Microeconomics

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

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Lecture 2

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- The presentation is based on the following sources:

- Snyder Christopher M Walter Nicholson and Robert Stewart. 2015. Microeconomic Theory : Basic Principles and Extensions EMEA ed. Andover: Cengage Learning. +
- Varian Hal R. 2020. Intermediate Microeconomics : A Modern Approach Ninth ed. New York: W.W. Norton & Company. +
- Excerpts from Gravelle H O Hart and R Rees. 2004. Advanced Microeconomics.

1. Indirect utility function
2. Expenditure function

1. Indirect utility function

- Gravelle & Rees, Chap 2.B

- Indirect utility function

$$v[\vec{p},m]=\max_{\vec{x}} u[\vec{x}] \text{ such that } \vec{p}\vec{x}\le m$$

How to write this in the less general notation?

$$v[p_1,\dots,p_n,m]=\max u[x_1,\dots,x_n] \text{ such that } \sum_{i=1}^{n} p_i x_i \le m$$

$$L=u[\vec{x}]-\lambda[\vec{p}\vec{x}-m]$$

$$FOC: \lambda\cdot p_1=u_1[x_1,x_2],\quad \lambda\cdot p_2=u_2[x_1,x_2]$$

$$\begin{cases} \dfrac{p_1}{p_2}=\dfrac{u_1[h_1,h_2]}{u_2[h_1,h_2]} \\ \lambda=\dfrac{u_1[h_1,h_2]}{p_1}=\dfrac{u_2[h_1,h_2]}{p_2} \end{cases}$$

Result is $\vec{x}[\vec{p},m]$,

$$\equiv\{x_1[\vec{p},m],\dots x_n[\vec{p},m]\}$$

$$\equiv\{x_1[p_1,\dots,p_n,m],\dots x_n[p_1,\dots,p_n,m]\}$$

The <u>Marshallian</u> demand function

(What other demand function do we have?)

(Hicksian demand function) $\vec{h}[\vec{p},u]$

$$v[\vec{p},m]=\max_{\vec{x}} u[\vec{x}] \text{ such that } \vec{p}\vec{x}=m$$

$$v[\vec{p},m]=u[\vec{x}[\vec{p},m]]$$

- Roy's identity: $x_i(\vec{p},m) = -\dfrac{\frac{\partial v(\vec{p},m)}{\partial p_i}}{\frac{\partial v(\vec{p},m)}{\partial m}}$

Envelope theory (p.490 in Varian Microeconomic Analysis):

$$M[a] = \max_x g[x,a] \rightarrow \frac{dM[a]}{da} = \left.\frac{dg[x,a]}{da}\right|_{x=x[a]}$$

$$v(\vec{p},m) = \max_{\vec{x}} u(\vec{x}) \text{ such that } \vec{p}\vec{x} = m$$

$$v(\vec{p},m) = \max_{\vec{x}} L = u(\vec{x}) - \lambda(\vec{p}\vec{x} - m)$$

$$\frac{dv(\vec{p},m)}{dp_i} = \left.\frac{dL}{dp_i}\right|_{\vec{x}=x(\vec{p},m)} = -\lambda x_i \,|_{\vec{x}=x(\vec{p},m)} = -\lambda x_i(\vec{p},m)$$

$$\frac{dv(\vec{p},m)}{dm} = \left.\frac{dL}{dm}\right|_{\vec{x}=x(\vec{p},m)} = \lambda \,|_{\vec{x}=x(\vec{p},m)} = \lambda$$

$$Thus\ -\frac{\frac{\partial v(\vec{p},m)}{\partial p_i}}{\frac{\partial v(\vec{p},m)}{\partial m}} = -\frac{-\lambda x_i(\vec{p},m)}{\lambda} = x_i(\vec{p},m)$$

$$-\frac{\frac{\partial v[\vec{p},m]}{\partial p_i}}{\frac{\partial v[\vec{p},m]}{\partial m}} = x_i[\vec{p},m]$$

