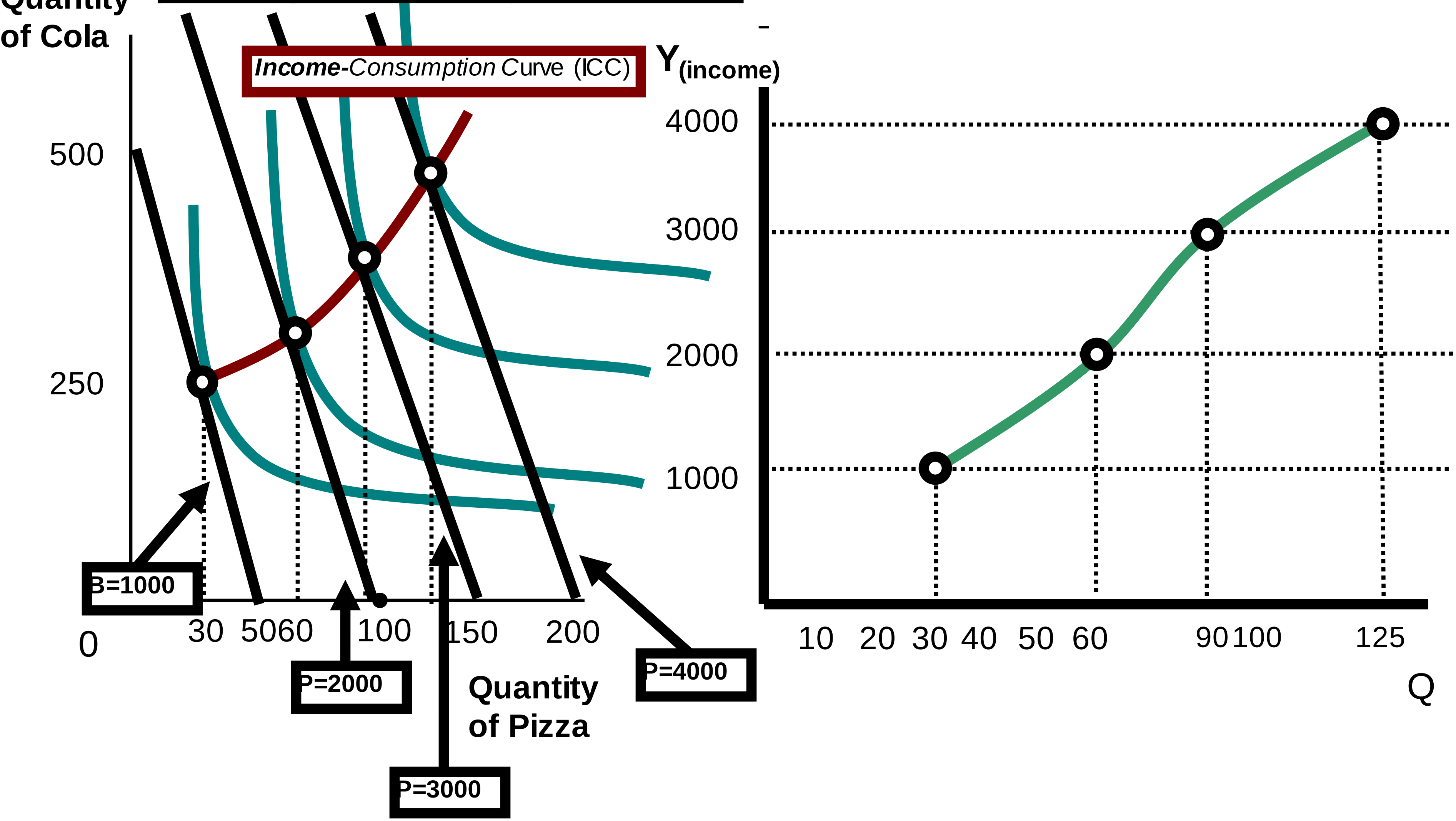
$$\boxed{\xi > 0}$$



“normal goods”

Budget	Price of Cola	Price of Pizza
$4000	$2	$20

Engel **curve for Pizza**



$$\xi = \frac{\Delta Q / Q}{\Delta Y / Y} = \frac{\Delta Q}{\Delta Y} \cdot \frac{Y}{Q}$$

Income elasticity of demand

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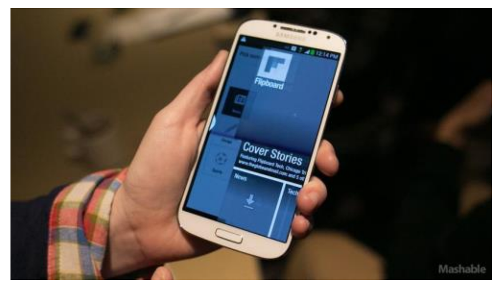
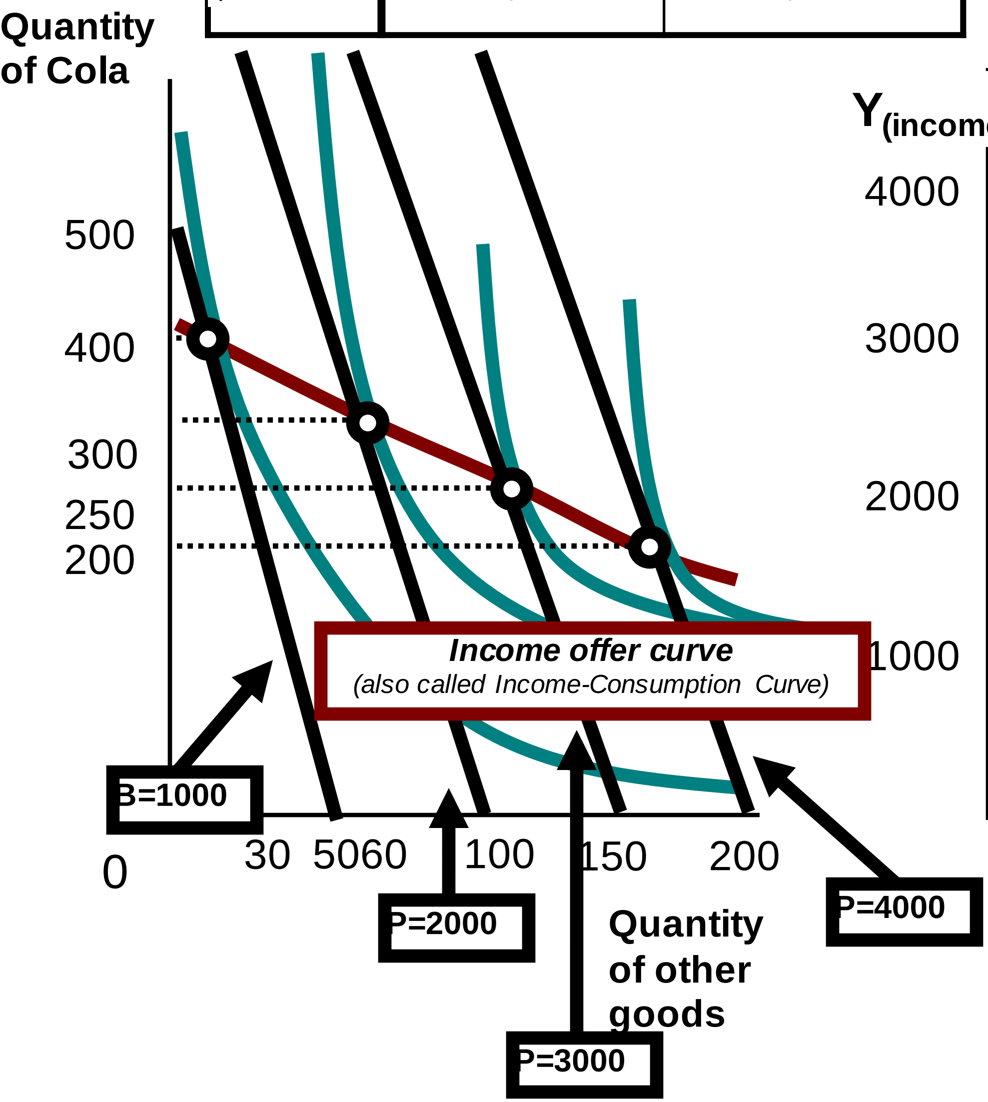


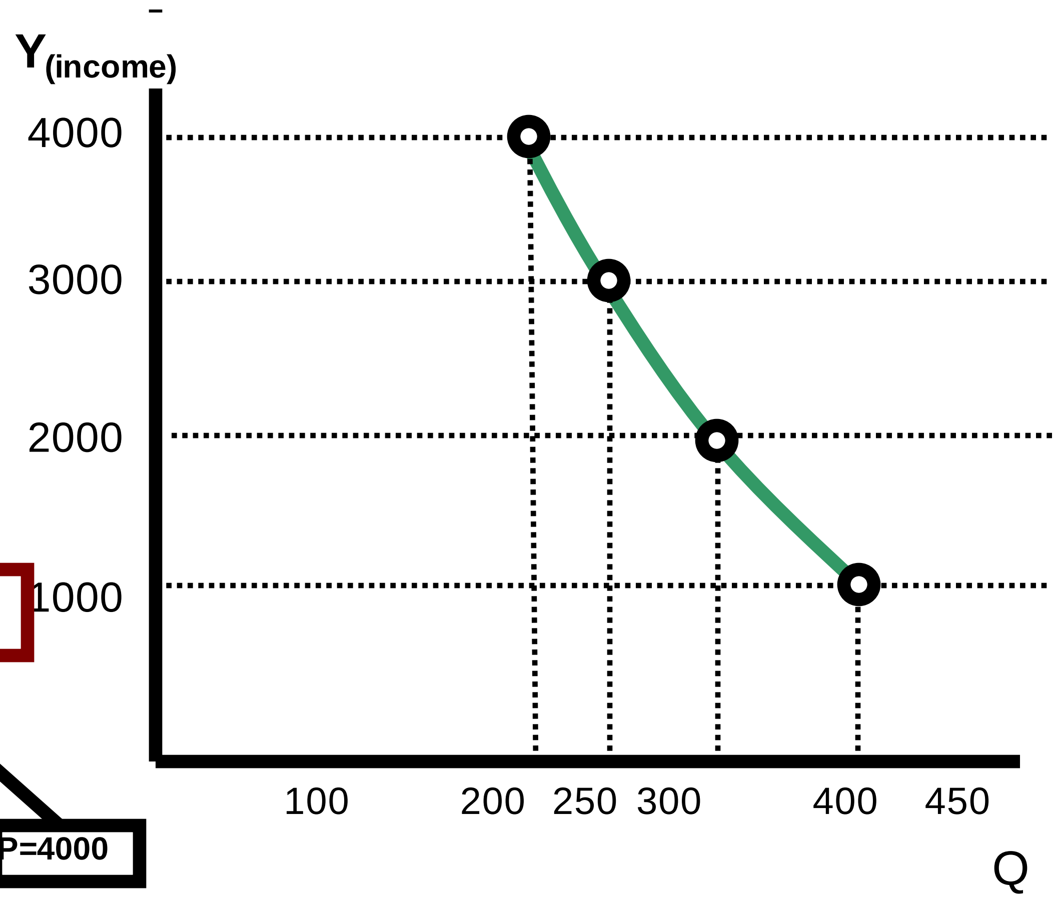
"normal goods"

$$\boxed{\xi < 0}$$

Budget	Price of Cola	Price of Pizza
$4000	$2	$20



Engel curve for Cola



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Income elasticity of demand

- Examples:
  - Beer: 0.88
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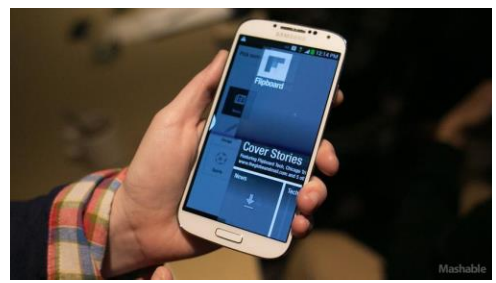


"normal goods"

$$\boxed{\xi < 0}$$

77 Calories Per 100 grams

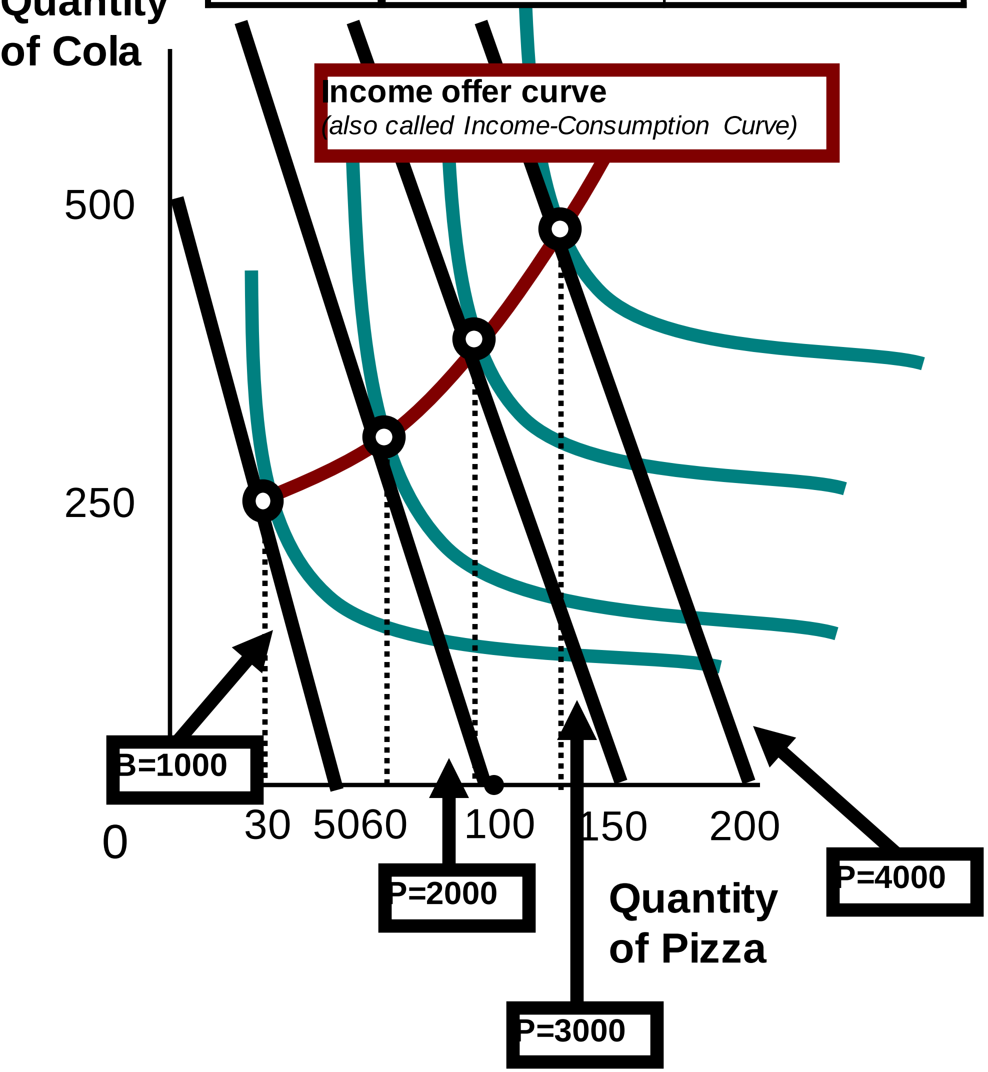
111 Calories Per 100 grams

The Pirate Bay

"inferior goods"

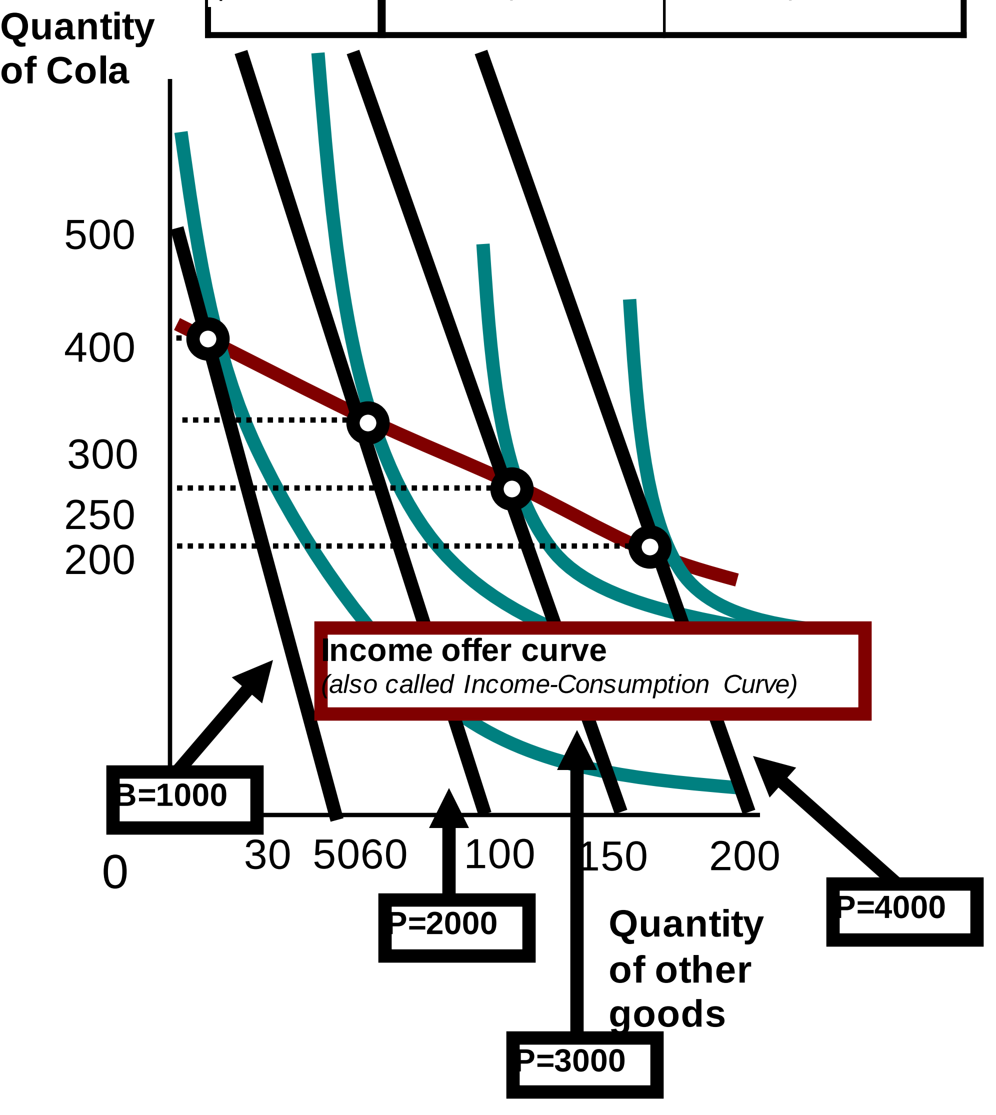
Cola and Pizza normal goods

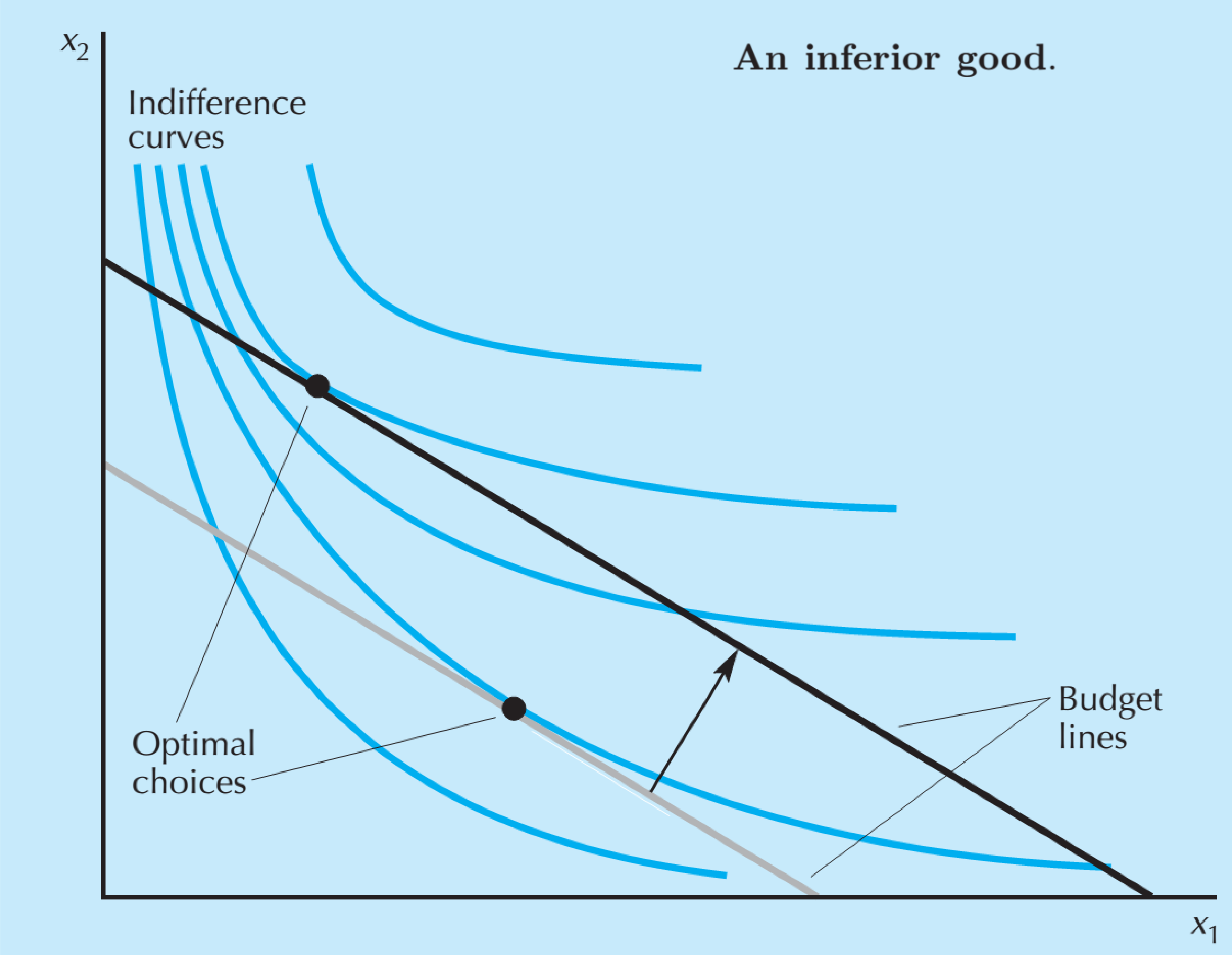
Budget	Price of Cola	Price of Pizza
$3000	$2	$20



Budget	Price of Cola	Price of Pizza
$4000	$2	$20

Pizza normal, Cola inferior




x_2
An inferior good.
Indifference curves
Optimal choices
Budget lines
x_1

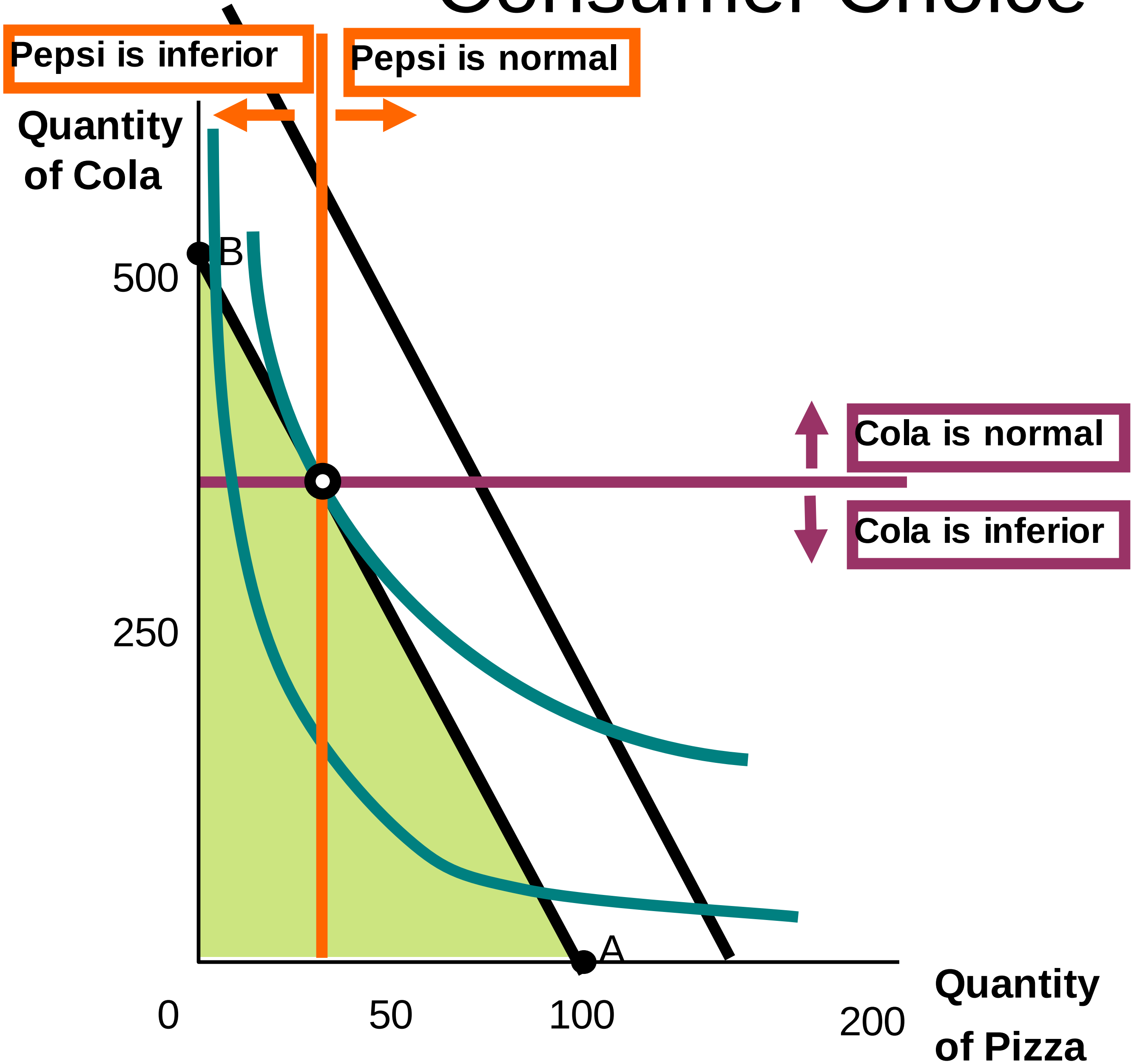
- Can a good be normal (or even luxurious) for some people, and inferior for others?



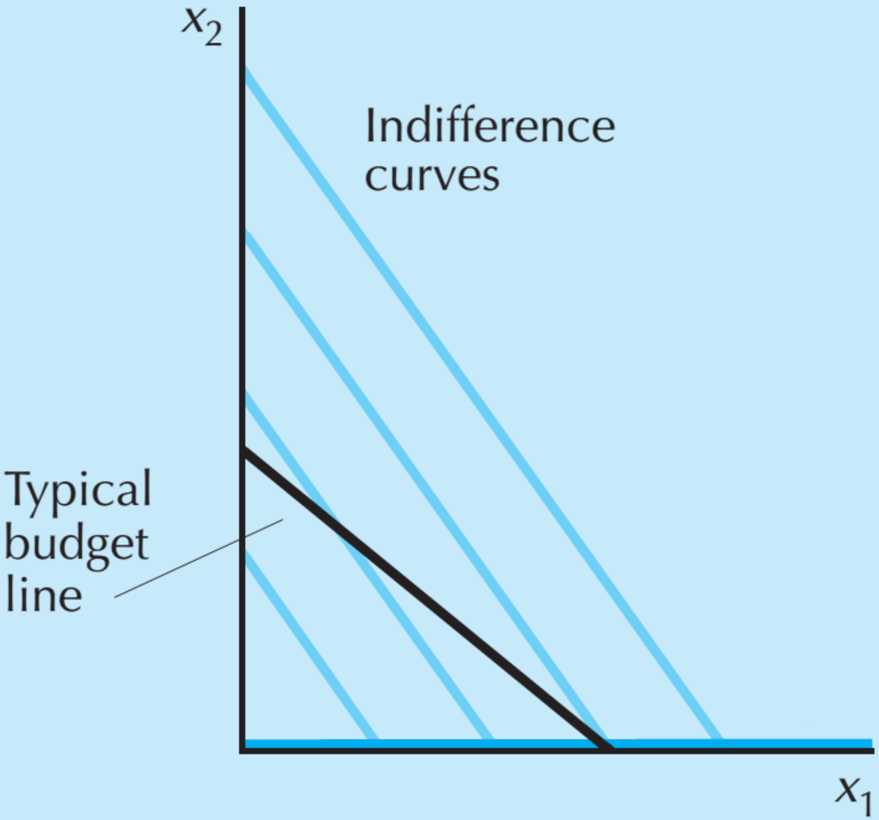





Consumer Choice



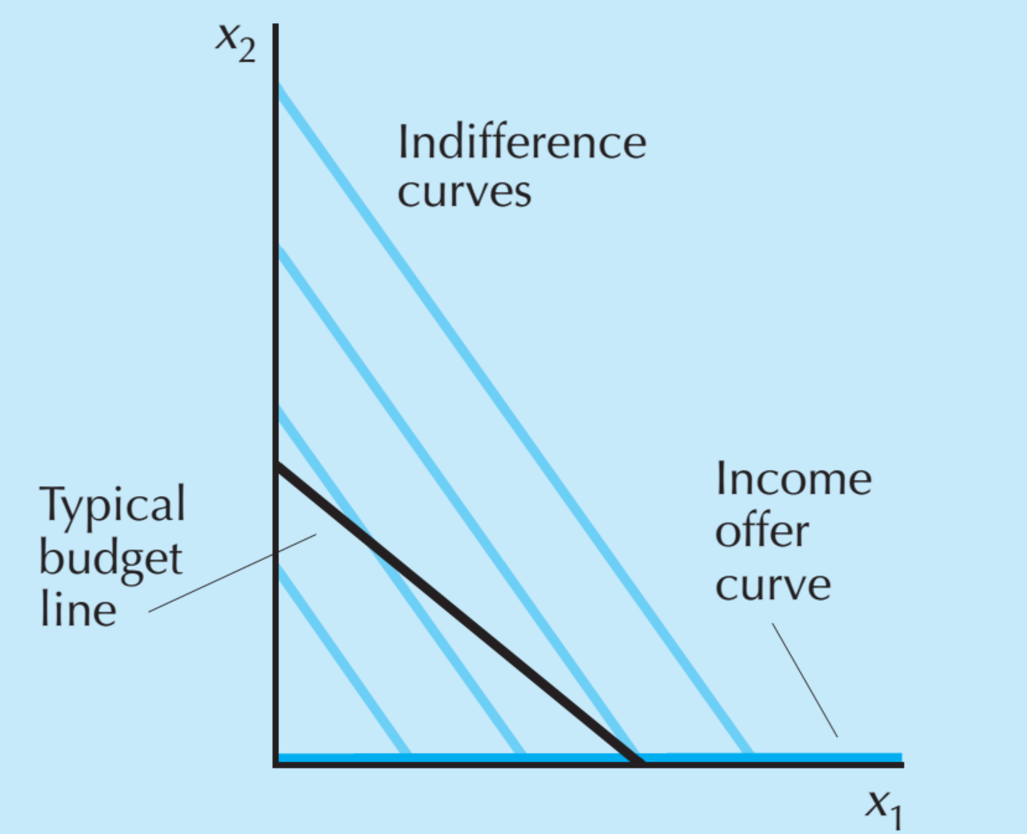
Perfect Substitutes



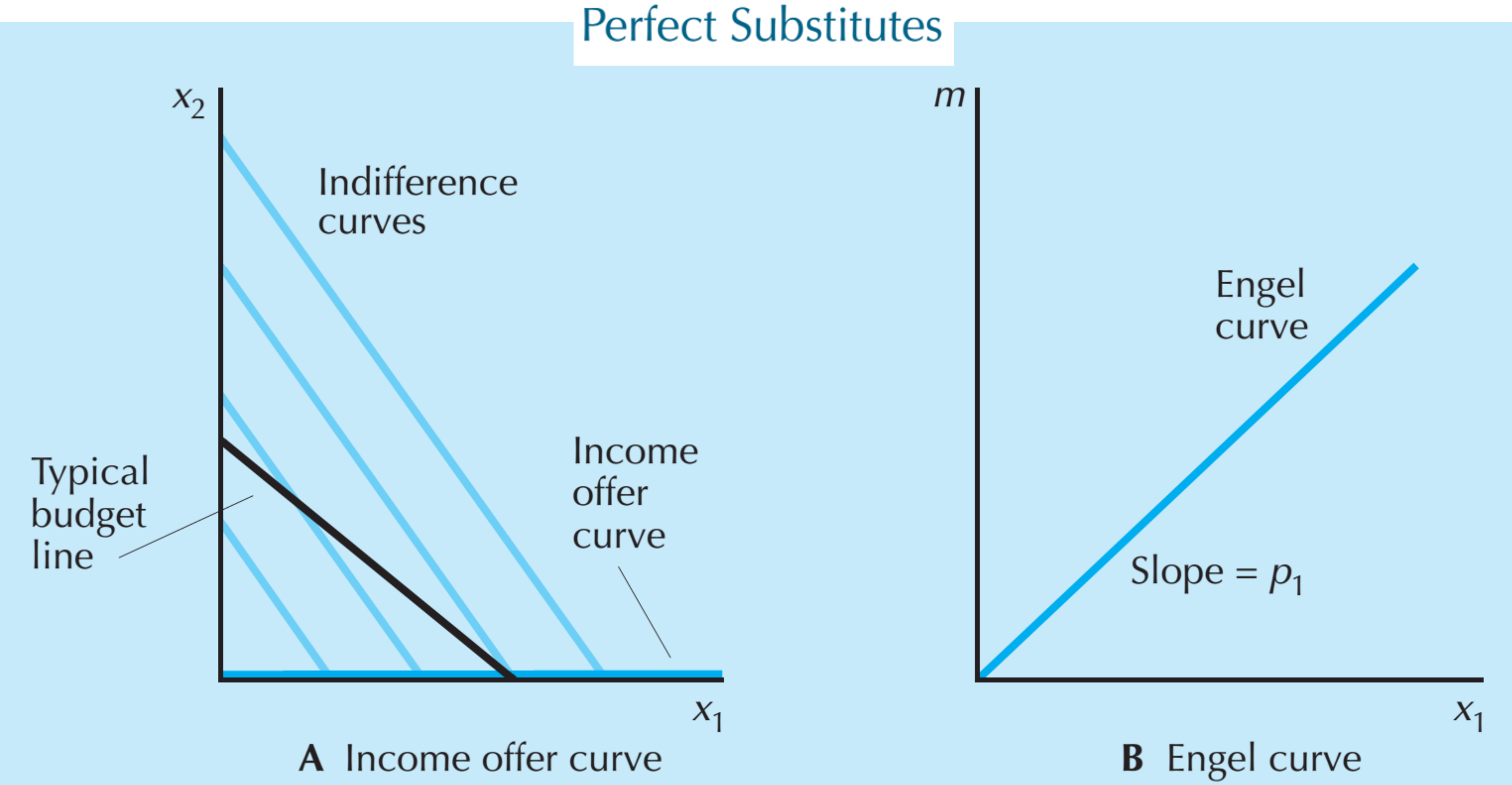
A Income offer curve

Perfect Substitutes

Engel curve

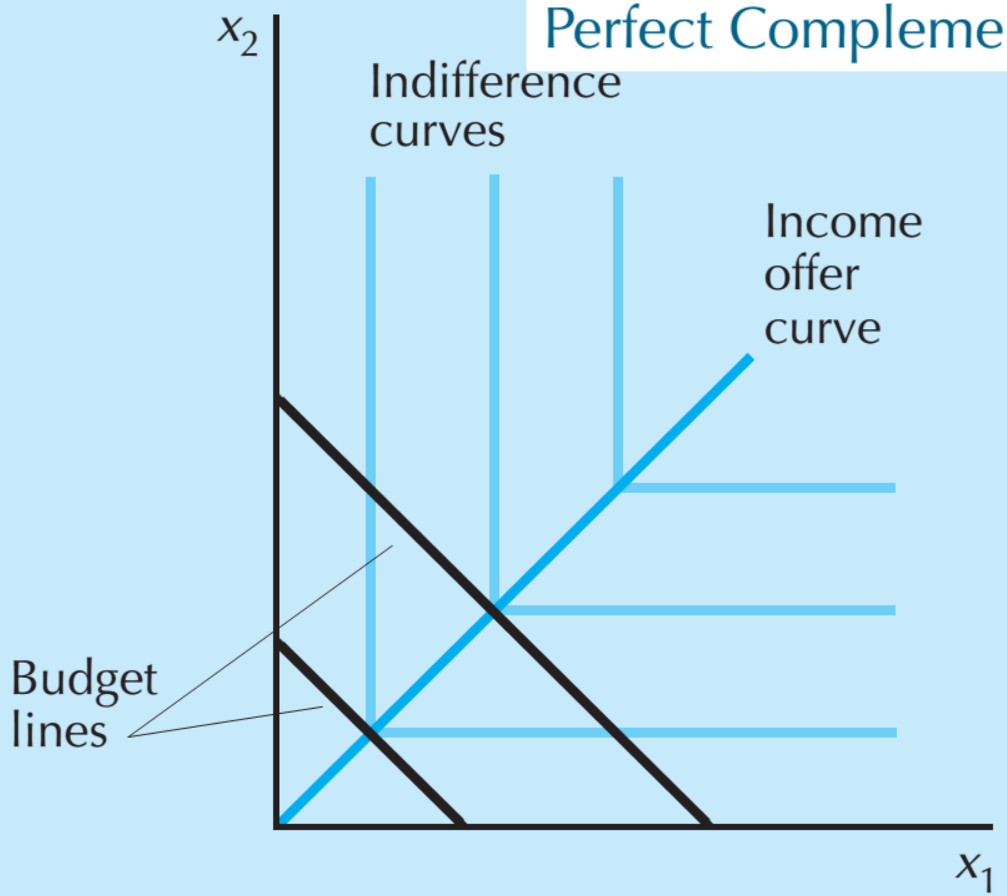


A Income offer curve



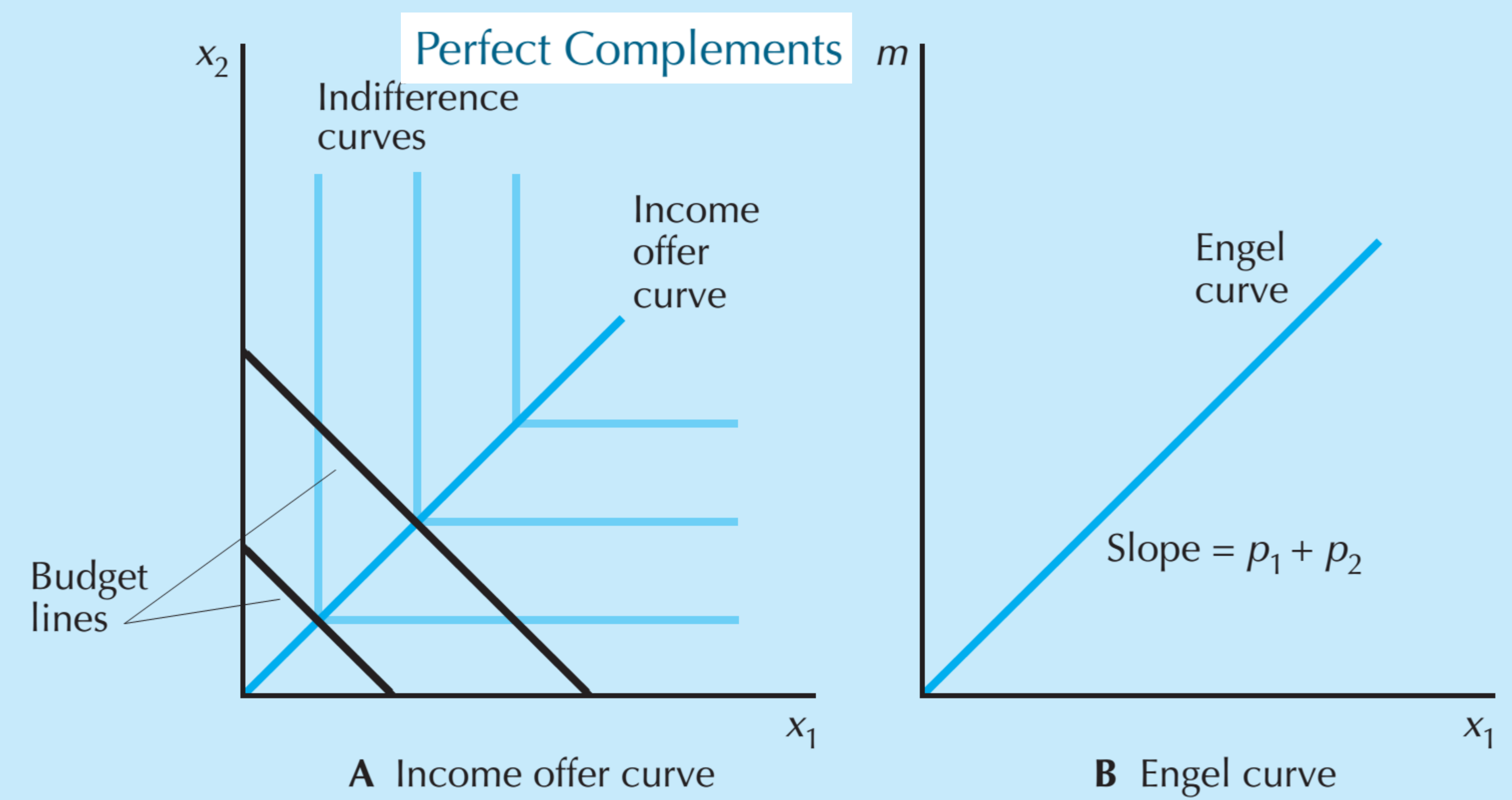
Since the demand for good 1 is $x_1 = m/p_1$ in this case, the Engel curve will be a straight line with a slope of p_1, as depicted in Figure 6.4B. (Since m is on the vertical axis, and x_1 on the horizontal axis, we can write $m = p_1 x_1$, which makes it clear that the slope is p_1.)