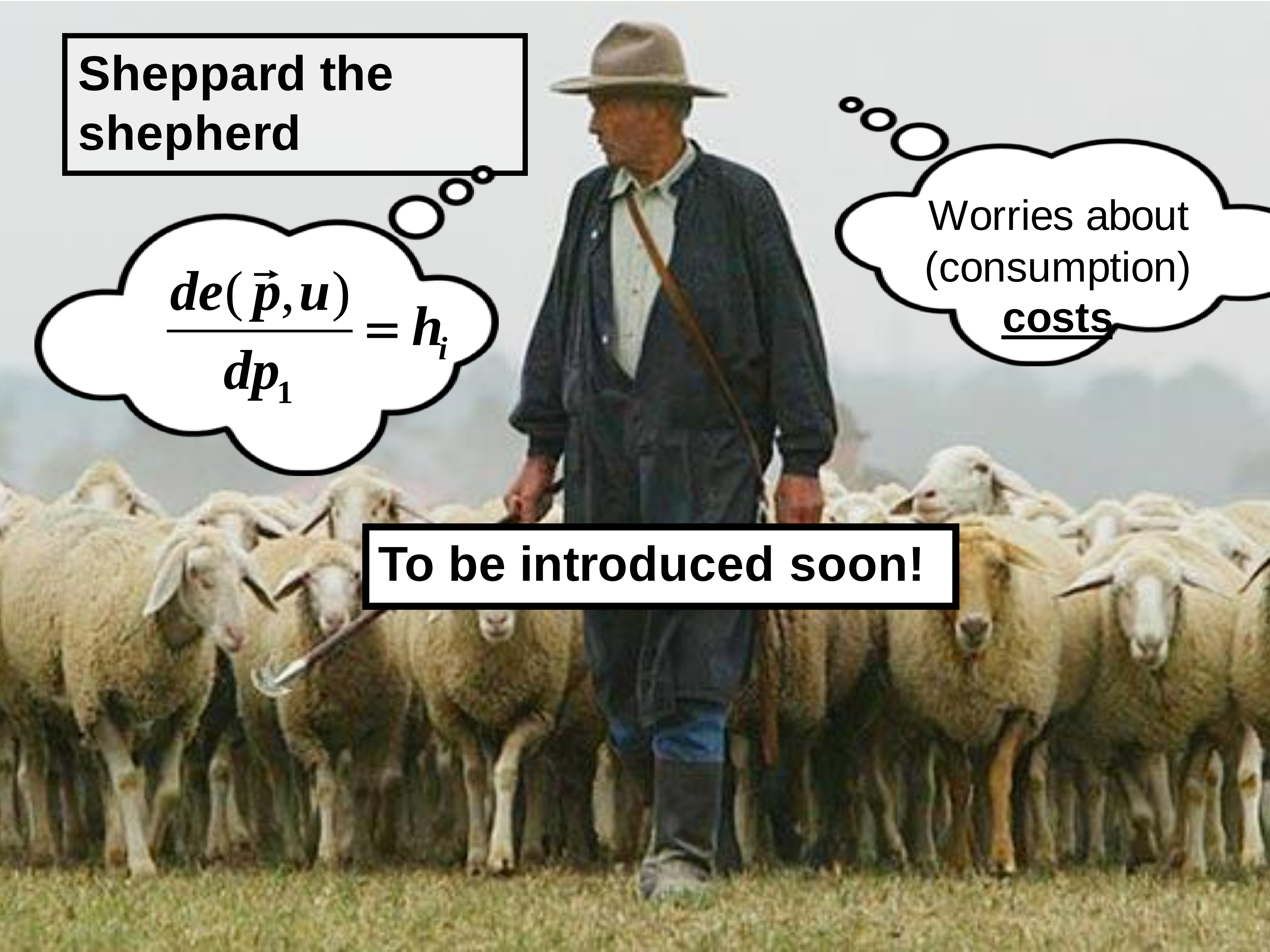
Utility

!!!

Roy the hotel manager

- We did this in the part on choice (L4 & L5_ Choice_ VSE_ 2016.ppt)
- Even proofed Roy’s identity using envelope theorem
- Did a few exercises
  - calculating x[p,m], v[p,m] and then getting back x[p,m] using Roy’s
  - For perf. Complements and perf. Substitutes.
  - But not yet for Cobb-Douglas!


$$\frac{\partial \pi[p,\vec{w}]}{\partial w_i} = x_i[p,\vec{w}]$$
$$\frac{\partial \pi[p,\vec{w}]}{\partial p} = y[p,\vec{w}]$$
PROFITS !!!
To be discussed in the part on production
Colleague Hotelling the hotel manager


Sheppard the shepherd
$$\frac{de(\vec{p},u)}{dp_1} = h_i$$
Worries about (consumption) **costs**
To be introduced soon!

- **Gravelle & Rees, Chap 2.B**

Exercise

EXAMPLE: The Cobb-Douglas utility function

Find the Marshallian Demand functions: **direct method** and using indirect utility function

$$u(x_1, x_2) = x_1 x_2^2$$

$$v[p_1, p_2, m] = \underset{x_1, x_2}{MAX} \; x_1 x_2^2 \text{ s.t. } p_1 x_1 + p_2 x_2 = m$$

$$v[p_1, p_2, m] = \underset{x_1, x_2, \lambda}{MAX} \; L(x_1, x_2, \lambda) = x_1 x_2^2 - \lambda(p_1 x_1 + p_2 x_2 - m)$$

$FOC:$

$$x_2^2 = \lambda p_1$$

$$2 x_1 x_2 = \lambda p_2$$

$$p_1 x_1 + p_2 x_2 = m$$

$$\frac{x_2^2}{2 x_1 x_2} = \frac{p_1}{p_2}$$

$$\frac{x_2}{2 x_1} = \frac{p_1}{p_2}$$

$$p_2 x_2 = 2 p_1 x_1$$

$$p_1 x_1 + 2 p_1 x_1 = m$$

$$3 p_1 x_1 = m$$

$$x_1 = \tfrac{1}{3} \frac{m}{p_1}$$

$$x_2 = \tfrac{2}{3} \frac{m}{p_2}$$

$$-\frac{\frac{\partial v(\vec{p},m)}{\partial p_i}}{\frac{\partial v(\vec{p},m)}{\partial m}} = x_i(\vec{p},m)$$

Utility

!!!