Perfect Complements



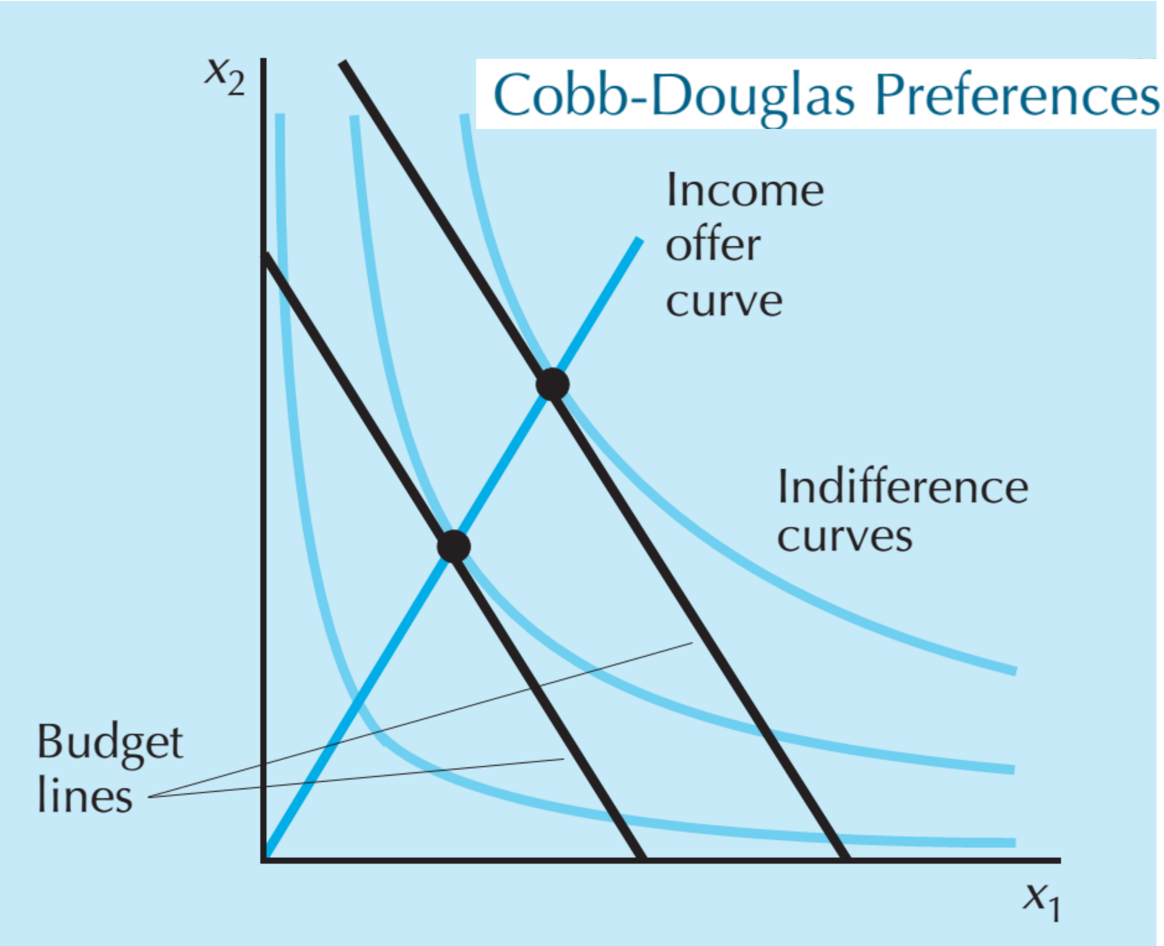
A Income offer curve

Engel curve



depicted in Figure 6.5A. We have seen that the demand for good 1 is $x_1 = m/(p_1 + p_2)$, so the Engel curve is a straight line with a slope of $p_1 + p_2$ as shown in Figure 6.5B.

Engel curve

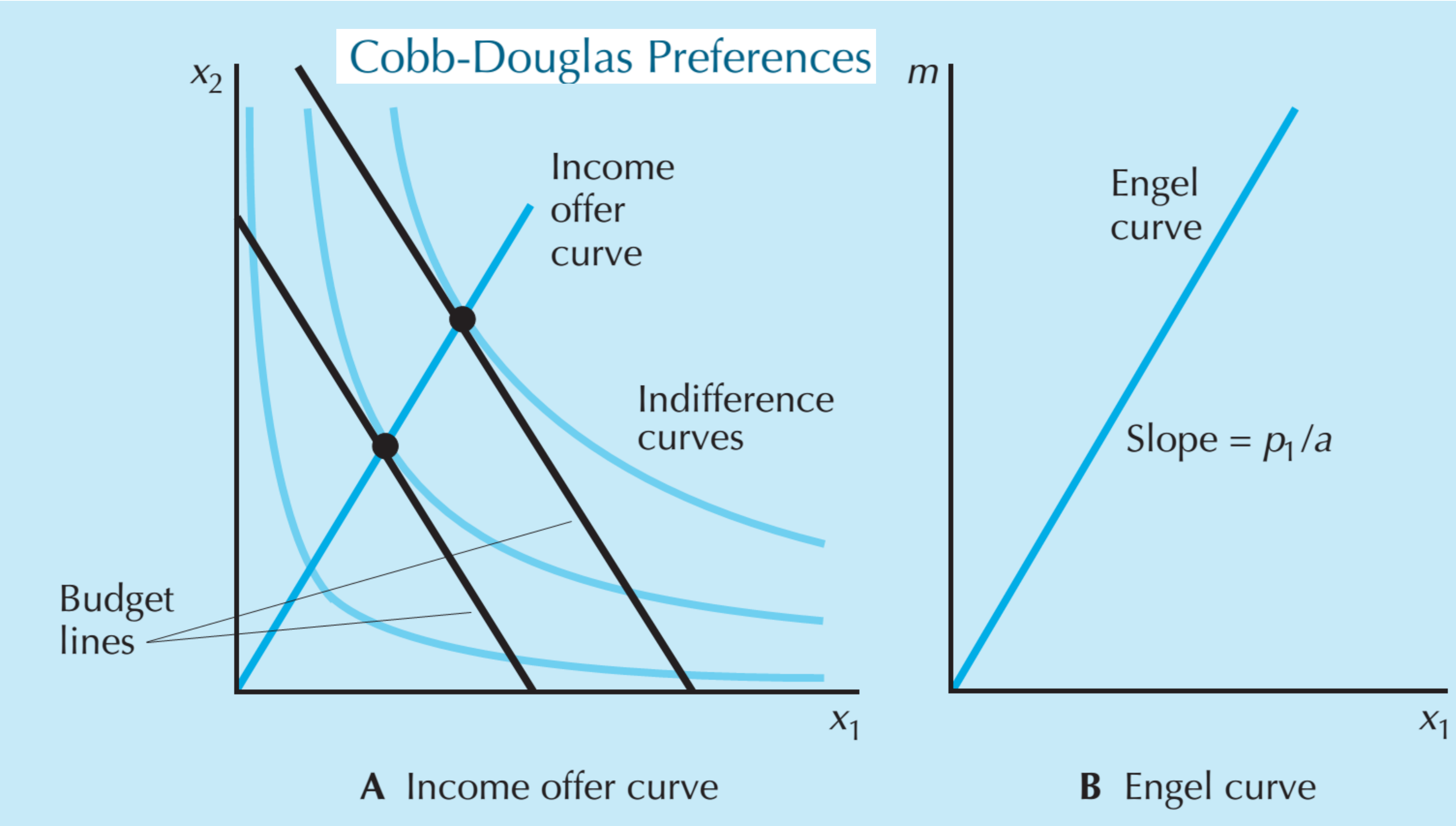


A Income offer curve

$$u(x_1, x_2) = x_1^a x_2^{1-a}$$

$$x_1 = am/p_1$$

$$x_2 = (1-a)m/p_2$$



$$u(x_1, x_2) = x_1^a x_2^{1-a}$$

$$x_1 = am/p_1$$

$$x_2 = (1-a)m/p_2$$

- Homothetic preferences

$$\text{If } \left(x_1, x_2\right) \text{ is preferred to } \left(y_1, y_2\right)$$

$$\Leftrightarrow$$

$$\left(t \cdot x_1, t \cdot x_2\right) \text{ is preferred to } \left(t \cdot y_1, t \cdot y_2\right)$$

$$\left(\text{where } t > 0\right)$$

Homothetic Preferences

Suppose that the consumer's preferences only depend on the *ratio* of good 1 to good 2. This means that if the consumer prefers (x_1, x_2) to (y_1, y_2), then she automatically prefers $(2x_1, 2x_2)$ to $(2y_1, 2y_2)$, $(3x_1, 3x_2)$ to $(3y_1, 3y_2)$, and so on, since the ratio of good 1 to good 2 is the same for all of these bundles. In fact, the consumer prefers (tx_1, tx_2) to (ty_1, ty_2) for any positive value of t. Preferences that have this property are known as **homothetic preferences**. It is not hard to show that the three examples of preferences given above—perfect substitutes, perfect complements, and Cobb-Douglas—are all homothetic preferences.

Homothetic Preferences

Cobb-Douglas?

$$u[x_1, x_2] = x_1^a x_2^b$$

$$u[tx_1, tx_2] = (tx_1)^a (tx_2)^b = x_1^a x_2^b \cdot t^{a+b}$$

$$u[tx_1, tx_2] - u[ty_1, ty_2] = u[x_1, x_2] \cdot t^{a+b} - u[y_1, y_2] \cdot t^{a+b}$$

$$= \left(u[x_1, x_2] - u[y_1, y_2]\right) \cdot t^{a+b} > 0$$

Homothetic Preferences

-perfect substitutes ?

$$u[x_1, x_2] = ax_1 + bx_2$$

$$u[tx_1, tx_2] = tax_1 + tbx_2$$
$$= \left(ax_1 + bx_2\right) \cdot t = u[x_1, x_2] \cdot t$$

$$u[tx_1, tx_2] - u[ty_1, ty_2] = u[x_1, x_2] \cdot t - u[y_1, y_2] \cdot t$$
$$= \left(u[x_1, x_2] - u[y_1, y_2]\right) \cdot t > 0$$

Homothetic Preferences

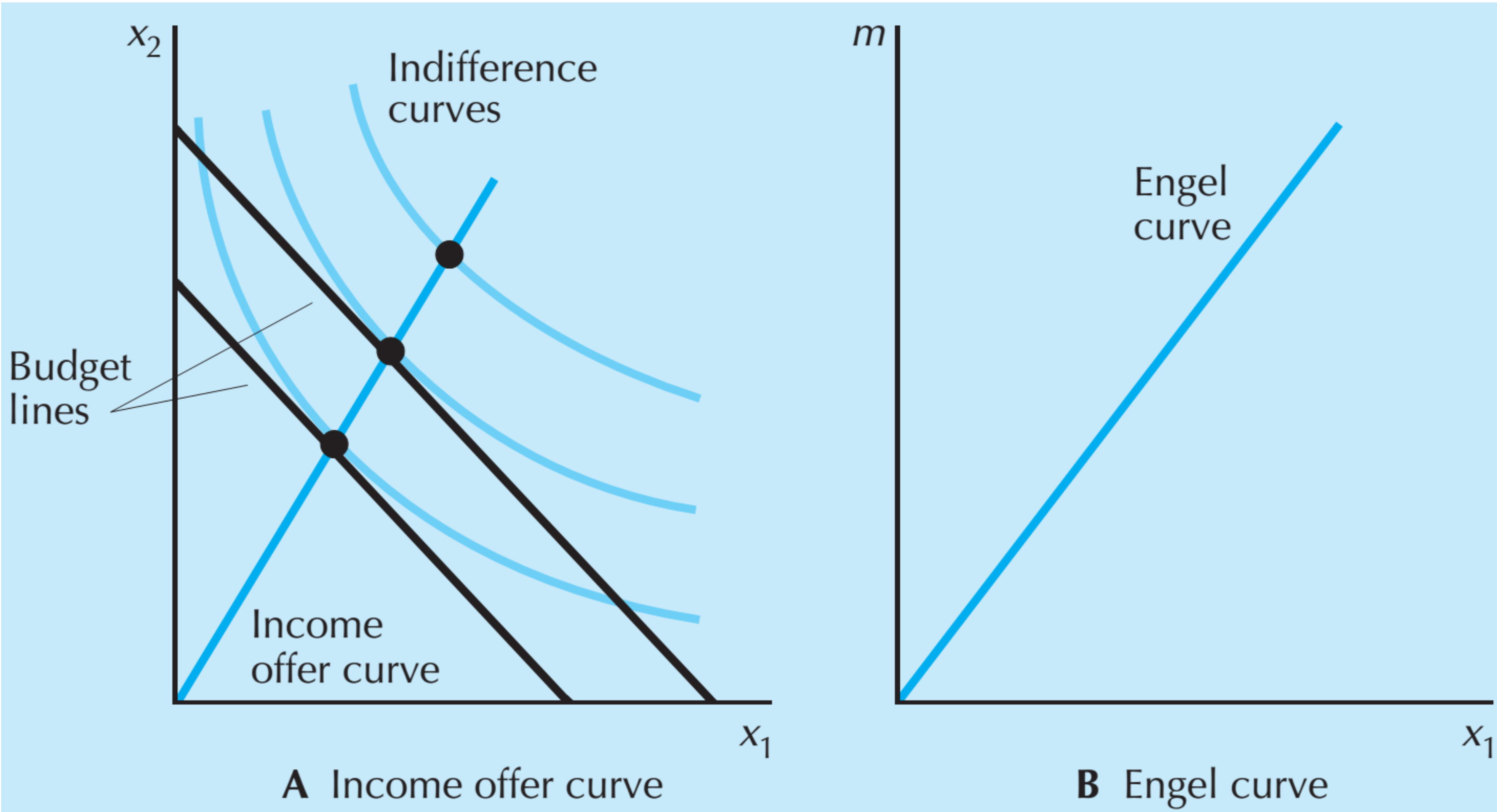
perfect complements ?

$$\boxed{u[x_1, x_2] = \min[ax_1, bx_2]}$$

$$\boxed{\begin{aligned} u[tx_1, tx_2] &= \min[tax_1, tbx_2] \\ &= \min[ax_1, bx_2]\cdot t = u[x_1, x_2]\cdot t \end{aligned}}$$

$$\begin{aligned} u[tx_1, tx_2] - u[ty_1, ty_2] &= u[x_1, x_2]\cdot t - u[y_1, y_2]\cdot t \\ &= \left(u[x_1, x_2] - u[y_1, y_2]\right)\cdot t > 0 \end{aligned}$$

Homothetic Preferences

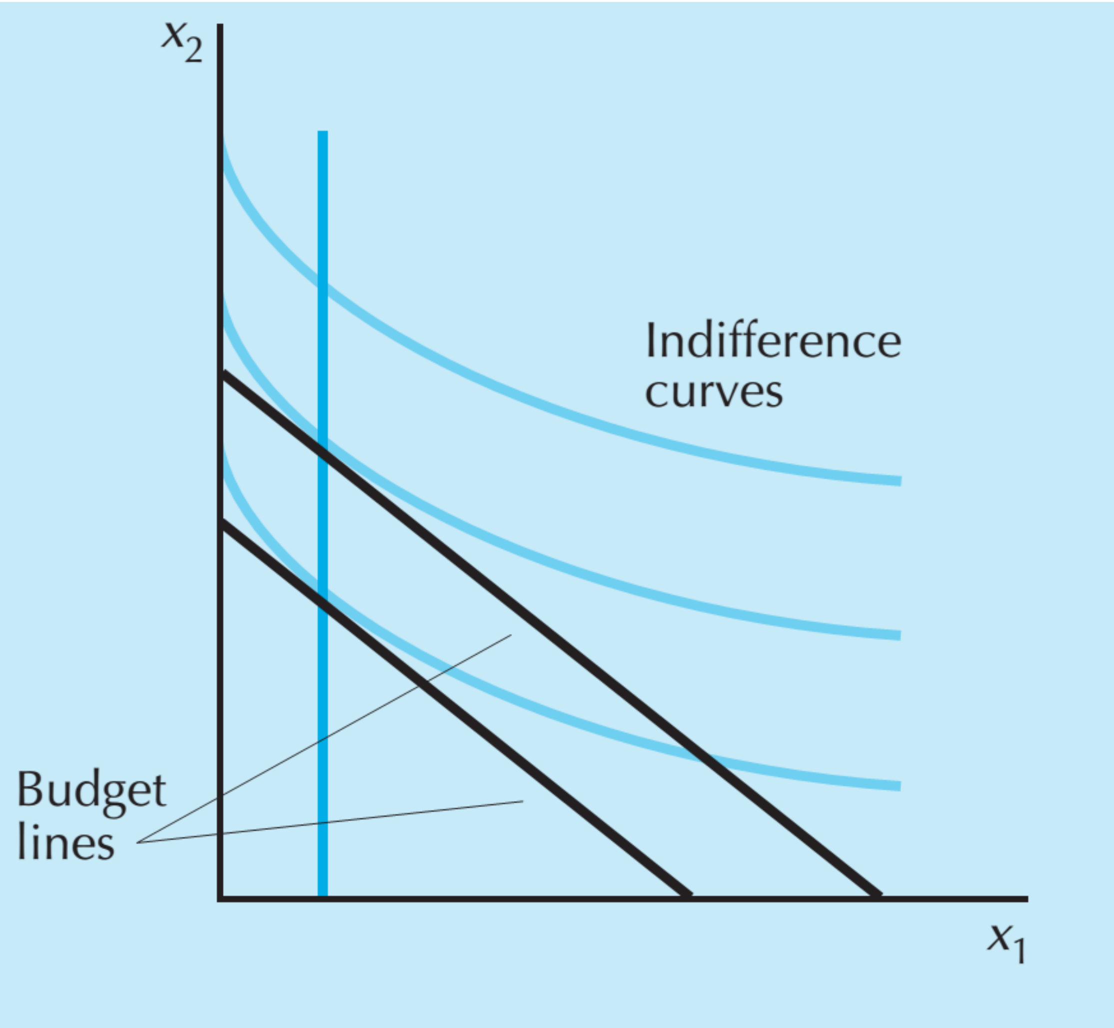


specifically, if preferences are homothetic, it means that when income is scaled up or down by any amount $t > 0$, the demanded bundle scales up or down by the same amount. This can be established rigorously, but it is