Colleague Roy the hotel manager

EXAMPLE: The Cobb-Douglas utility function

Find the Marshallian Demand functions: direct method and **using indirect utility function**

$$u(x_1, x_2) = x_1 x_2^2 \qquad x_1 = \tfrac{1}{3}\frac{m}{p_1} \qquad x_2 = \tfrac{2}{3}\frac{m}{p_2}$$

$$v[p_1, p_2, m] = \tfrac{1}{3}\frac{m}{p_1} x_2 \cdot \left(\tfrac{2}{3}\frac{m}{p_2}\right)^2 = \tfrac{4}{27}\frac{m^3}{p_1 p_2^2}$$

$$\frac{dv[p_1, p_2, m]}{dp_1} = -\tfrac{4}{27}\frac{m^3}{p_1^2 p_2^2}$$

$$\frac{dv[p_1, p_2, m]}{dm} = -\tfrac{12}{27}\frac{m^2}{p_1 p_2^2}$$

$$\frac{\frac{dv[p_1, p_2, m]}{dp_1}}{\frac{dv[p_1, p_2, m]}{dm}} = \frac{-\tfrac{4}{27}\frac{m^3}{p_1^2 p_2^2}}{-\tfrac{12}{27}\frac{m^2}{p_1 p_2^2}}$$

$$= \frac{\tfrac{1}{3}\frac{m}{p_1}}{\frac{1}{1}}$$

- More general example
  - Go through by yourself at home

Now for Lagrange's method. Set up the Lagrangian

$$L = c \ln x_1 + d \ln x_2 - \lambda(p_1 x_1 + p_2 x_2 - m)$$

$$\frac{\partial L}{\partial x_1} = \frac{c}{x_1} - \lambda p_1 = 0$$
$$\frac{\partial L}{\partial x_2} = \frac{d}{x_2} - \lambda p_2 = 0$$
$$\frac{\partial L}{\partial \lambda} = p_1 x_1 + p_2 x_2 - m = 0.$$

$$c = \lambda p_1 x_1$$
$$d = \lambda p_2 x_2.$$

$$c + d = \lambda(p_1 x_1 + p_2 x_2) = \lambda m$$

$$\lambda = \frac{c + d}{m}$$

$$\lambda = \frac{c+d}{m}$$

$$x_1 = \frac{c}{c+d}\frac{m}{p_1}$$

$$x_2 = \frac{d}{c+d}\frac{m}{p_2},$$

$$c = \lambda p_1 x_1$$

$$d = \lambda p_2 x_2.$$

$$v(p_1, p_2, m) = c\ln[\frac{c}{c+d}\frac{m}{p_1}] + d\ln[\frac{d}{c+d}\frac{m}{p_2}]$$

$$\left(v(p_1, p_2, m) = c\ln[\frac{c}{c+d}\frac{m}{p_1}] + d\ln[\frac{d}{c+d}\frac{m}{p_2}] \right)$$

$$x_1 = \frac{c}{c+d}\frac{m}{p_1}$$
$$x_2 = \frac{d}{c+d}\frac{m}{p_2},$$

$$-\frac{\frac{\partial v(\vec{p},m)}{\partial p_i}}{\frac{\partial v(\vec{p},m)}{\partial m}} = x_i(\vec{p},m)$$

$$v(p_1, p_2, m) = c\ln[\frac{c}{c+d}\frac{m}{p_1}] + d\ln[\frac{d}{c+d}\frac{m}{p_2}]$$

$$\frac{\partial v(\vec{p},m)}{\partial p_1} = c\left(\frac{c}{c+d}\frac{m}{p_1}\right)^{-1}\frac{c}{c+d}m\frac{-1}{p_1^2}$$

$$= \frac{-c}{p_1}$$

$$\frac{\partial v(\vec{p},m)}{\partial m} = c\left(\frac{c}{c+d}\frac{m}{p_1}\right)^{-1}\frac{c}{c+d}\frac{1}{p_1} + d\left(\frac{d}{c+d}\frac{m}{p_2}\right)^{-1}\frac{d}{c+d}\frac{1}{p_2}$$

$$= \frac{c}{m} + \frac{d}{m}$$

$$x_1 = \frac{c}{c+d}\frac{m}{p_1}$$

$$x_2 = \frac{d}{c+d}\frac{m}{p_2},$$

$$-\frac{\frac{\partial v(\vec{p},m)}{\partial p_1}}{\frac{\partial v(\vec{p},m)}{\partial m}} = -\frac{\frac{-c}{p_1}}{\frac{c}{m}+\frac{d}{m}} = \frac{c}{c+d}\frac{m}{p_1}$$

$$-\frac{\frac{\partial v(\vec{p},m)}{\partial p_2}}{\frac{\partial v(\vec{p},m)}{\partial m}} = -\frac{\frac{-d}{p_2}}{\frac{c}{m}+\frac{d}{m}} = \frac{d}{c+d}\frac{m}{p_2}$$

2. Expenditure function

- **Gravelle & Rees, Chap 3A & 3B (up to mid p.55)**

- Expenditure function   Minimize expenditure given utility level

$$e[p_1,\ldots,p_n,u] = \min_{h_i} \sum_{i=1}^{n} p_i h_i \text{ such that } \bar{u} \le u[h_1,\ldots,h_n]$$