Quasilinear Preferences

$$u(x_1, x_2) = v(x_1) + x_2$$

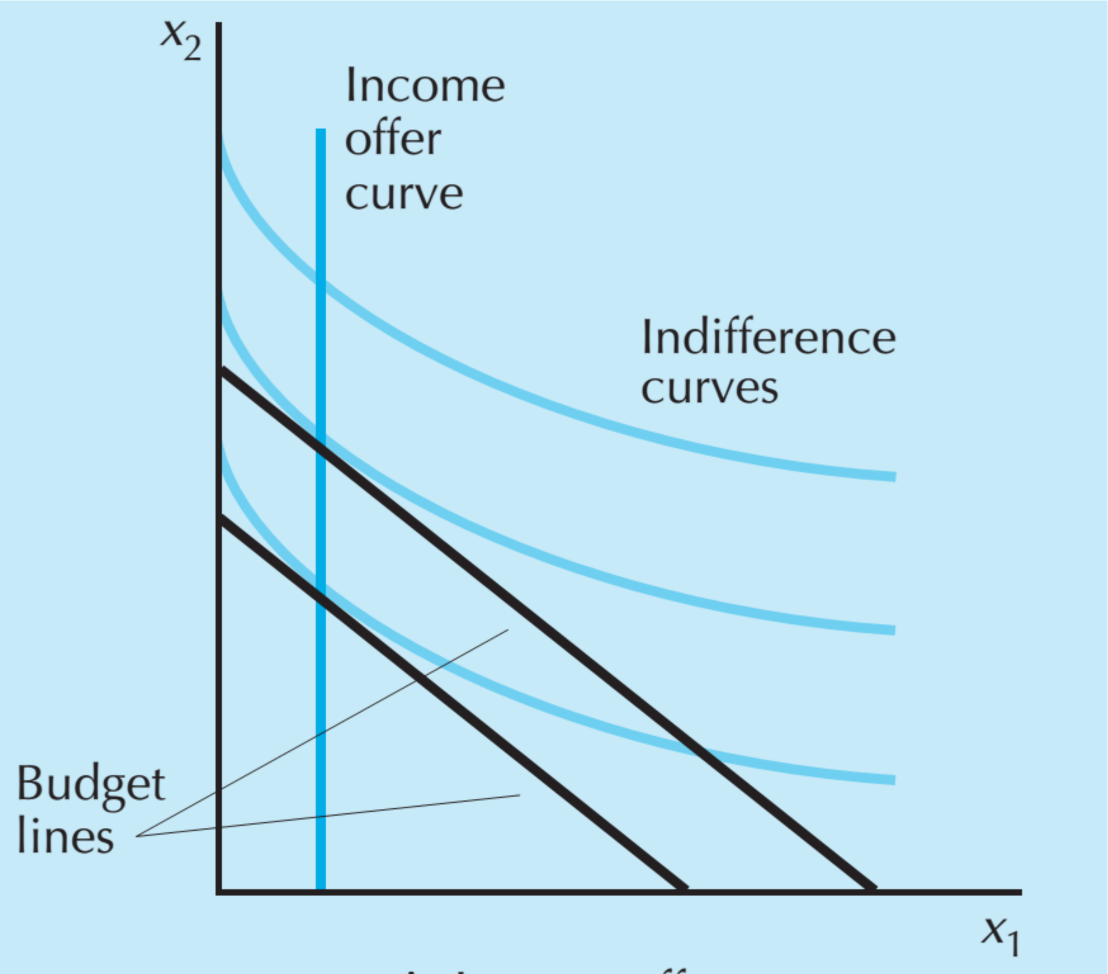
Quasilinear Preferences



Income-offer curve

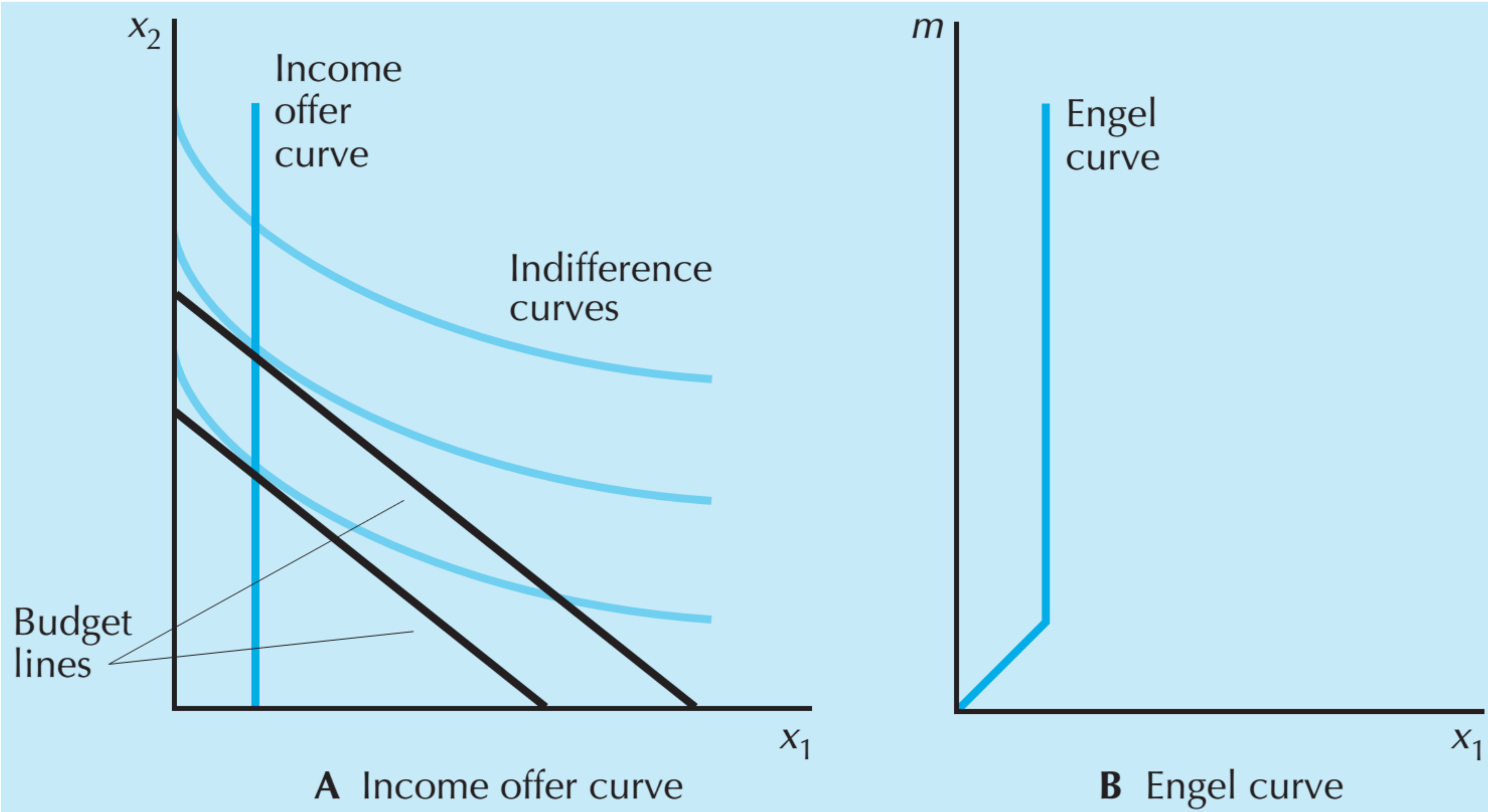
Quasilinear Preferences

Engel curve



A Income offer curve

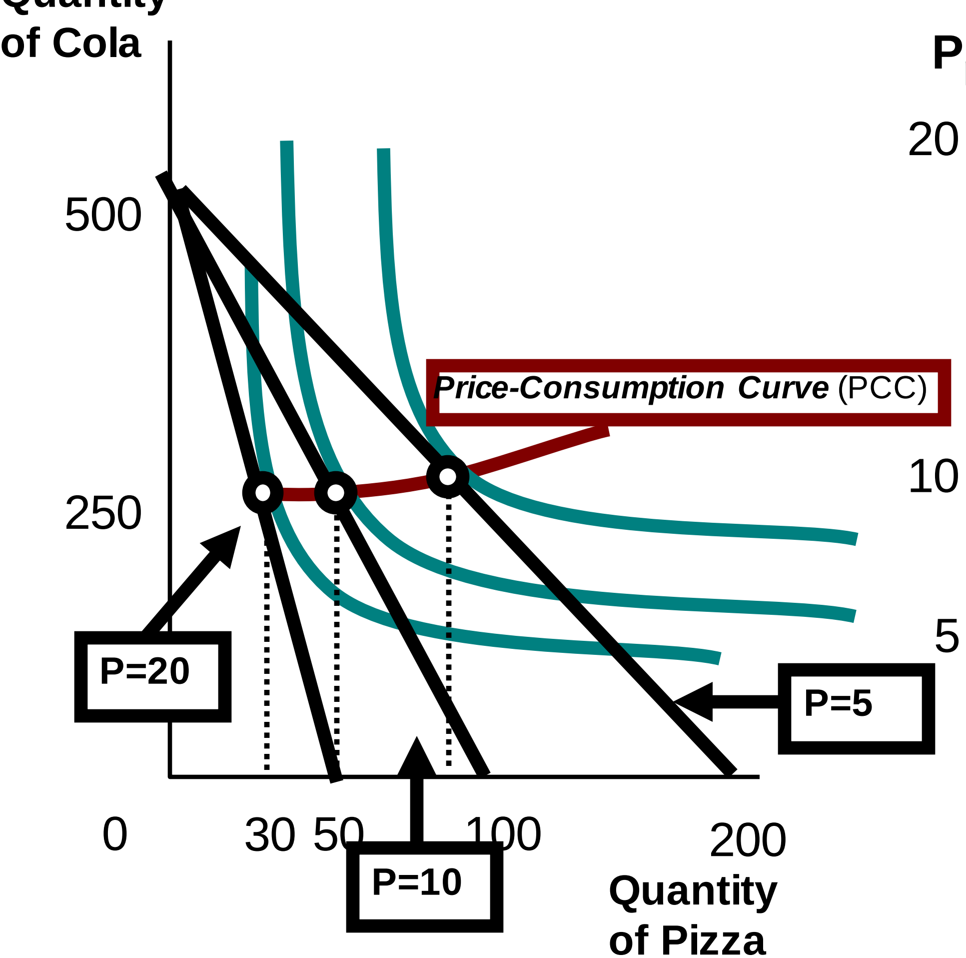
Quasilinear Preferences



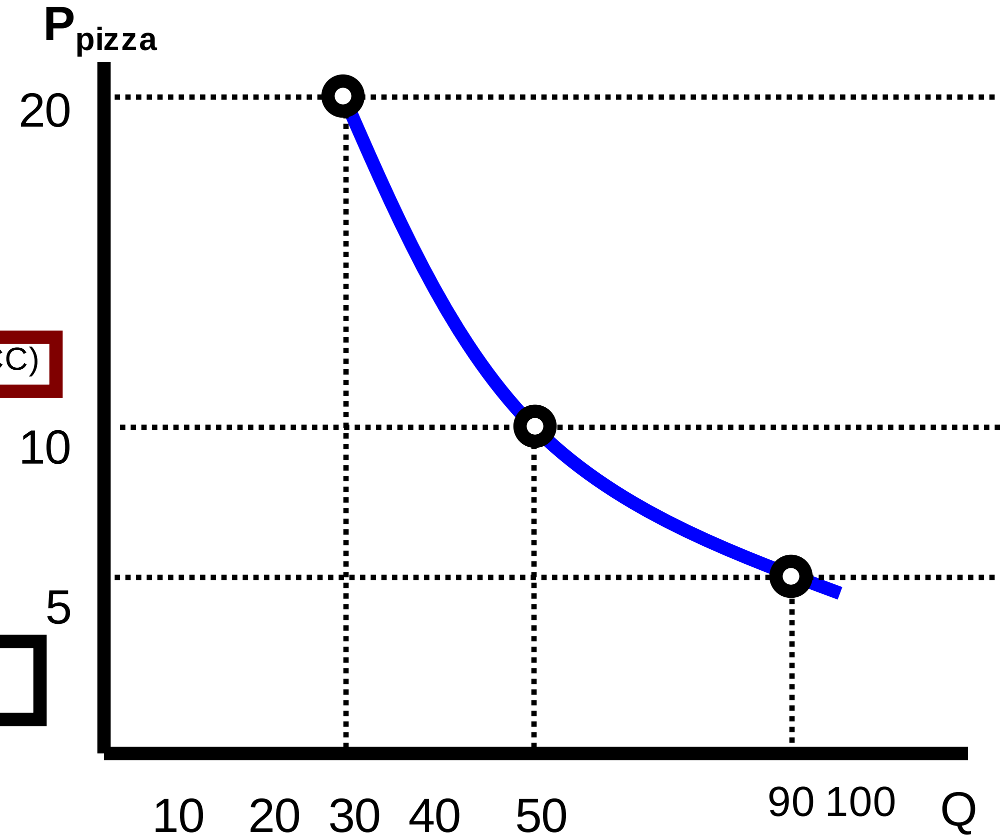
6.4 Price changes

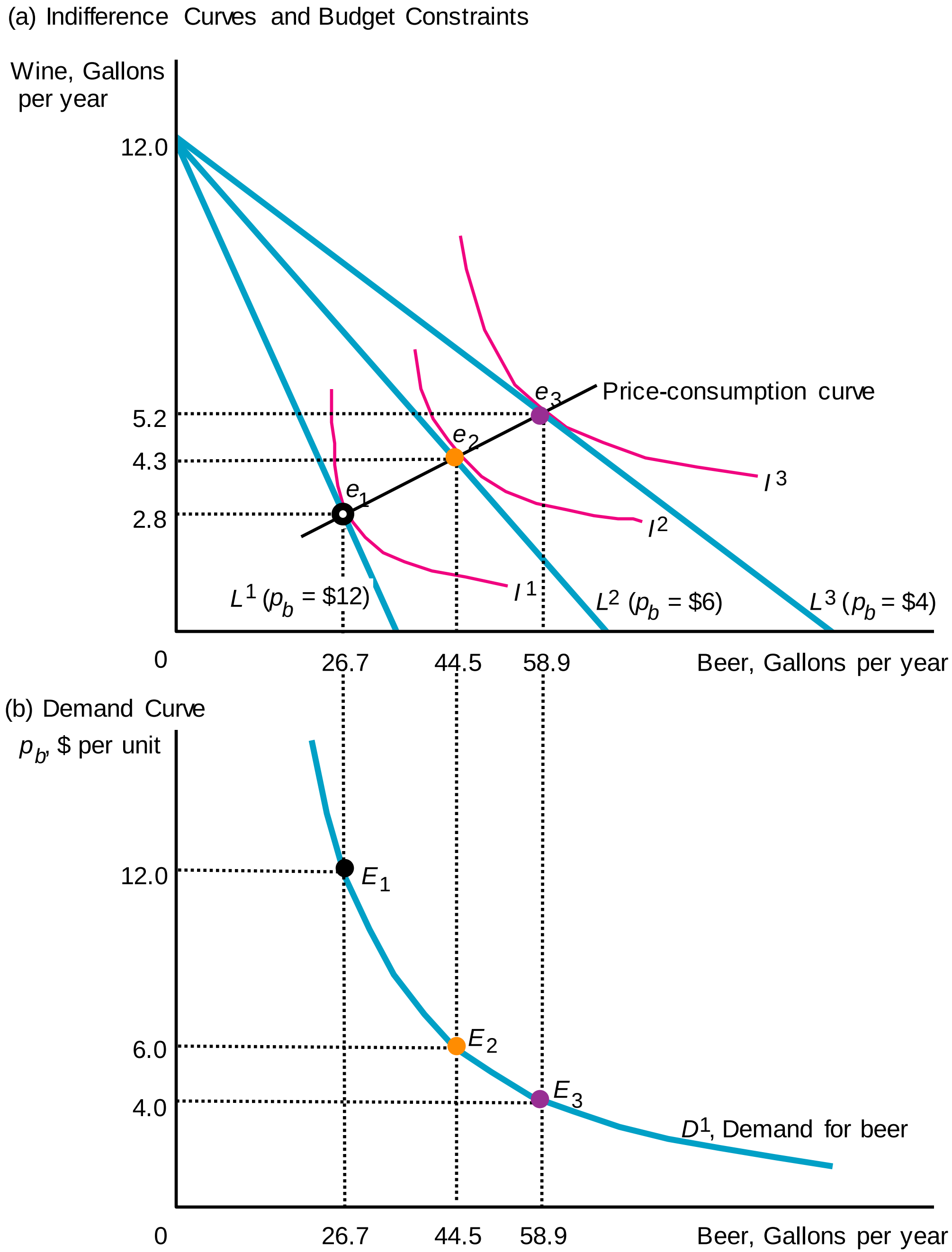
Deriving demand functions: effect of price on demand

Budget	Price of Cola	Price of Pizza
1000	$2	$5

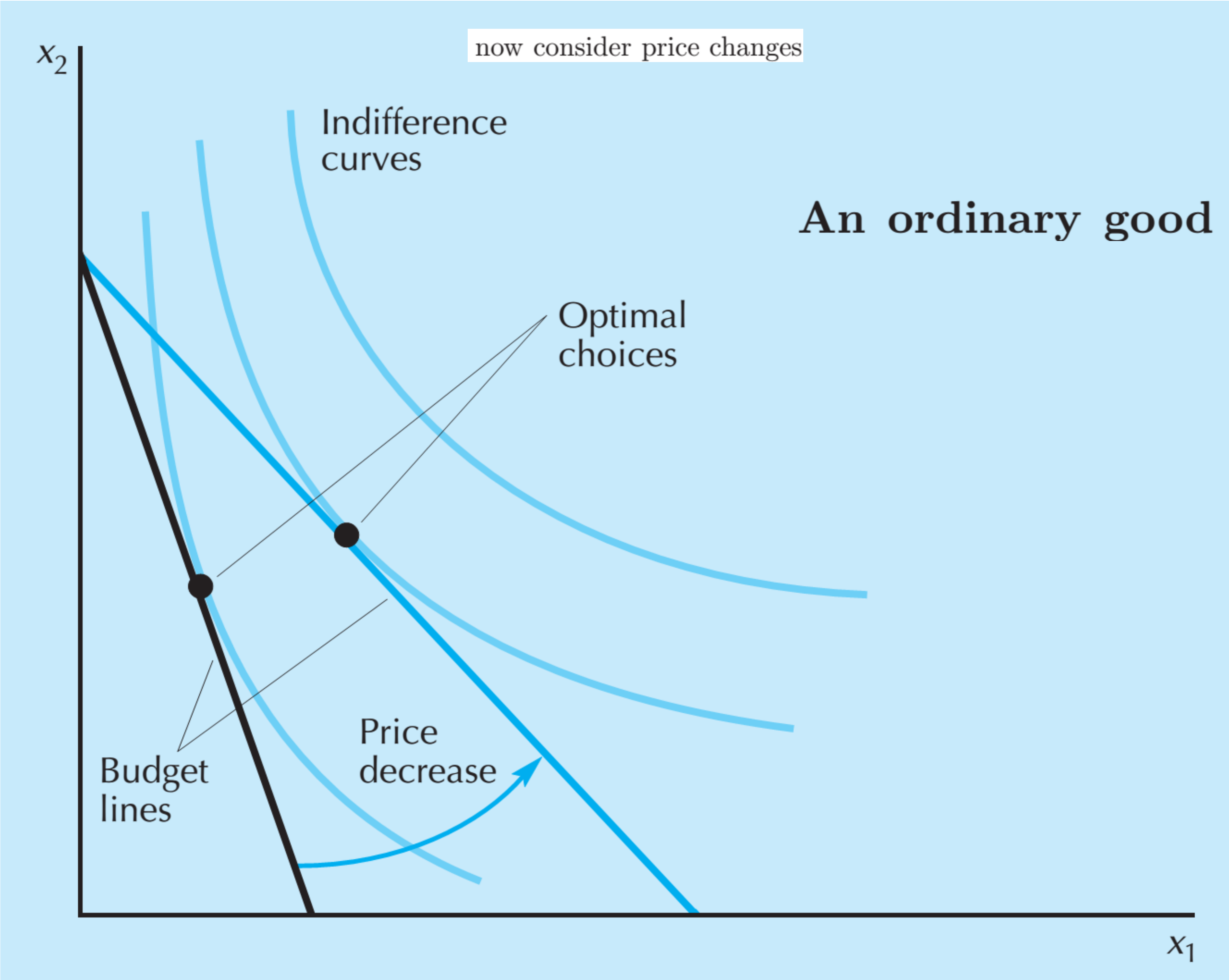


Pizza ***demand*** curve



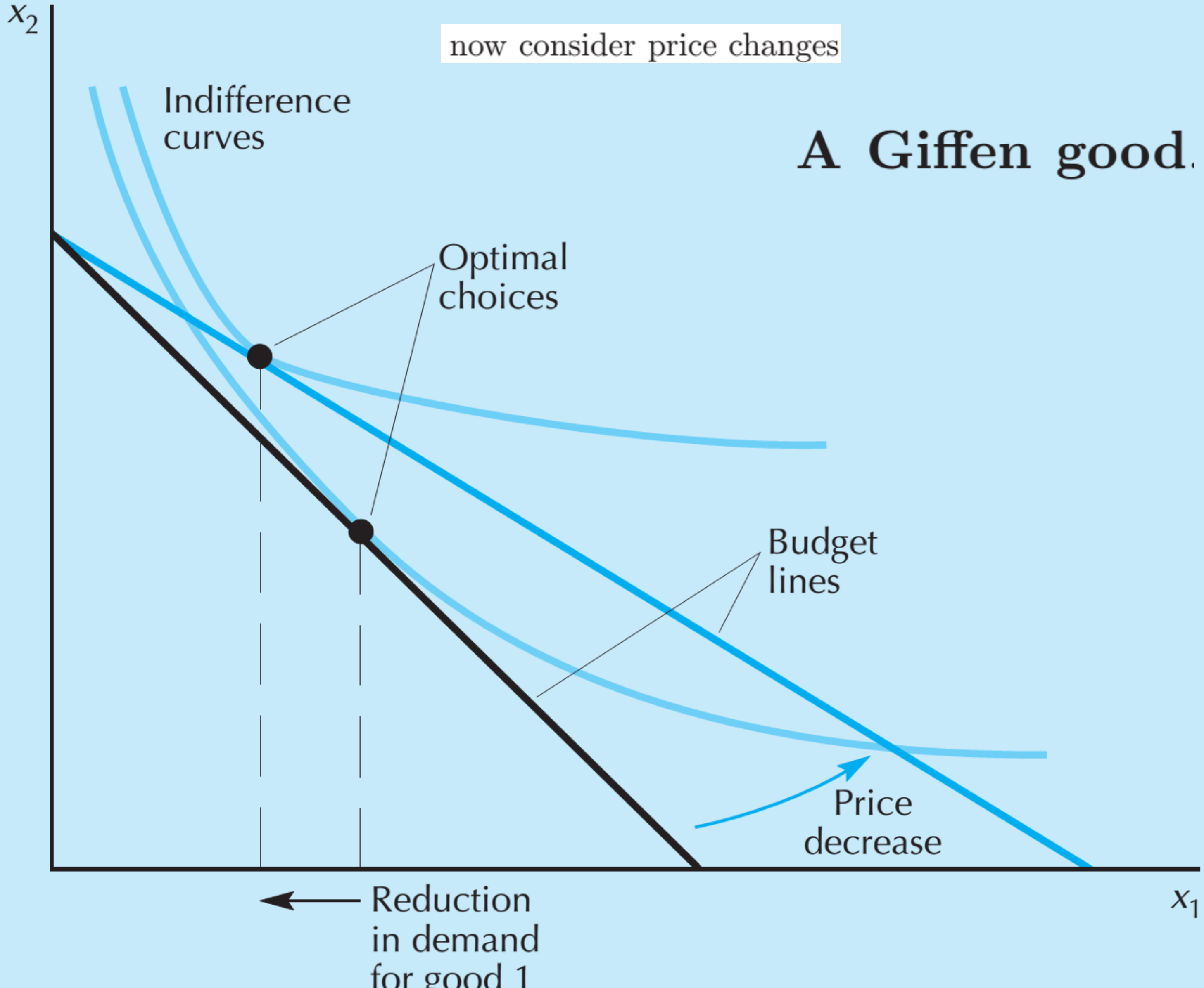

(a) Indifference Curves and Budget Constraints
Wine, Gallons per year
12.0
5.2
4.3
2.8
0
26.7
44.5
58.9
Beer, Gallons per year
Price-consumption curve
e1
e2
e3
I 1
I 2
I 3
L 1 (pb = $12)
L2 (pb = $6)
L3 (pb = $4)
(b) Demand Curve
pb, $ per unit
12.0
6.0
4.0
0
26.7
44.5
58.9
Beer, Gallons per year
E1
E2
E3
D1, Demand for beer

now consider price changes



now consider price changes

A Giffen good.