- Expenditure function   Minimize expenditure given utility level

$$e[p_1, p_2, u] = \min_{h_1, h_2} p_1 x_1 + p_2 x_2 \text{ such that } \bar{u} \le u[h_1, \dots, h_n]$$

$$e[p_1, p_2, u] = \min_{h_1, h_2, \mu} L[h_1, h_2, \mu]$$

$$= \min_{h_1, h_2, \mu} \left(p_1 h_1 + p_2 h_2 + \mu \left(u - u[h_1, h_2] \right) \right)$$

$$FOC: \; p_1 = \mu \cdot u_1[h_1, h_2], \quad p_2 = \mu \cdot u_2[h_1, h_2]$$

$$\frac{p_1}{p_2} = \frac{u_1[h_1, h_2]}{u_2[h_1, h_2]} \qquad \mu = \frac{p_1}{u_1[h_1, h_2]} = \frac{p_2}{u_2[h_1, h_2]}$$

From utility maximization

$$L = u[\vec{x}] - \lambda[\vec{p}\vec{x} - m]$$

$$\begin{cases} \dfrac{p_1}{p_2} = \dfrac{u_1[h_1, h_2]}{u_2[h_1, h_2]} \\ \lambda = \dfrac{u_1[h_1, h_2]}{p_1} = \dfrac{u_2[h_1, h_2]}{p_2} \end{cases}$$

$$\lambda = \frac{1}{\mu}$$

The expenditure function

$$\min_{x_1,\ldots,x_n} \sum p_i x_i \quad \text{s.t.} \quad \text{(i) } u(x_1, \ldots, x_n) \geqslant u$$

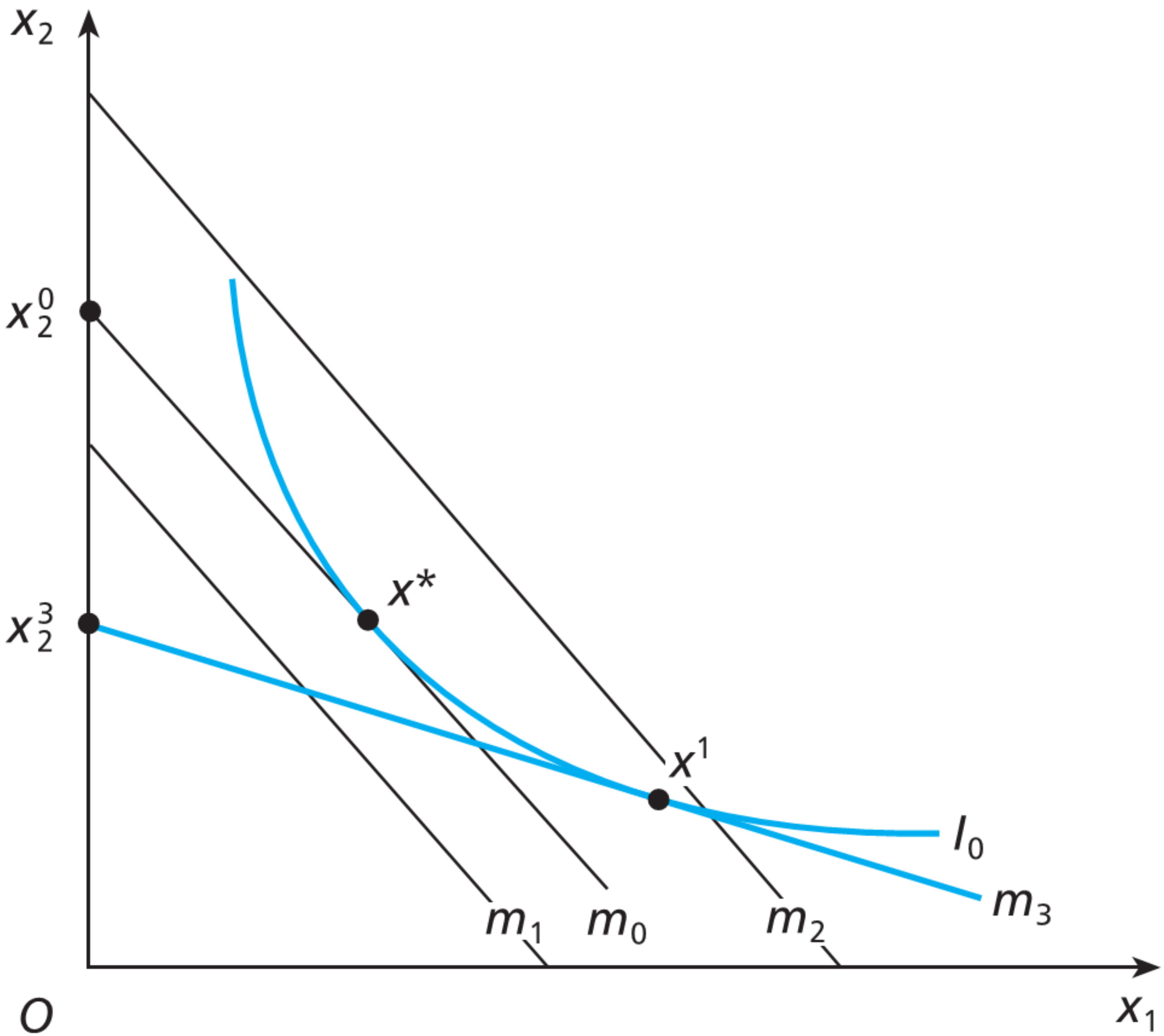
$$\text{(ii) } x_i \geqslant 0, \qquad i = 1, \ldots, n$$

$$L = \sum p_i x_i + \mu[u - u(x_1, \ldots, x_n)]$$

$$\frac{\partial L}{\partial x_i} = p_i - \mu u_i = 0 \qquad i = 1, \ldots, n$$

$$\frac{\partial L}{\partial \mu} = u - u(x_1, \ldots, x_n) = 0$$

$$\frac{p_i}{p_j} = \frac{u_i}{u_j}$$



x_2
x_2^0
x_2^3
x^*
x^1
I_0
m_3
m_1
m_0
m_2
O
x_1

Duality

$$\max_{x_1,\ldots,x_n} u(x_1, x_2, \ldots, x_n) \quad \text{s.t.} \sum_i p_i x_i \leqslant M, \qquad x_i \geqslant 0, \qquad i = 1, \ldots, n$$

$$\min_{x_1,\ldots,x_n} \sum p_i x_i \quad \text{s.t.} \quad \text{(i) } u(x_1, \ldots, x_n) \geqslant u$$

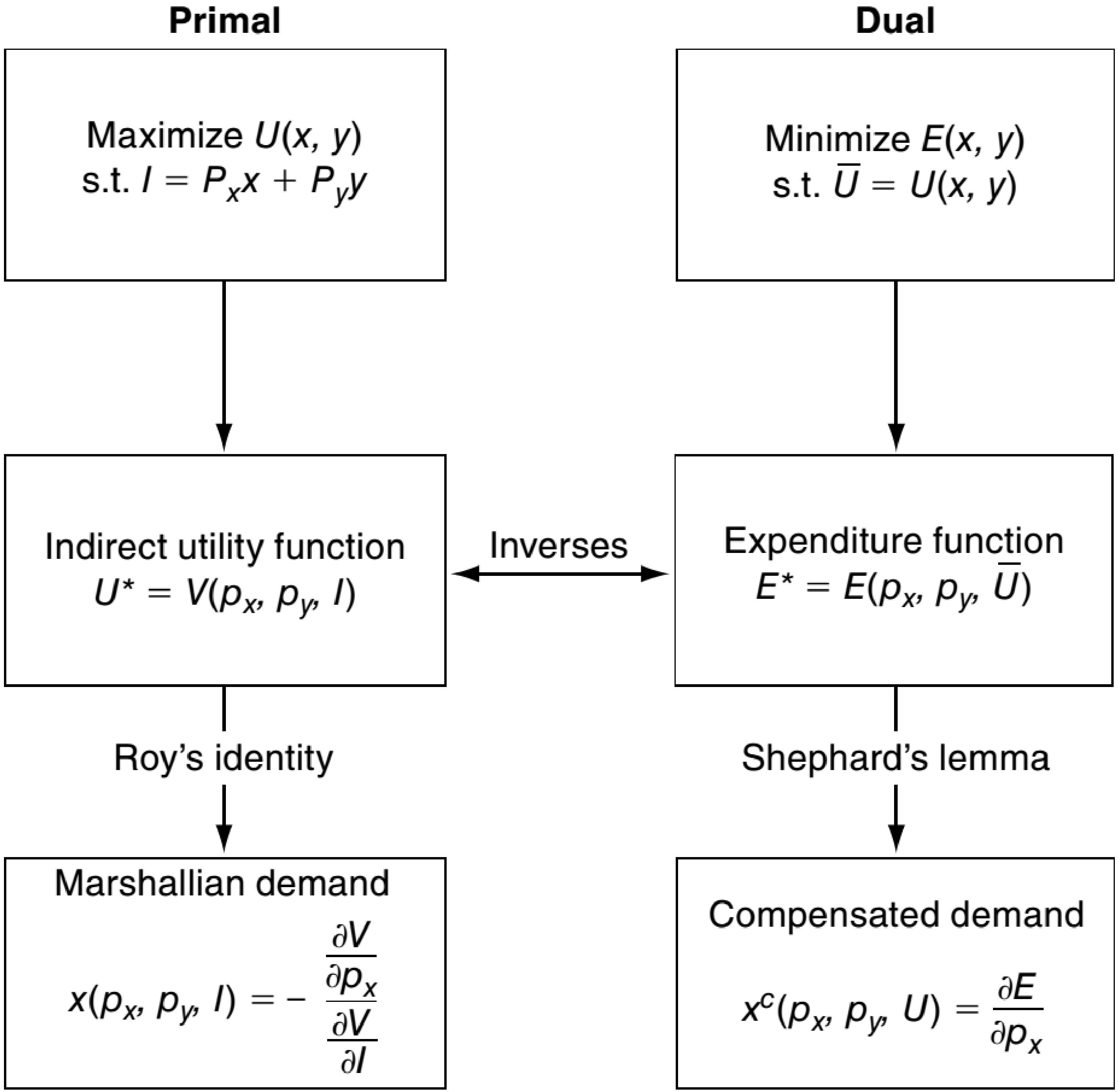
$$\text{(ii) } x_i \geqslant 0, \qquad i = 1, \ldots, n$$