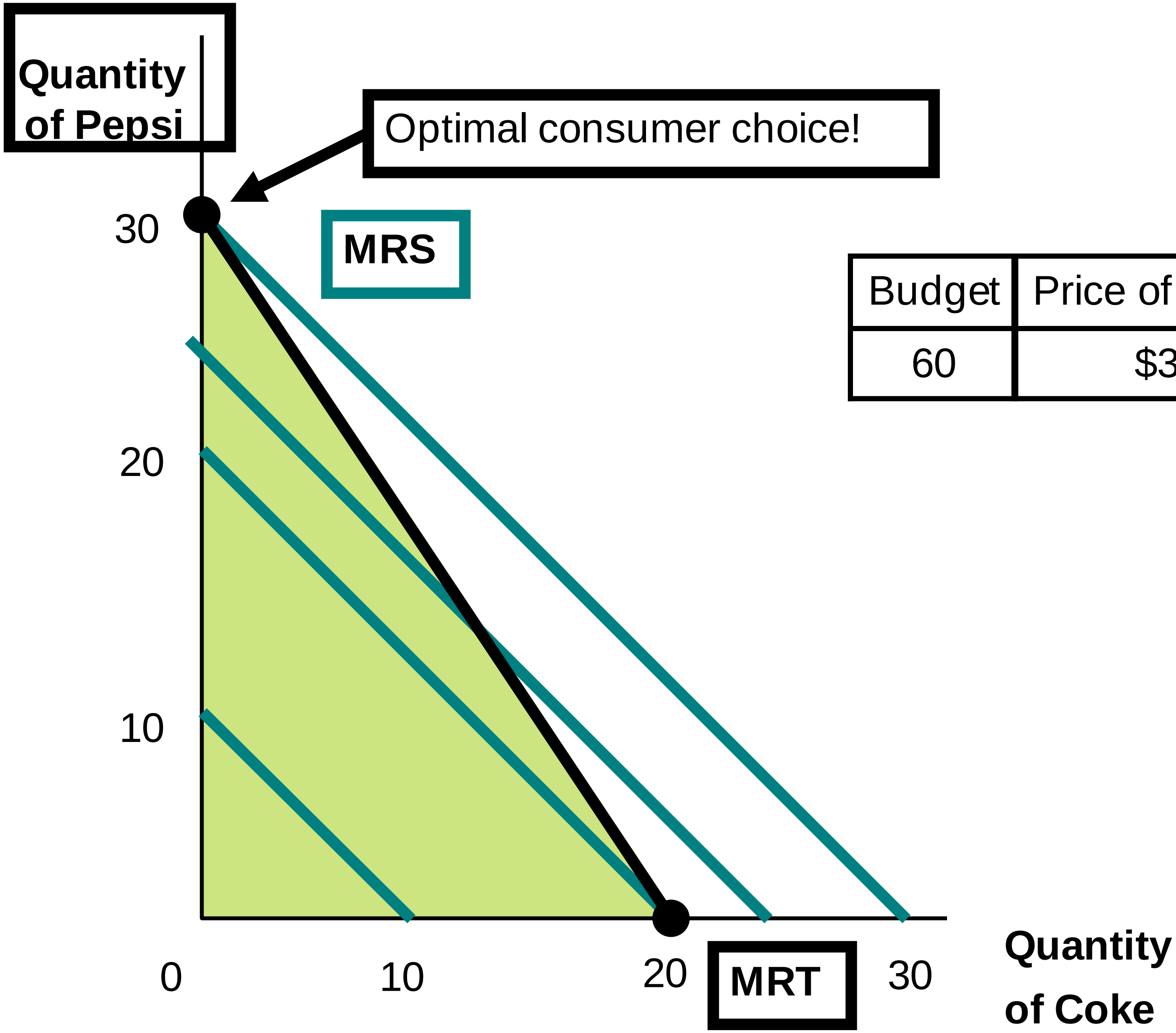
A Giffen good. Good 1 is a Giffen good, since the demand for it decreases when its price decreases.

6.5 Demand curve

- Mahdu views Coke, q, and Pepsi as perfect substitutes: He is indifferent as to which one he drinks.
  - The price of Coke is 3
  - The price of Pepsi is 2
  - Mahdu's weekly cola budget is 60.
- Derive Mahdu's demand curve for Coke

Budget	Price of Coke	Price of Pepsi
60	$3	$2

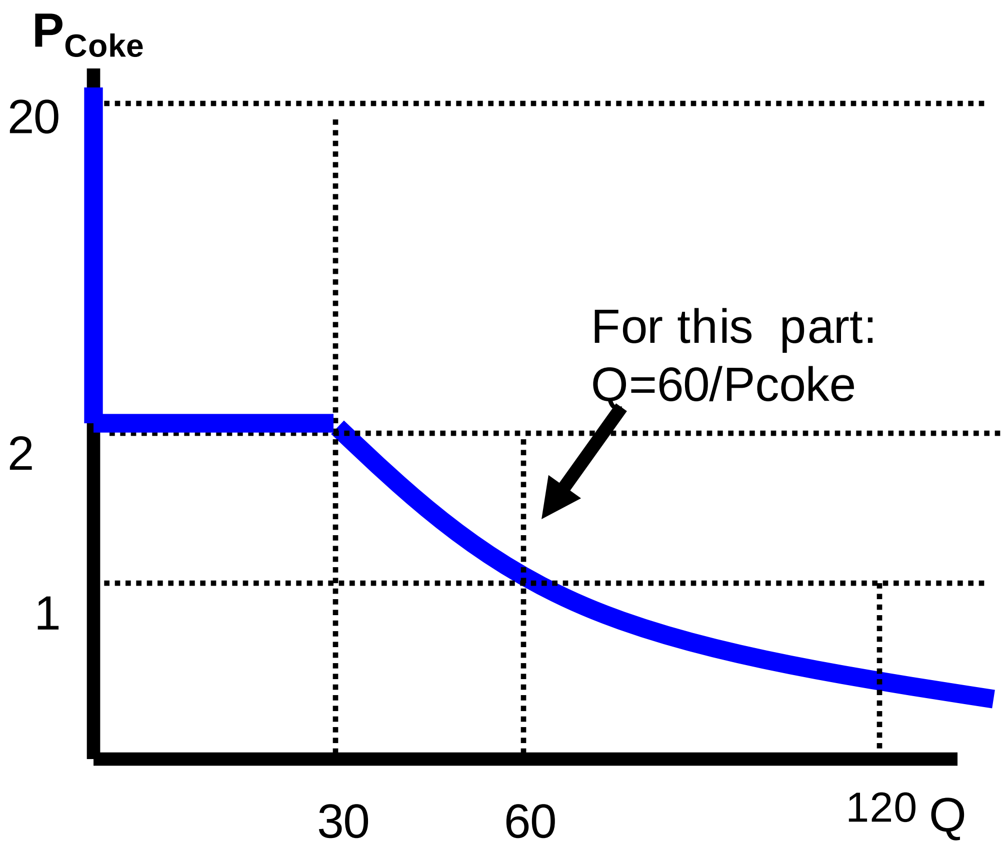
Consumer Choice



Budget	Price of Coke	Price of Pepsi
60	$3	$2

Budget	Price of Coke	Price of Pepsi
60	$3	$2

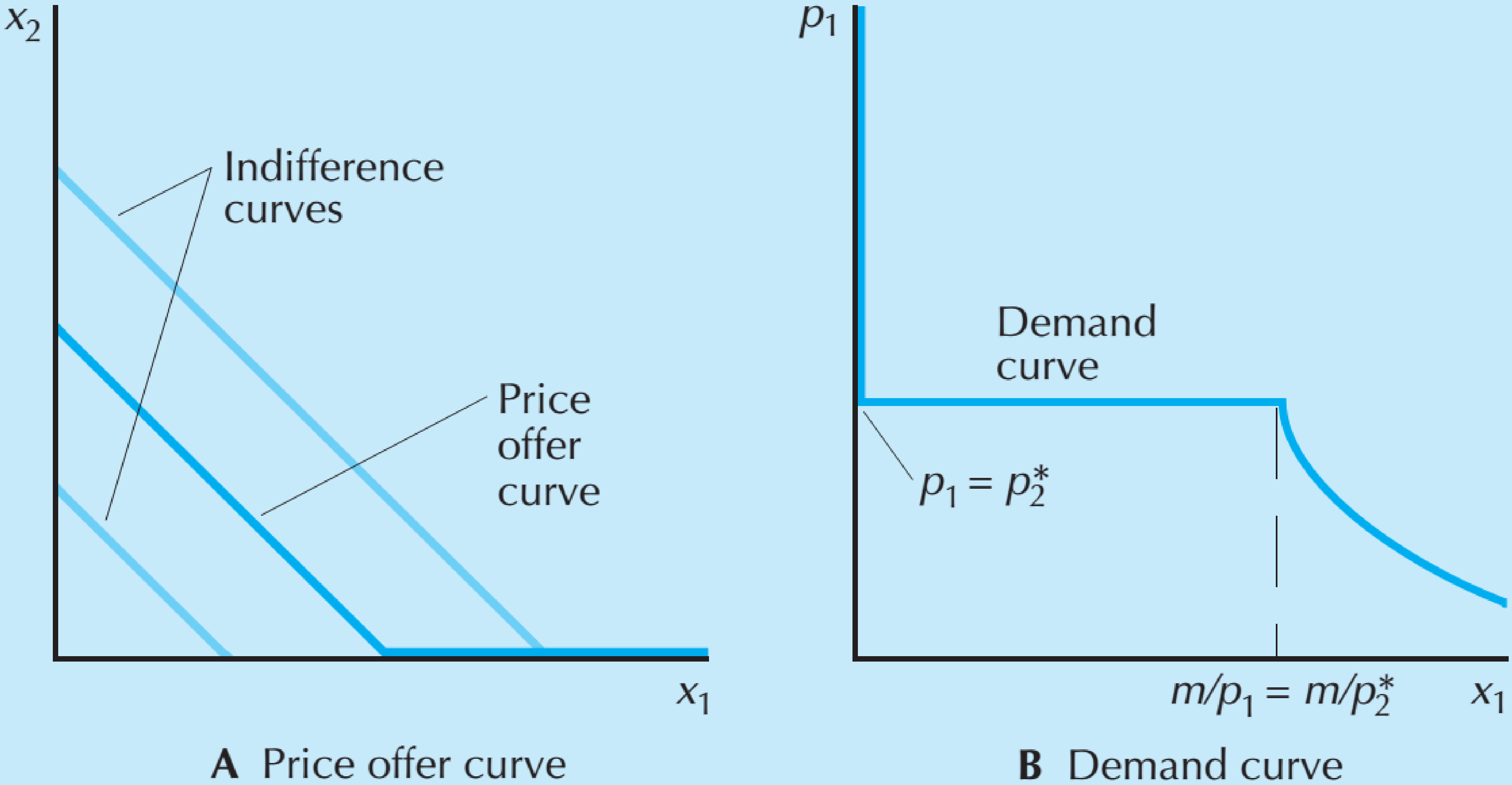
Coke ***demand*** curve



Perfect Substitutes

The offer curve and demand curve for perfect substitutes—the red and blue pencils example—are illustrated in Figure 6.12. As we saw in Chapter 5, the demand for good 1 is zero when $p_1 > p_2$, any amount on the budget line when $p_1 = p_2$, and m/p_1 when $p_1 < p_2$. The offer curve traces out these possibilities.

In order to find the demand curve, we fix the price of good 2 at some price p_2^* and graph the demand for good 1 versus the price of good 1 to get the shape depicted in Figure 6.12B.



Perfect substitutes. Price offer curve (A) and demand curve (B) in the case of perfect substitutes.

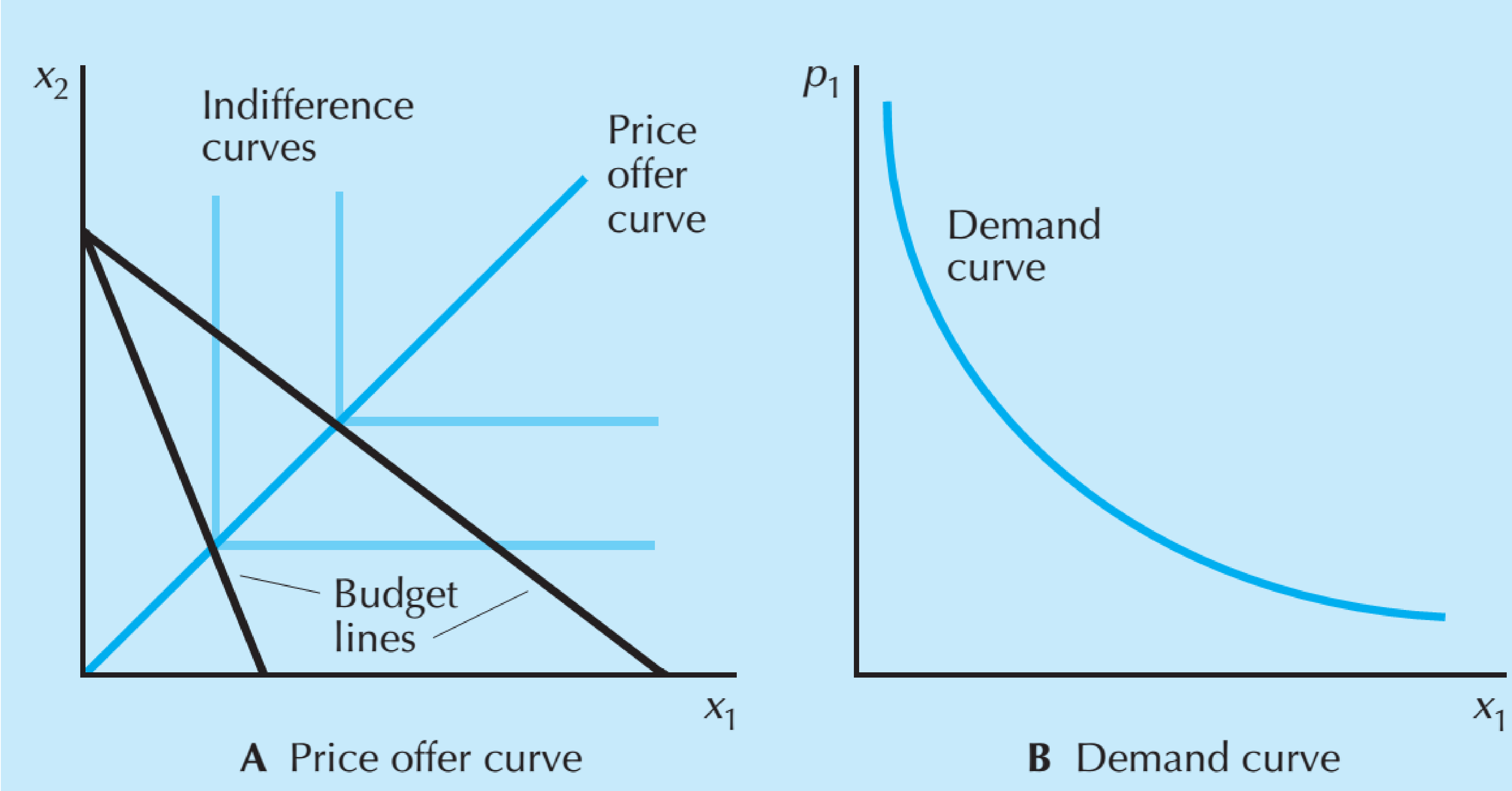
Perfect Complements

The case of perfect complements—the right and left shoes example—is depicted in Figure 6.13. We know that whatever the prices are, a consumer will demand the same amount of goods 1 and 2. Thus his offer curve will be a diagonal line as depicted in Figure 6.13A.

We saw in Chapter 5 that the demand for good 1 is given by

$$x_1 = \frac{m}{p_1 + p_2}.$$

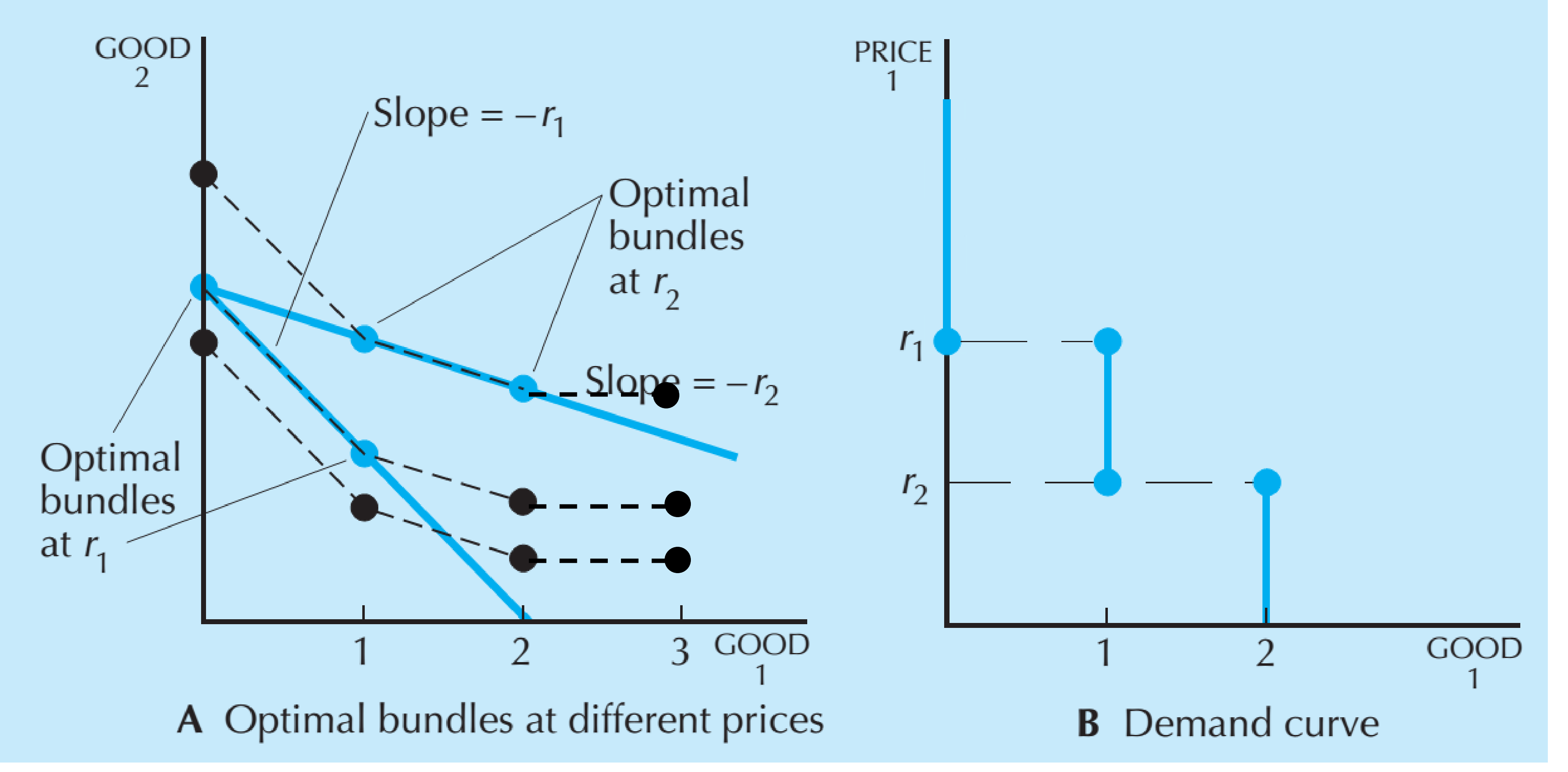
If we fix m and p_2 and plot the relationship between x_1 and p_1, we get the curve depicted in Figure 6.13B.



x_2
Indifference curves
Price offer curve
Budget lines
x_1
A Price offer curve
p_1
Demand curve
x_1
B Demand curve

- Discrete good

Suppose that good 1 is a discrete good. If p_1 is very high then the consumer will strictly prefer to consume zero units; if p_1 is low enough the consumer will strictly prefer to consume one unit. At some price r_1, the consumer will be indifferent between consuming good 1 or not consuming it. The price



Something missing in left picture

$$u(0, m) = u(1, m - r_1).$$

$$u(1, m - r_2) = u(2, m - 2r_2)$$

These prices can be described in terms of the original utility function. For example, r_1 is the price where the consumer is just indifferent between consuming 0 or 1 unit of good 1, so it must satisfy the equation

$$u(0, m) = u(1, m - r_1).$$

Similarly r_2 satisfies the equation

$$u(1, m - r_2) = u(2, m - 2r_2).$$

If the utility function is quasilinear, then the formulas describing the reservation prices become somewhat simpler. If $u(x_1, x_2) = v(x_1) + x_2$, and $v(0) = 0$, then we can write equation (6.1) as

$$v(0) + m = m = v(1) + m - r_1.$$
$$r_1 = v(1).$$

$$v(1) + m - r_2 = v(2) + m - 2r_2.$$

$$r_2 = v(2) - v(1)$$

$$r_3 = v(3) - v(2)$$

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$$v(1) + m - r_2 = v(2) + m - 2r_2.$$

$$r_2 = v(2) - v(1)$$

$$r_3 = v(3) - v(2)$$

In each case, the reservation price measures the increment in utility necessary to induce the consumer to choose an additional unit of the good.