$$x_i^* = H_i(p_1, \ldots, p_n, u) = H_i(p, u) \qquad i = 1, \ldots, n$$

$H_i(p, u)$ is the *Hicksian* demand function for x_i

Also called compensated demand function

$$\sum p_i x_i^* = \sum p_i H_i(p, u) = m(p, u)$$


Primal
Dual
Maximize $U(x, y)$
s.t. $I = P_x x + P_y y$
Minimize $E(x, y)$
s.t. $\bar{U} = U(x, y)$
Indirect utility function
$U^* = V(p_x, p_y, I)$
Inverses
Expenditure function
$E^* = E(p_x, p_y, \bar{U})$
Roy's identity
Shephard's lemma
Marshallian demand
$x(p_x, p_y, I) = -\frac{\frac{\partial V}{\partial p_x}}{\frac{\partial V}{\partial I}}$
Compensated demand
$x^c(p_x, p_y, U) = \frac{\partial E}{\partial p_x}$

- Expenditure function   Minimize expenditure given utility level

$$e[p_1,p_2,u] = \min_{h_1,h_2} p_1 x_1 + p_2 x_2 \text{ such that } \bar{u} \le u[h_1,\ldots,h_n]$$

$$e[p_1,p_2,u] = \min_{h_1,h_2,\mu} L[h_1,h_2,\mu]$$

$$= \min_{h_1,h_2,\mu}\left(p_1 h_1 + p_2 h_2 + \mu\left(u - u[h_1,h_2]\right)\right)$$

$$\boxed{FOC: \; p_1 = \mu \cdot u_1[h_1,h_2], \quad p_2 = \mu \cdot u_2[h_1,h_2]}$$

$$e[p_1,p_2,u] = e[h_1^*,h_2^*],$$

$$\text{where } h_1^* = \operatorname{argmin}_{h_1,h_2,\mu}\left(p_1 h_1 + p_2 h_2 + \mu\left(u - u[h_1,h_2]\right)\right)$$

$$= h_1[p_1,p_2,u]$$

$$\text{What is } \frac{de[p_1,p_2,u]}{dp_1}? \qquad = \frac{de\left[h_1[p_1,p_2,u],h_2[p_1,p_2,u]\right]}{dp_1}$$

From here on, see Varian (Microecon analysis), p.501-502 (envelope theorem)

Envelope theory (p.490 in Varian Microeconomic Analysis):

$$M[a] = \max_x g[x,a] \rightarrow \frac{dM[a]}{da} = \frac{dg[x,a]}{da}\bigg|_{x=x[a]}$$

$$L[h_1, h_2, \mu] = p_1 h_1 + p_2 h_2 + \mu\left(u - u[h_1, h_2]\right)$$

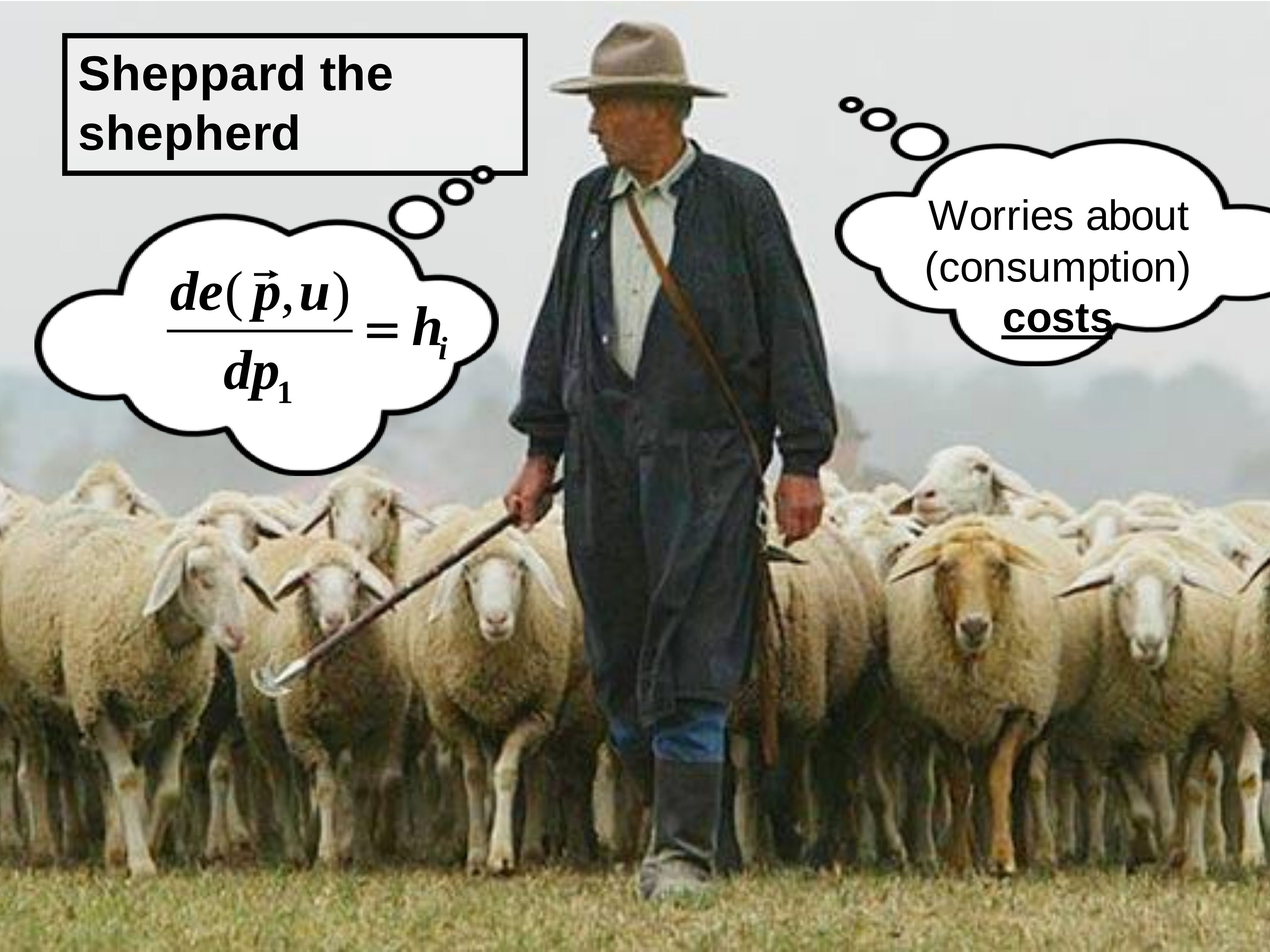
$$e[p_1, p_2, u] = \max_{h_1, h_2, \mu}\left(L[h_1, h_2, \mu]\right)$$