Example

Suppose that the consumer demands 6 units of good 1.
We want to show that we must have $r_6 \geq p \geq r_7$.
If the consumer is maximizing utility, then we must have

$\forall x_1:$

$$v(6) + m - 6p \geq v(x_1) + m - px_1$$

Thus certainly for $x_1 = 5:$

$$v(6) + m - 6p \geq v(5) + m - 5p.$$

$$r_6 = v(6) - v(5) \geq p.$$

Thus certainly for $x_1 = 7:$

$$v(6) + m - 6p \geq v(7) + m - 7p.$$

$$p \geq v(7) - v(6) = r_7,$$

6.8 Inverse demand function

If we hold p_2 and m fixed and plot p_1 against x_1 we get the **demand curve**. As suggested above, we typically think that the demand curve slopes downwards, so that higher prices lead to less demand, although the Giffen example shows that it could be otherwise.

Recall, for example, the Cobb-Douglas demand for good 1, $x_1 = am/p_1$. We could just as well write the relationship between price and quantity as $p_1 = am/x_1$. The first representation is the direct demand function; the second is the inverse demand function.

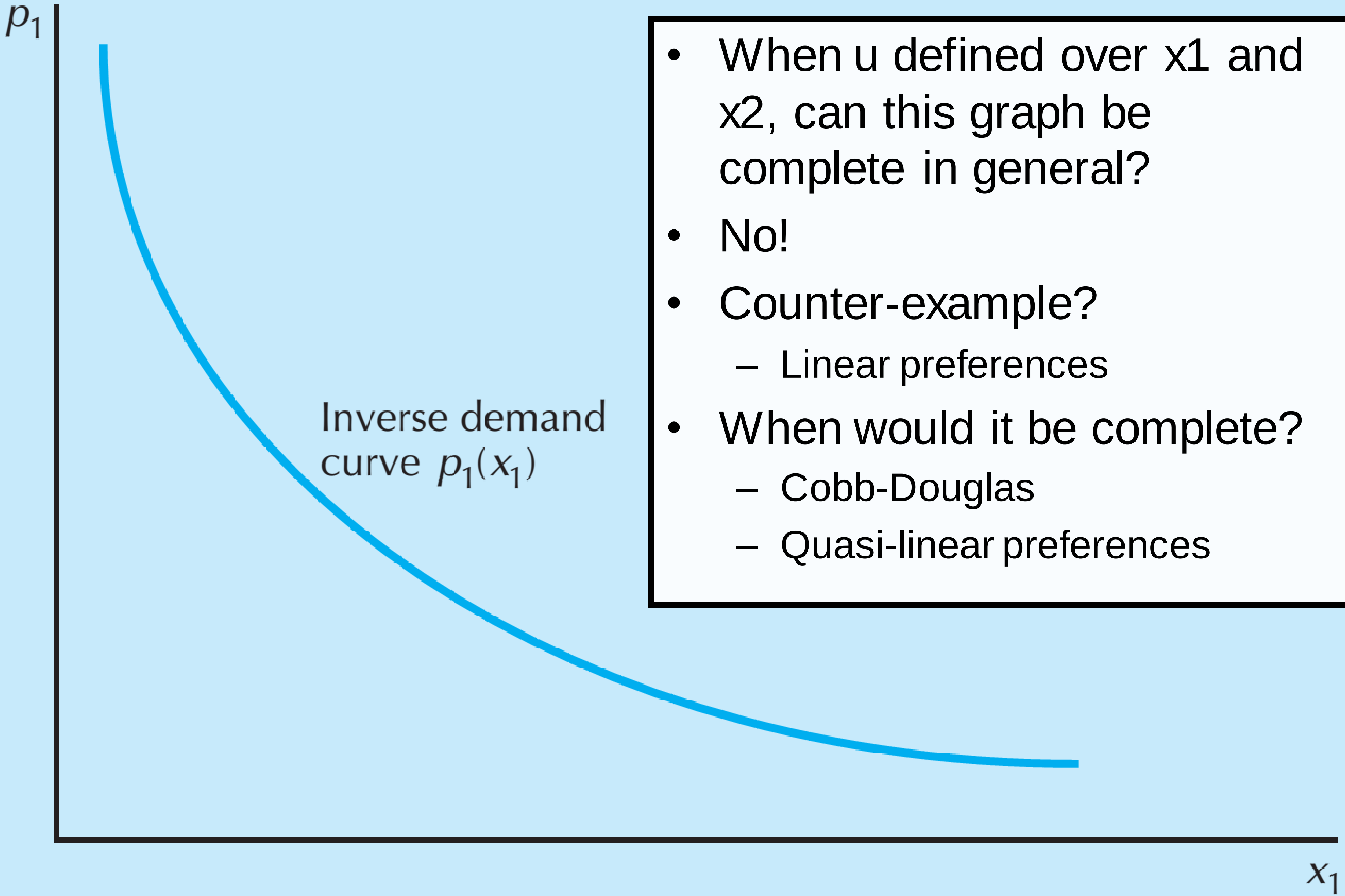
$$|\text{MRS}| = \frac{p_1}{p_2}$$

$$p_1 = p_2|\text{MRS}|$$

Suppose for simplicity that the price of good 2 is one. Then equation (6.4) tells us that at the optimal level of demand, the price of good 1 measures how much the consumer is willing to give up of good 2 in order to get a little more of good 1. In this case the inverse demand function is simply measuring the absolute value of the MRS. For any optimal level of x_1 the inverse demand function tells how much of good 2 the consumer would want to have to compensate him for a small reduction in the amount of good 1. Or, turning this around, the inverse demand function measures how much the consumer would be willing to sacrifice of good 2 to make him just indifferent to having a little more of good 1.

Looked at in this way, the downward-sloping demand curve has a new meaning. When x_1 is very small, the consumer is willing to give up a lot of money—that is, a lot of other goods, to acquire a little bit more of good 1. As x_1 is larger, the consumer is willing to give up less money, on the margin, to acquire a little more of good 1. Thus the marginal willingness to pay, in the sense of the marginal willingness to sacrifice good 2 for good 1, is decreasing as we increase the consumption of good 1.

- When u defined over x1 and x2, can this graph be complete in general?
- No!
- Counter-example?
  - Linear preferences
- When would it be complete?
  - Cobb-Douglas
  - Quasi-linear preferences



$$u(x_1, x_2) = v(x_1) + x_2$$

$$\max_{x_1, x_2} v(x_1) + x_2 \quad \text{s.t. } p_1 x_1 + p_2 x_2 = m$$

$$\max_{x_1} v(x_1) + m/p_2 - p_1 x_1/p_2$$

$$v'(x_1^*) = \frac{p_1}{p_2}$$

The inverse demand curve is given by

$$p_1(x_1) = v'(x_1) p_2$$

Example

$$u(x_1, x_2) = \ln x_1 + x_2$$

Applying the first-order condition gives

$$\frac{1}{x_1} = \frac{p_1}{p_2}$$

so the direct demand function for good 1 is

$$x_1 = \frac{p_2}{p_1}$$

and the inverse demand function is

$$p_1(x_1) = \frac{p_2}{x_1}$$

The direct demand function for good 2 comes from substituting $x_1 = p_2/p_1$ into the budget constraint:

$$x_2 = \frac{m}{p_2} - 1.$$

A warning is in order concerning these demand functions. Note that the demand for good 1 is independent of income in this example. This is a general feature of a quasilinear utility function—the demand for good 1 remains constant as income changes. However, this can only be true for some values of income. A demand function can't literally be independent of income for *all* values of income; after all, when income is zero, all demands are zero. It turns out that the quasilinear demand function derived above is only relevant when a positive amount of each good is being consumed.

1. If the consumer is consuming exactly two goods, and she is always spending all of her money, can both of them be inferior goods?

6.1. No. If her income increases, and she spends it all, she must be purchasing more of at least one good.

2. Show that perfect substitutes are an example of homothetic preferences.

6.2. The utility function for perfect substitutes is $u(x_1, x_2) = x_1 + x_2$. Thus if $u(x_1, x_2) > u(y_1, y_2)$, we have $x_1 + x_2 > y_1 + y_2$. It follows that $tx_1 + tx_2 > ty_1 + ty_2$, so that $u(tx_1, tx_2) > u(ty_1, ty_2)$.

3. Show that Cobb-Douglas preferences are homothetic preferences.

6.3. The Cobb-Douglas utility function has the property that

$$u(tx_1, tx_2) = (tx_1)^a (tx_2)^{1-a} = t^a t^{1-a} x_1^a x_2^{1-a} = t x_1^a x_2^{1-a} = tu(x_1, x_2).$$

Thus if $u(x_1, x_2) > u(y_1, y_2)$, we know that $u(tx_1, tx_2) > u(ty_1, ty_2)$, so that Cobb-Douglas preferences are indeed homothetic.

5. If the preferences are concave will the consumer ever consume both of the goods together?

6.5. No. Concave preferences can only give rise to optimal consumption bundles that involve zero consumption of one of the goods.

7. What is the form of the inverse demand function for good 1 in the case of perfect complements?

6.7. We know that $x_1 = m/(p_1 + p_2)$. Solving for p_1 as a function of the other variables, we have

$$p_1 = \frac{m}{x_1} - p_2.$$

- Chap 7

The Principle of Revealed Preference. *Let (x_1, x_2) be the chosen bundle when prices are (p_1, p_2), and let (y_1, y_2) be some other bundle such that $p_1x_1 + p_2x_2 \geq p_1y_1 + p_2y_2$. Then if the consumer is choosing the most preferred bundle she can afford, we must have $(x_1, x_2) \succ (y_1, y_2)$.*

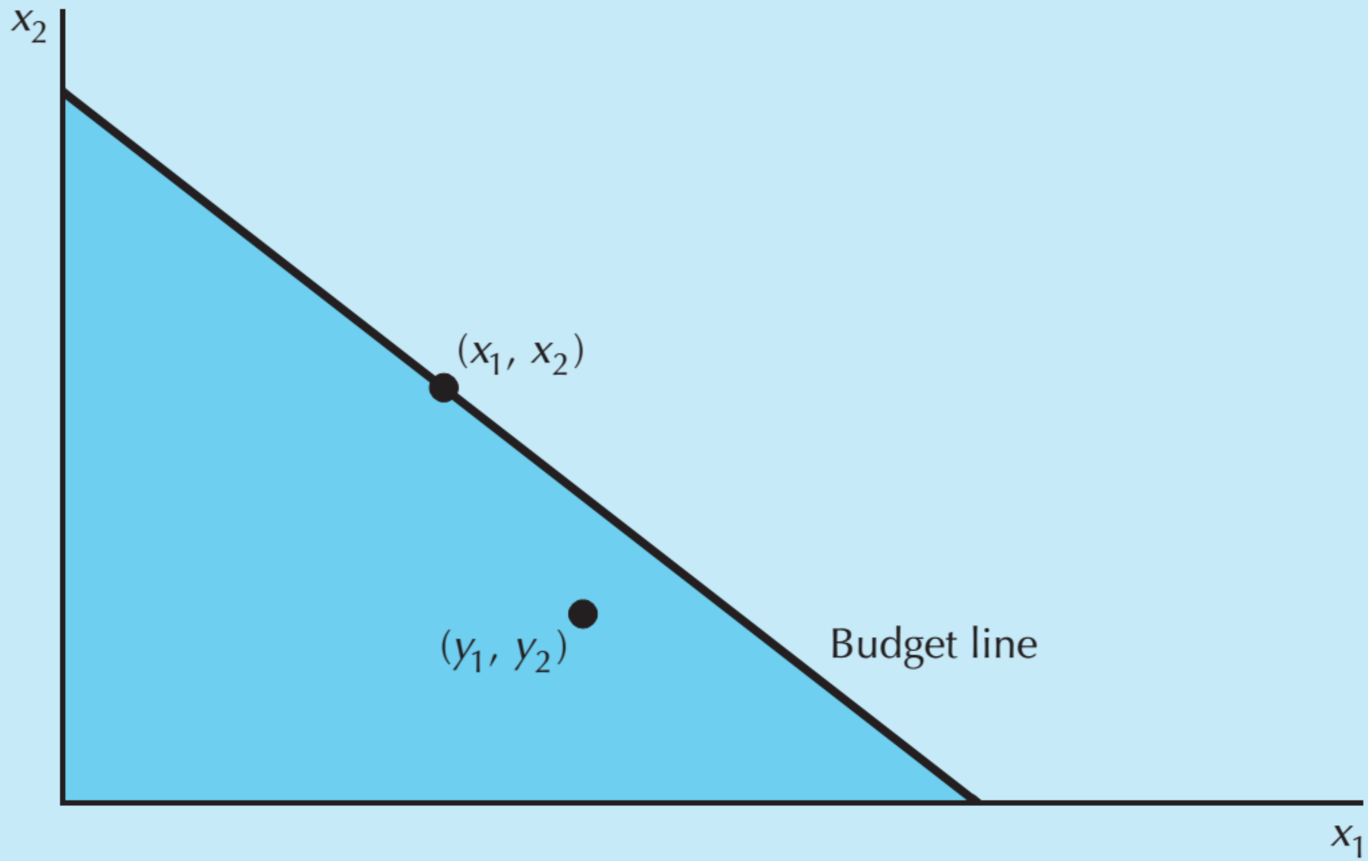
$$p_1y_1 + p_2y_2 \leq m$$

$$p_1x_1 + p_2x_2 = m$$

$$p_1x_1 + p_2x_2 \geq p_1y_1 + p_2y_2$$

Before we begin this investigation, let's adopt the convention that in this chapter, the underlying preferences—whatever they may be—are known to be strictly convex. Thus there will be a *unique* demanded bundle at each budget. This assumption is not necessary for the theory of revealed preference, but the exposition will be simpler with it.

Consider Figure 7.1, where we have depicted a consumer's demanded bundle, (x_1, x_2), and another arbitrary bundle, (y_1, y_2), that is beneath the consumer's budget line. Suppose that we are willing to postulate that this consumer is an optimizing consumer of the sort we have been studying. What can we say about the consumer's preferences between these two bundles of goods?



Revealed preference. The bundle (x_1, x_2) that the consumer chooses is revealed preferred to the bundle (y_1, y_2), a bundle that he could have chosen.

In Figure 7.1 all of the bundles in the shaded area underneath the budget line are revealed worse than the demanded bundle (x_1, x_2). This is because they could have been chosen, but were rejected in favor of (x_1, x_2). We will now translate this geometric discussion of revealed preference into algebra.

Let (x_1, x_2) be the bundle purchased at prices (p_1, p_2) when the consumer has income m. What does it mean to say that (y_1, y_2) is affordable at those prices and income? It simply means that (y_1, y_2) satisfies the budget constraint

$$p_1 y_1 + p_2 y_2 \leq m$$

$$p_1 x_1 + p_2 x_2 = m$$

$$p_1 x_1 + p_2 x_2 \geq p_1 y_1 + p_2 y_2$$

If the above inequality is satisfied and (y_1, y_2) is actually a different bundle from (x_1, x_2), we say that (x_1, x_2) is **directly revealed preferred** to (y_1, y_2).

bundle that is *actually chosen* at prices (p_1, p_2). Thus revealed preference is a relation that holds between the bundle that is actually demanded at some budget and the bundles that *could have been* demanded at that budget.

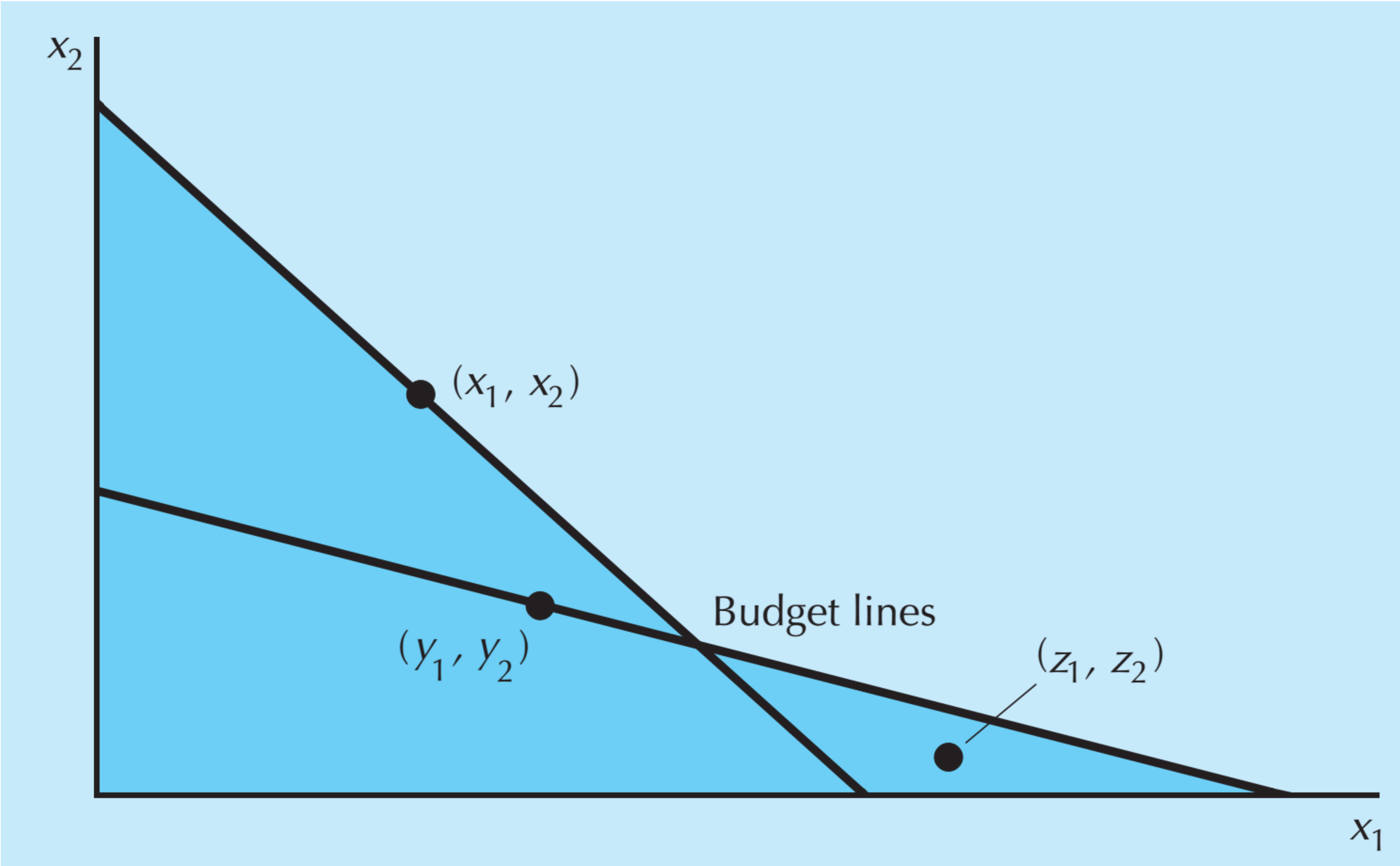
The Principle of Revealed Preference. *Let (x_1, x_2) be the chosen bundle when prices are (p_1, p_2), and let (y_1, y_2) be some other bundle such that $p_1x_1 + p_2x_2 \geq p_1y_1 + p_2y_2$. Then if the consumer is choosing the most preferred bundle she can afford, we must have $(x_1, x_2) \succ (y_1, y_2)$.*

Now suppose that we happen to know that (y_1, y_2) is a demanded bundle at prices (q_1, q_2) and that (y_1, y_2) is itself revealed preferred to some other bundle (z_1, z_2). That is,

$$q_1y_1 + q_2y_2 \geq q_1z_1 + q_2z_2.$$

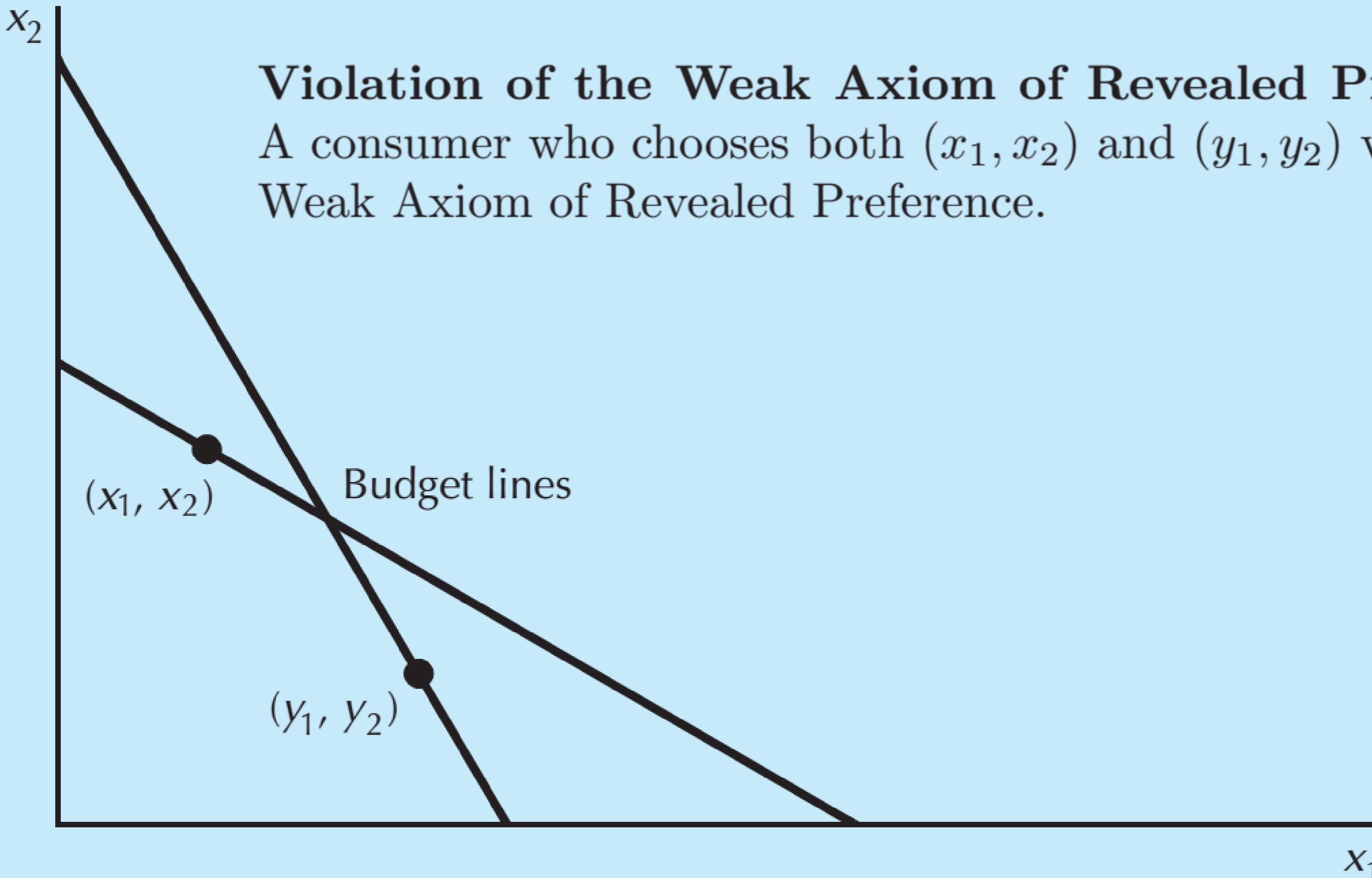
Then we know that $(x_1, x_2) \succ (y_1, y_2)$ and that $(y_1, y_2) \succ (z_1, z_2)$. From the transitivity assumption we can conclude that $(x_1, x_2) \succ (z_1, z_2)$.

It is natural to say that in this case (x_1, x_2) is **indirectly revealed preferred** to (z_1, z_2). Of course the "chain" of observed choices may be

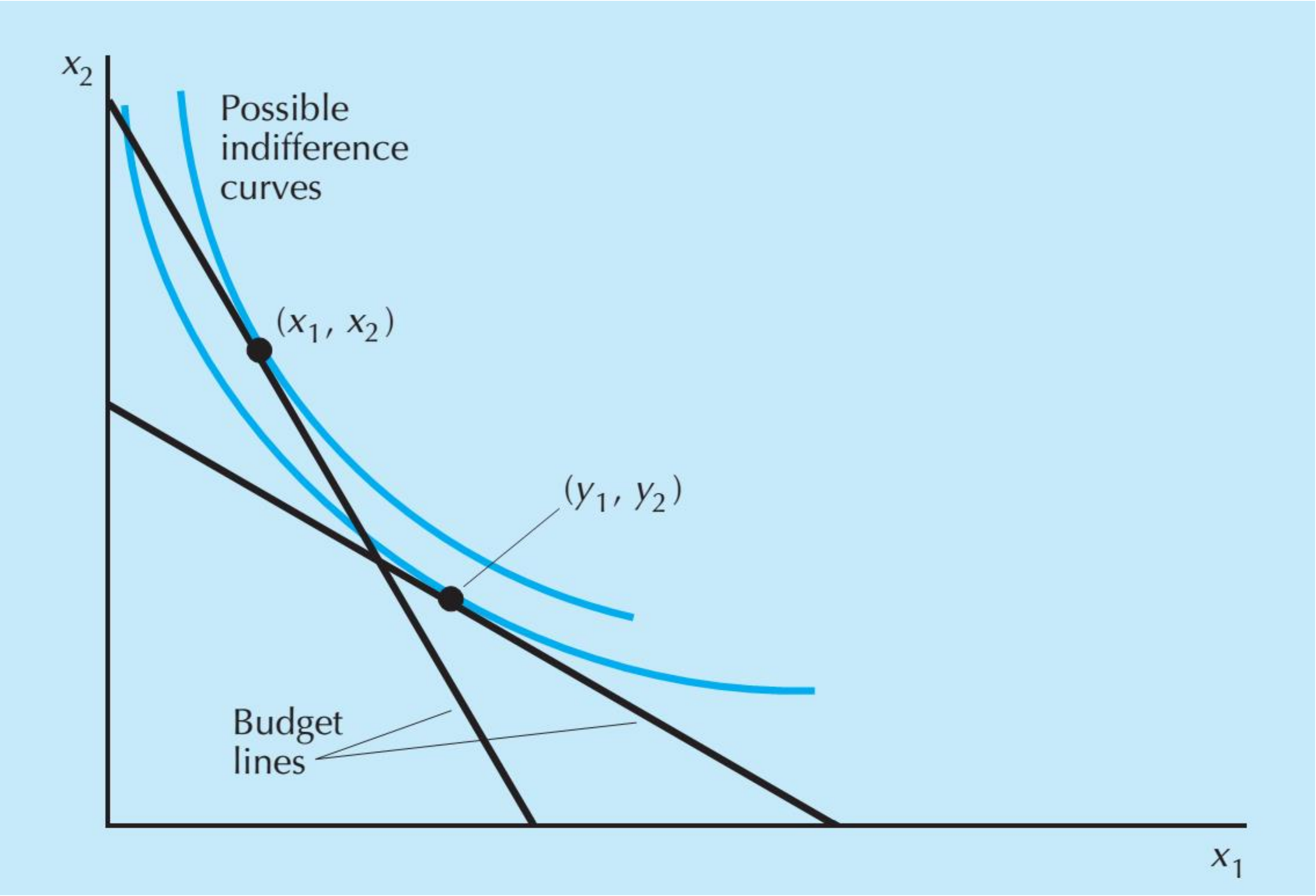


Indirect revealed preference. The bundle (x_1, x_2) is indirectly revealed preferred to the bundle (z_1, z_2).

Weak Axiom of Revealed Preference (WARP). *If (x_1, x_2) is directly revealed preferred to (y_1, y_2), and the two bundles are not the same, then it cannot happen that (y_1, y_2) is directly revealed preferred to (x_1, x_2).*



Violation of the Weak Axiom of Revealed Preference. A consumer who chooses both (x_1, x_2) and (y_1, y_2) violates the Weak Axiom of Revealed Preference.



Satisfying WARP. Consumer choices that satisfy the Weak Axiom of Revealed Preference and some possible indifference curves.

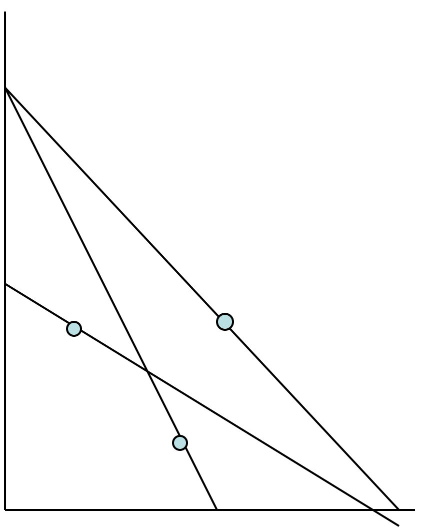
7.5 Checking WARP

Some consumption data.

Observation	p_1	p_2	x_1	x_2
1	1	2	1	2
2	2	1	2	1
3	1	1	2	2

Cost of each bundle at each set of prices.

		Bundles 1	Bundles 2	Bundles 3
Prices	1	5	4*	6
	2	4*	5	6
	3	3*	3*	4



Weak Axiom of Revealed Preference (WARP). *If* (x_1, x_2) *is directly revealed preferred to* (y_1, y_2)*, and the two bundles are not the same, then it cannot happen that* (y_1, y_2) *is directly revealed preferred to* (x_1, x_2)*.*

Strong Axiom of Revealed Preference (SARP). *If* (x_1, x_2) *is revealed preferred to* (y_1, y_2) *(either directly or indirectly) and* (y_1, y_2) *is different from* (x_1, x_2)*, then* (y_1, y_2) *cannot be directly or indirectly revealed preferred to* (x_1, x_2)*.*

1. When prices are $(p_1, p_2) = (1, 2)$ a consumer demands $(x_1, x_2) = (1, 2)$, and when prices are $(q_1, q_2) = (2, 1)$ the consumer demands $(y_1, y_2) = (2, 1)$. Is this behavior consistent with the model of maximizing behavior?

7.1. No. This consumer violates the Weak Axiom of Revealed Preference since when he bought (x_1, x_2) he could have bought (y_1, y_2) and vice versa. In symbols:

$$p_1 x_1 + p_2 x_2 = 1 \times 1 + 2 \times 2 = 5 > 4 = 1 \times 2 + 2 \times 1 = p_1 y_1 + p_2 y_2$$

and

$$q_1 y_1 + q_2 y_2 = 2 \times 2 + 1 \times 1 = 5 > 4 = 2 \times 1 + 1 \times 2 = q_1 x_1 + q_2 x_2.$$

2. When prices are $(p_1, p_2) = (2, 1)$ a consumer demands $(x_1, x_2) = (1, 2)$, and when prices are $(q_1, q_2) = (1, 2)$ the consumer demands $(y_1, y_2) = (2, 1)$. Is this behavior consistent with the model of maximizing behavior?

7.2. Yes. No violations of WARP are present, since the y-bundle is not affordable when the x-bundle was purchased and vice versa.

3. In the preceding exercise, which bundle is preferred by the consumer, the x-bundle or the y-bundle?

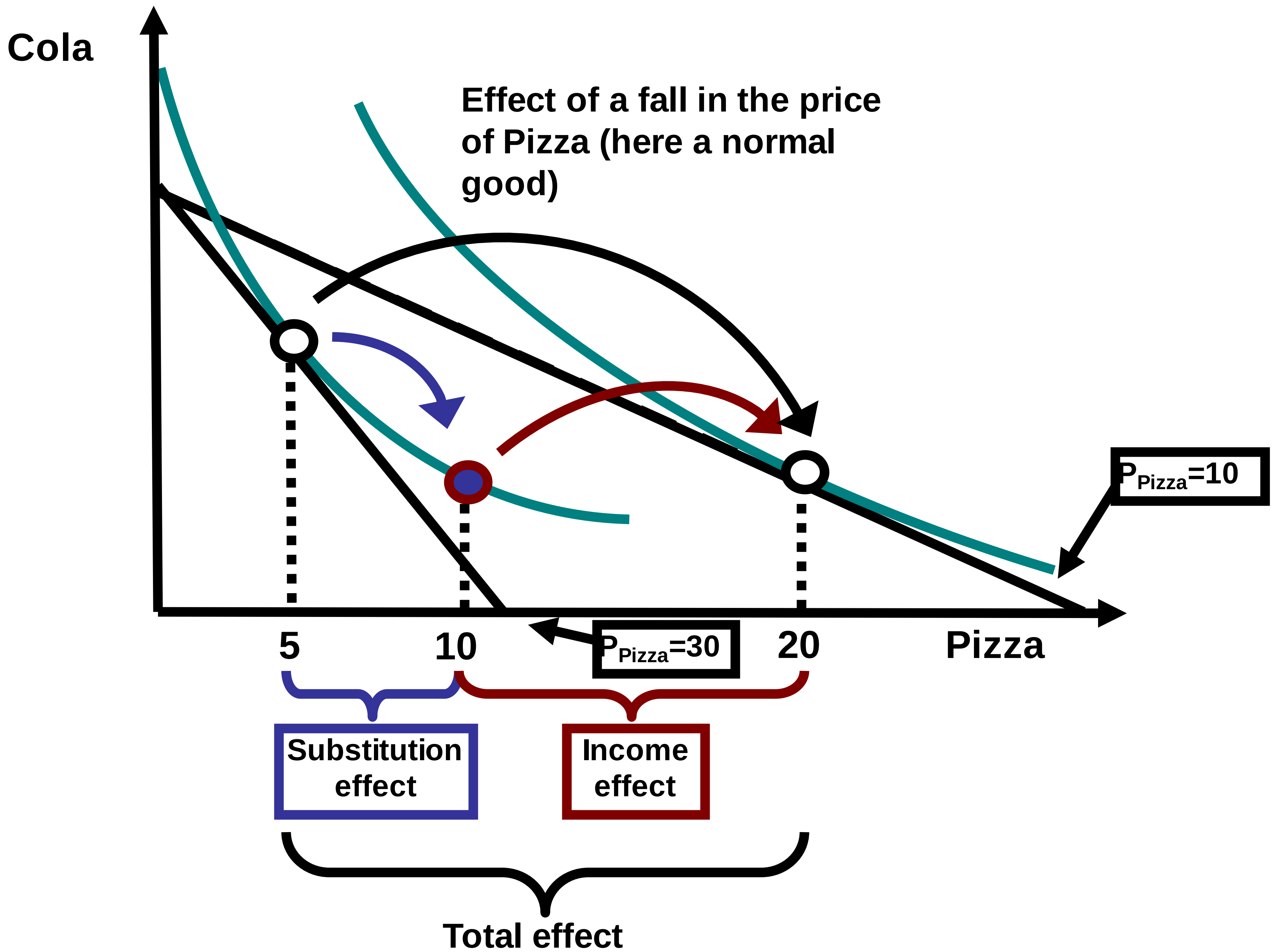
7.3. Since the y-bundle was more expensive than the x-bundle when the x-bundle was purchased and vice versa, there is no way to tell which bundle is preferred.

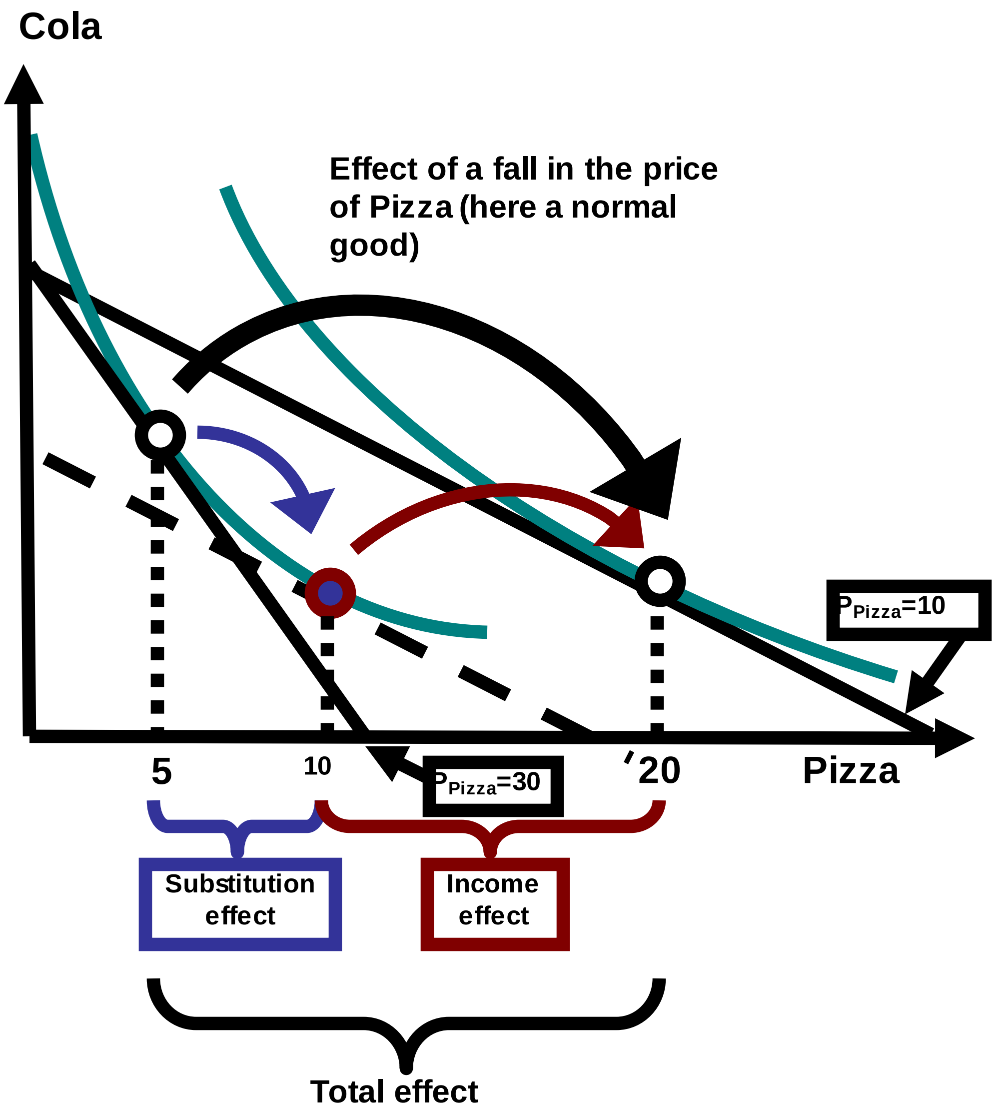
- End lecture 2016.10.24

- **Chap 8**

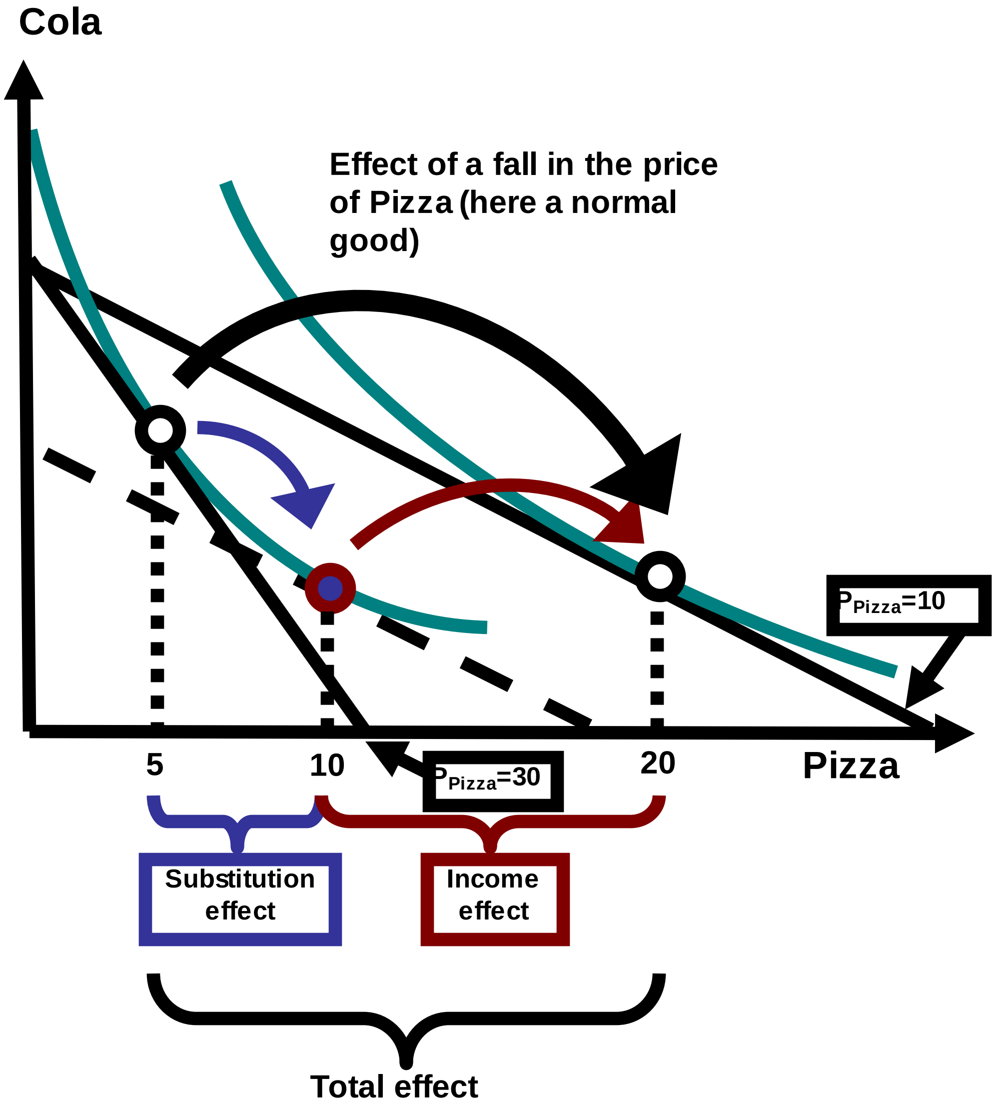
- It is natural to think that when the price of a good rises the demand for it will fall.
- Giffen good.
  - But possible to construct examples where the optimal demand for a good decreases when its price falls.

- What is going on here? How is it that changes in price can have these ambiguous effects on demand?


Cola
Effect of a fall in the price of Pizza (here a normal good)
P Pizza =10
P Pizza =30
5
10
20
Pizza
Substitution effect
Income effect
Total effect

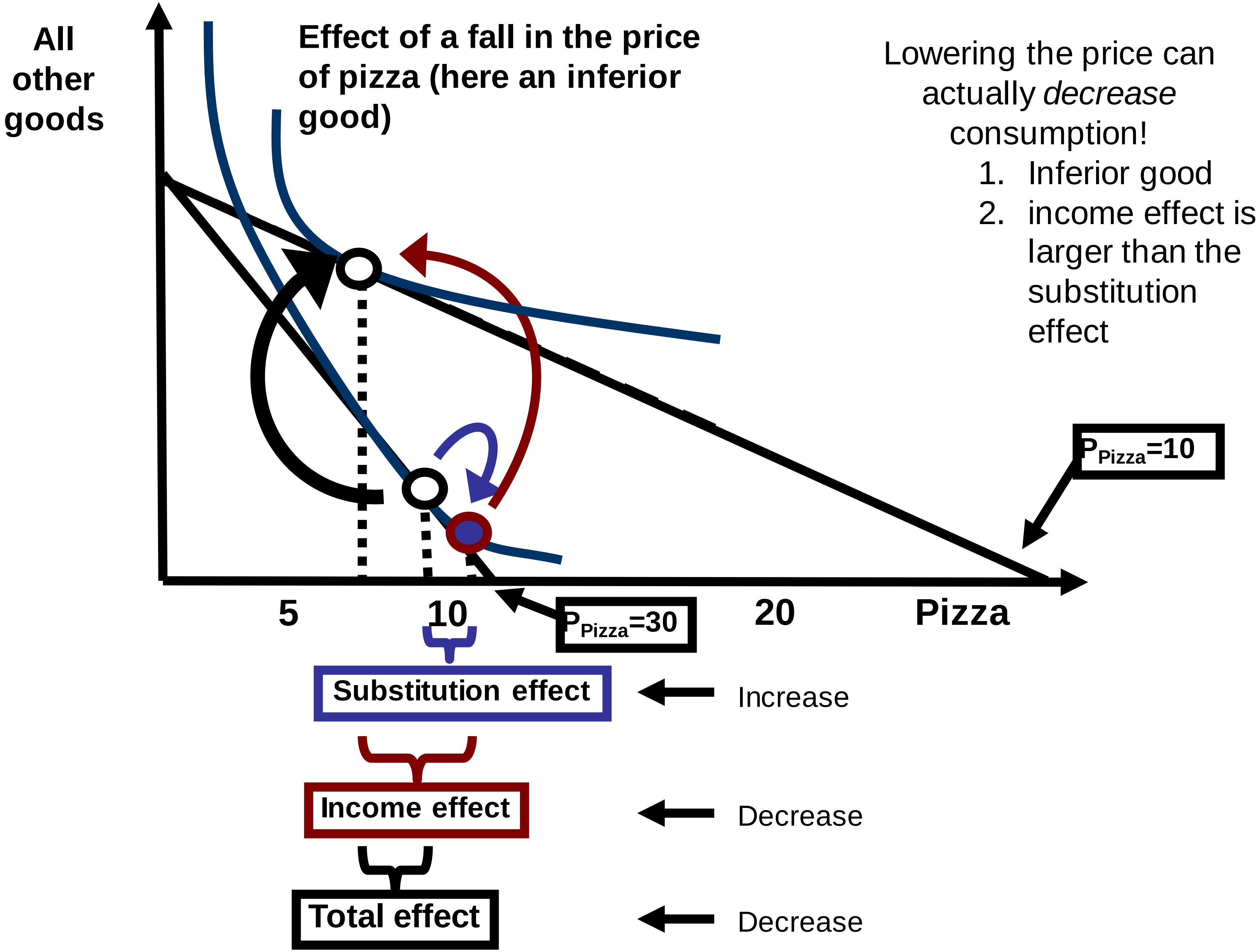


1. Draw the new price line (P_{Pizza}=10)
2. Find the point where a IC touches the new price line
3. The difference between the original point and the new point is **the total effect.**
4. Move the new price line, until it touches the old IC.
5. Mark the point where it touches with a special “in-between point”.
6. Now:
   - The difference between the original point and the “in-between” point is the substitution effect. (a movement along the IC)
   - The difference between the “in-between” point and new point the income effect (a movement between the two parallel budged lines).

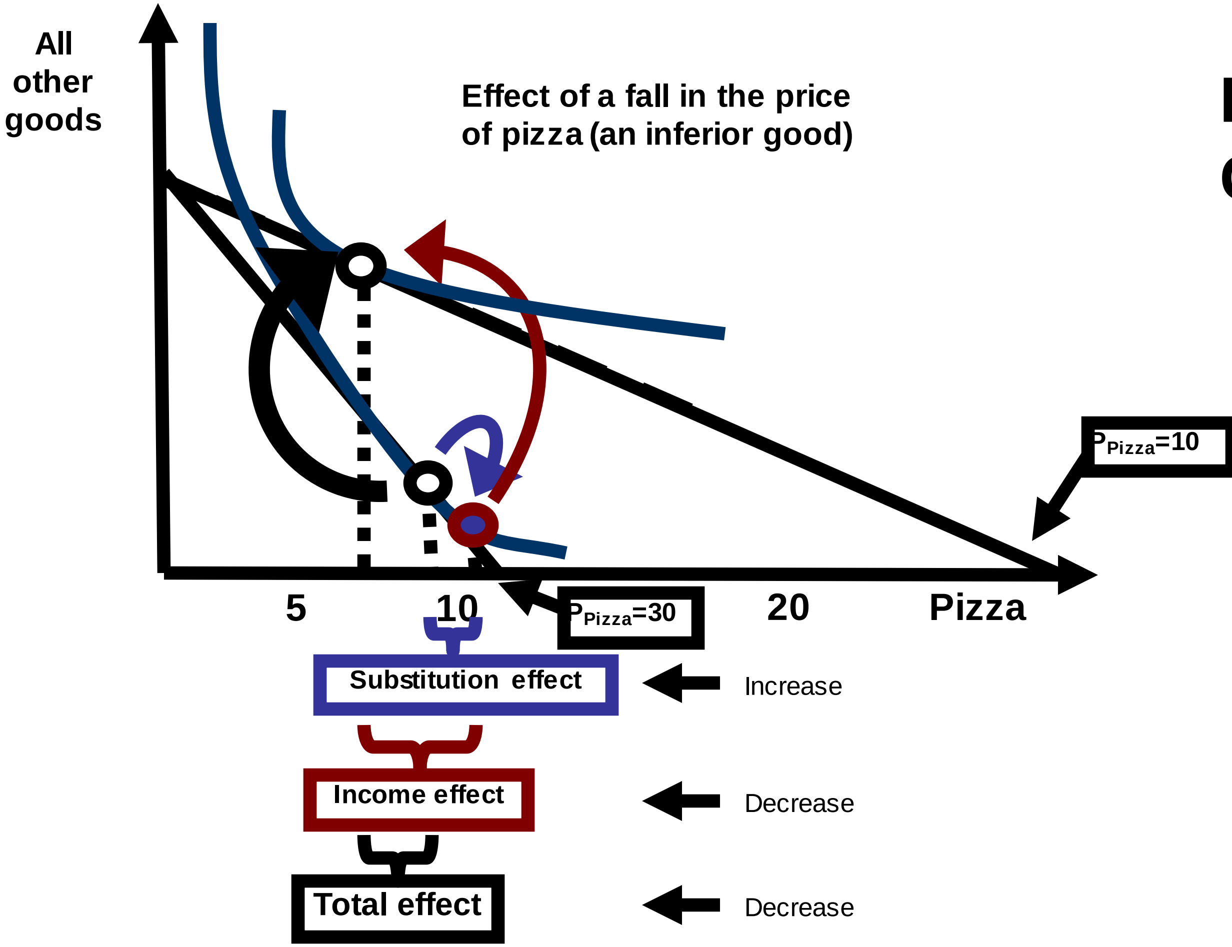


When lowering the price:

- The substitution effect always gives an increase
- What about the income effect?
  - Normal goods: increase
  - Inferior good: decrease

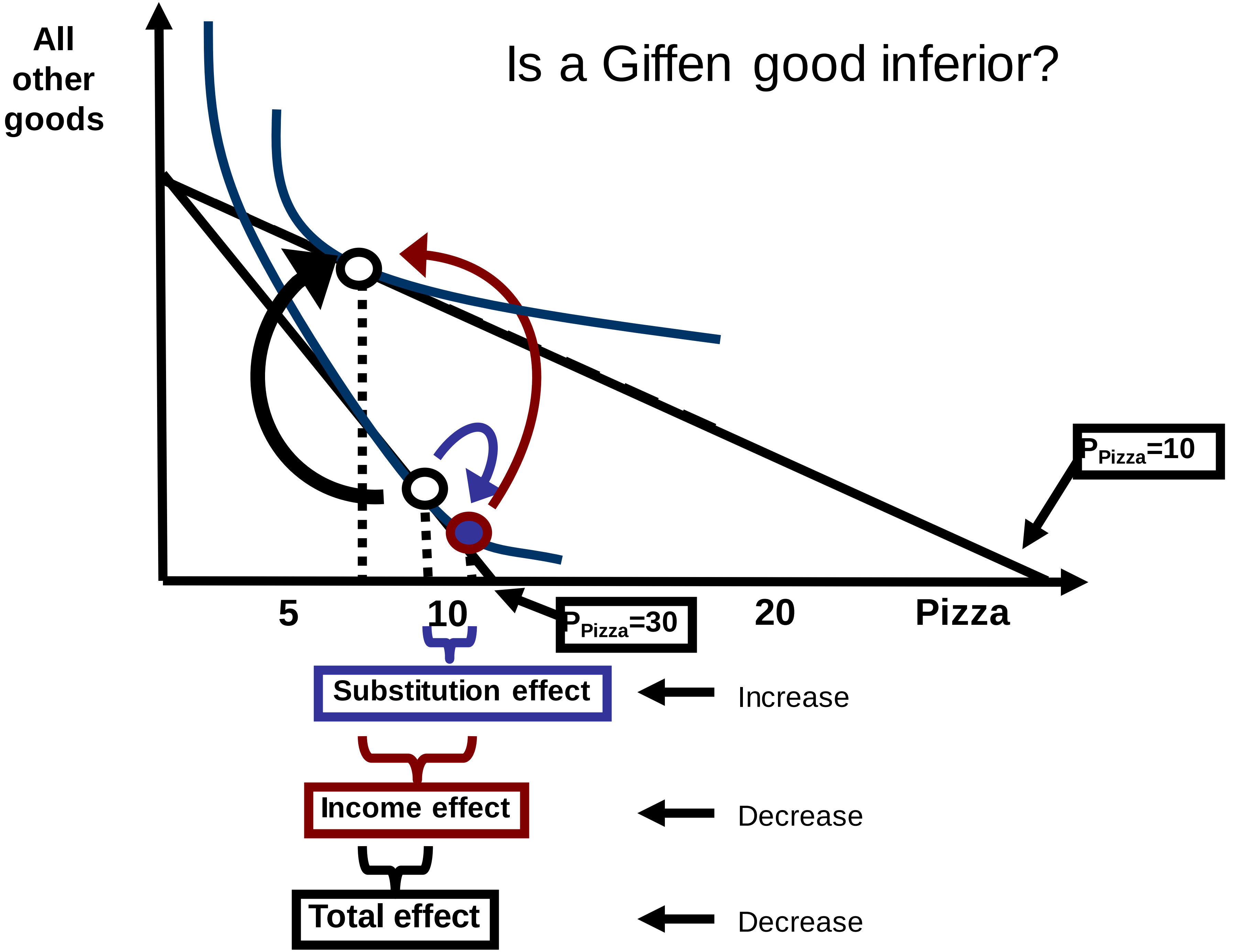

Effect of a fall in the price of pizza (here an inferior good)
All other goods
Pizza
5
10
20
P_{Pizza}=10
P_{Pizza}=30
Lowering the price can actually *decrease* consumption!
1. Inferior good
2. income effect is larger than the substitution effect
Substitution effect
Increase
Income effect
Decrease
Total effect
Decrease

Pizza is here a Giffen good



- The income effect is negative
  - (inferior good)
- The income effect is larger than the substitution effect
- Thus, a decrease in price *decreases* consumption
- Does an increase in price thus increase consumption?
  - Yes!
- Such goods are called “Giffen goods”

Is a Giffen good inferior?



- Example, calculating Hicks substitution and income effect

- Wanda & Slutsky:

$$U(x,y) = x + 72y - 3y^2$$

  - Her income is 300,
  - the price of x is 2
  - the price of y is 24.

a) what is the MRT?

b) what is the MRS?

c) How many units of good y and x will Wanda demand?

a) $MRT = \frac{\Delta y}{\Delta x}\Big|_{Budget}$

Budget:

$$2x + 24y = 300 \Leftrightarrow 24y = 300 - 2x \Leftrightarrow y = \frac{300}{24} - \frac{2}{24}x \Leftrightarrow y = \frac{300}{24} - \frac{1}{12}x$$

Thus $MRT = \frac{\Delta y}{\Delta x}\Big|_{Budget} = -\frac{1}{12}$

b) $MRS = \frac{\Delta y}{\Delta x}\Big|_{Indifference\,Curve} = -\frac{MU_x}{MU_y}$

$$MU_x = \frac{\Delta U}{\Delta x} = \frac{\Delta(x + 72y - 3y^2)}{\Delta x} = 1$$

$$MU_y = \frac{\Delta U}{\Delta y} = \frac{\Delta(x + 72y - 3y^2)}{\Delta y} = 72 - 6y$$

Thus

$$MRS = \frac{\Delta y}{\Delta x}\Big|_{Indifference\,Curve} = -\frac{MU_x}{MU_y} = -\frac{1}{72 - 6y}$$

c) Maximizing utility, Wanda chooses the optimal point, where $MRT = MRS$

Thus $-\frac{1}{12} = -\frac{1}{72-6y} \Leftrightarrow 12 = 72 - 6y \Leftrightarrow 6y = 60 \Leftrightarrow y = 10$

And as $2x + 24y = 207$ **(budget!) it follows that**

$2x = 300 - 24y \Leftrightarrow x = 150 - 12 \cdot 10 \Leftrightarrow x = 30$

NOTE: For question h I also calculate the utility! $U(x, y) = x + 72y - 3y^2$, thus

$U(30,10) = 30 + 72 \cdot 10 - 3 \cdot 10^2 = 30 + 720 - 300 = 450$

Summarizing: x=30, y=10 & U=450

- Now the price of x becomes 4
- the price of y stays 24 and her income stays 300.

d) what is the MRT?

e) what is the MRS?

f) How many units of good y and x will Wanda demand?

g) What is the total effect?

Answer:

d) $MRT = \frac{\Delta y}{\Delta x}\Big|_{Budget}$

Budget: $4x + 24y = 300 \Leftrightarrow 24y = 300 - 4x \Leftrightarrow y = \frac{300}{24} - \frac{4}{24}x \Leftrightarrow y = \frac{300}{24} - \frac{1}{6}x$

Thus $MRT = \frac{\Delta y}{\Delta x}\Big|_{Budget} = -\frac{1}{6}$

e) $MRS = \frac{\Delta y}{\Delta x}\Big|_{Indifference\,Curve} = -\frac{MU_x}{MU_y}$

$$MU_x = \frac{\Delta U}{\Delta x} = \frac{\Delta(x + 72y - 3y^2)}{\Delta x} = 1$$

$$MU_y = \frac{\Delta U}{\Delta y} = \frac{\Delta(x + 72y - 3y^2)}{\Delta y} = 72 - 6y$$

Thus $MRS = \frac{\Delta y}{\Delta x}\Big|_{Indifference\,Curve} = -\frac{MU_x}{MU_y} = -\frac{1}{72 - 6y}$

f) Maximizing utility, Wanda chooses the optimal point, where $MRT = MRS$

Thus $-\dfrac{1}{6} = -\dfrac{1}{72-6y} \Leftrightarrow 6 = 72 - 6y \Leftrightarrow 6y = 66 \Leftrightarrow y = 11$

And as $4x + 24y = 300$ **(budget!) it follows that**

$4x = 300 - 24y \Leftrightarrow x = 75 - 6 \cdot 11 \Leftrightarrow x = 9$

Thus our new x is given by: $x_{NEW} = 9$

Summarizing: xNEW=9 & y=11

g) **X changes from 30 to 9: total effect is 9 – 30 = -21 (21 fewer units of x)**

- Now the price of x becomes 4
- the price of y stays 24 and her income stays 300.

h) What is the Hicks substitution effect?

i) What is the Hicks income effect?

I need to determine how much x would the consumer choose when Px=4 but the consumer has the budget to have the same utility as under the old price, when Px=2.

Under the old price, Px=2, utility was:

$$U(30,10) = 30 + 72 \cdot 10 - 3 \cdot 10^2 = 30 + 720 - 300 = 450$$

Under the old prices, Px=2 and Py=4, $MRT = \frac{\Delta y}{\Delta x}\Big|_{Budget} = -\frac{1}{6}$

Both for the old and new prices, MRS is given by

$$MRS = \frac{\Delta y}{\Delta x}\Big|_{Indifference\,Curve} = -\frac{MU_x}{MU_y} = -\frac{1}{72-6y}$$

In the optimal point: $-\frac{1}{6} = -\frac{1}{72-6y} \Leftrightarrow 6 = 72 - 6y \Leftrightarrow 6y = 66 \Leftrightarrow y = 11$

Because we should still be on the old indifference curve, we must have

$$U(x_S, 11) = x_S + 72 \cdot 11 - 3 \cdot 11^2 = 450$$

$$x_S = 450 - 72 \cdot 11 + 3 \cdot 11^2 = 450 - 792 + 363 = 21$$

The substitution effect is thus the difference between the old x $(x = 30)$ **and the x on the same IC but with the new prices** $(x_S = 21)$**. This is equal to 21- 30= -9. The substitution effect is thus equal to -9. (*an increase in price from Px=2 to Px=4 results in a substitution effect of 9 less units of x*)**

- A bit more formal
  - Using infinitesimal small changes (dp_i)
- We need to look at
  - the indirect utility function
  - the expenditure function

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