$$\frac{de[p_1, p_2, u]}{dp_1} = \frac{dL}{dp_1}\bigg|_{h=h[p_1, p_2, u]} = h_1 \big|_{h=h[p_1, p_2, u]} = h_1[p_1, p_2, u]$$

$$\frac{de[p_1, p_2, u]}{du} = \frac{dL}{du}\bigg|_{h=h[p_1, p_2, u]} = \mu \big|_{h=h[p_1, p_2, u]} = \mu$$

This is called Sheppard's lemma!

And the above is proof 1 (using envelope theorem)


Sheppard the shepherd
$\frac{de(\vec{p},u)}{dp_1} = h_i$
Worries about (consumption) **costs**

Consumption theory

Roy's identity: $$-\frac{\frac{\partial v(\vec{p},m)}{\partial p_i}}{\frac{\partial v(\vec{p},m)}{\partial m}} = x_i(\vec{p},m)$$

Sheppard's lemma: $$h_i(\vec{p},u) = \frac{\delta e(\vec{p},u)}{\delta p_i}$$

5) Sheppard’s lemma for consumption theory:

- If h(p,u) is the expenditure-minimizing bundle necessary to achieve utility level u at prices p, then:

$$h_i(\vec{p},u) = \frac{\delta e(\vec{p},u)}{\delta p_i}$$

Proof 2

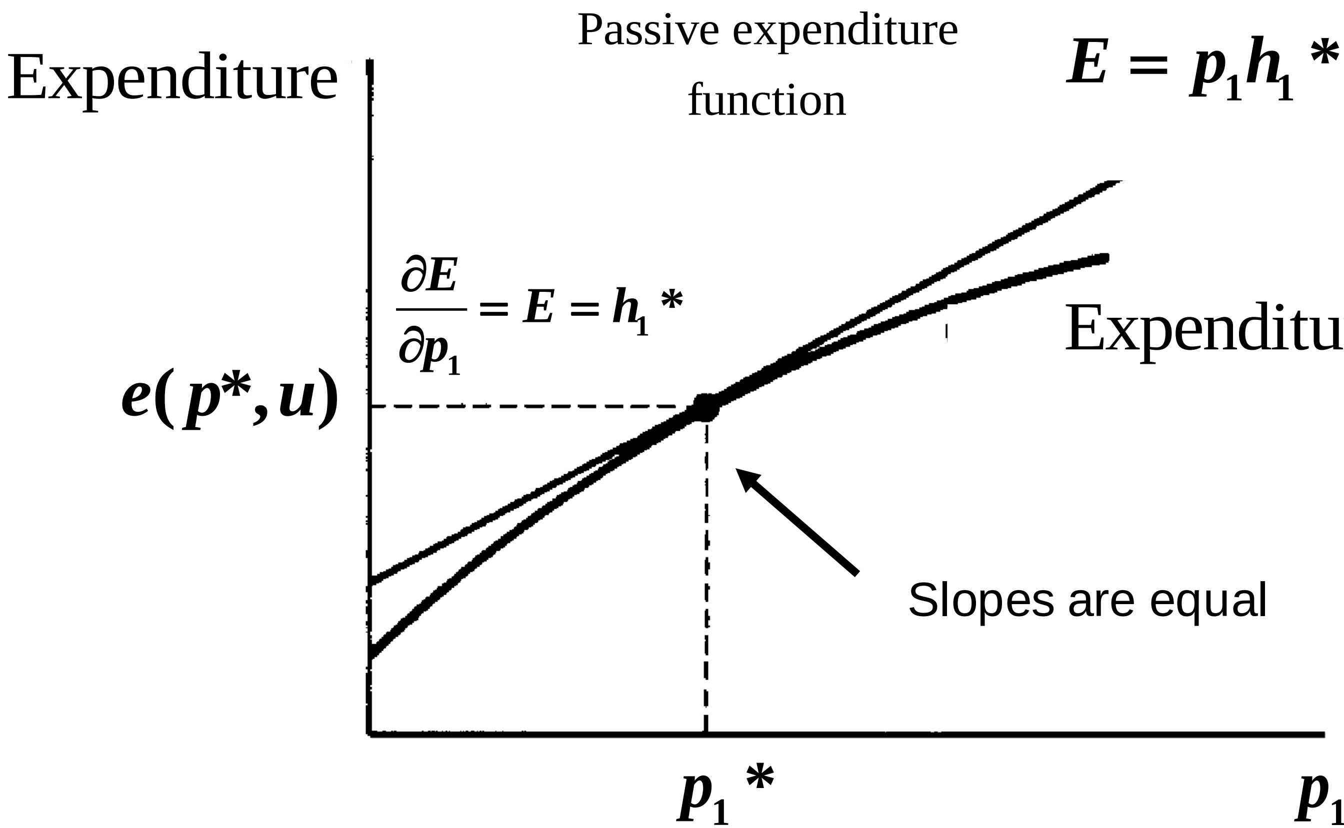
5) Sheppard's lemma for consumption theory:

- If h(p,u) is the expenditure-minimizing bundle necessary to achieve utility level u at prices p, then:

$$h_i(\vec{p},u) = \frac{\delta e(\vec{p},u)}{\delta p_i}$$

Which of the two graphs is the expenditure function?

$$E = p_1 h_1 * + \sum_{i=2}^{n} p_i * h_i *$$



Expenditure function $e(p,u)$

$$\frac{\partial e(p*,u)}{\partial p_1}$$

Proof 3

Shephard's lemma. (The derivative property) for n=2

- Let $h_i(p, u)$ be the firm's conditional factor demand for input i. Then if the exp function is differentiable at (p, u), and $p_i > 0$ for i = 1,…,2 then

$$h_i(\vec{p},u)=\frac{\delta e(\vec{p},u)}{\delta p_i} \text{ for } i=1,...,2$$

Proof:

Let $\vec{h}*$ minimize costs at prices $\vec{p}*$ and at utility u. Thus $\vec{h}*=h(\vec{p}*,\mathbf{u})$

Define function $g(\vec{p})=c(\vec{p},\mathbf{u})-\left(p_1h_1*+p_2h_2*\right)$

$$=\left(p_1h_1+p_2h_2\right)-\left(p_1h_1*+p_2h_2*\right)$$

Thus $g(\vec{p})\le 0$ and $g(\vec{p}*)=0$ $\leftrightarrow g(\vec{p})$ reaches a max at $\vec{p}=\vec{p}*$

$$0=\frac{dg(\vec{p},u)}{dp_1}=\frac{de(\vec{p},u)-\left(p_1h_1*+p_2h_2*\right)}{dp_1}$$

$$=\frac{de(\vec{p},u)}{dp_1}-h_1* \quad \leftrightarrow \frac{de(\vec{p},u)}{dp_1}=h_1*=h_1(\vec{p},u)$$

EXAMPLE: The Cobb-Douglas utility function

Find the expenditure function

$$u(x_1, x_2) = x_1 x_2^2$$

$$e[p_1, p_2, \bar{u}] = \underset{x_1, x_2}{MIN} \; p_1 x_1 + p_2 x_2 \quad \text{s.t. } x_1 x_2^2 = \bar{u}$$

$$e[p_1, p_2, \bar{u}] = \underset{x_1, x_2, \lambda}{MIN} \; L(x_1, x_2, \mu) = \underset{x_1, x_2}{MIN} \; p_1 x_1 + p_2 x_2 + \mu(\bar{u} - x_1 x_2^2)$$

$FOC:$

$$p_1 = \mu x_2^2$$

$$p_2 = \mu 2 x_1 x_2$$

$$x_1 x_2^2 = \bar{u}$$

$$\frac{x_2^2}{2 x_1 x_2} = \frac{p_1}{p_2}$$

$$\frac{x_2}{2 x_1} = \frac{p_1}{p_2}$$

$$x_2 = 2 \frac{p_1}{p_2} x_1$$

$$x_1 \left(2 \frac{p_1}{p_2} x_1 \right)^2 = \bar{u}$$

$$x_1^3 = \tfrac{1}{4} \bar{u} \left(\frac{p_2}{p_1} \right)^2$$

$$x_1 = \tfrac{1}{4}^{\frac{1}{3}} (\bar{u})^{\frac{1}{3}} \left(\frac{p_2}{p_1} \right)^{\frac{2}{3}}$$

$$x_2 = 2 \frac{p_1}{p_2} \tfrac{1}{4}^{\frac{1}{3}} (\bar{u})^{\frac{1}{3}} \left(\frac{p_2}{p_1} \right)^{\frac{2}{3}}$$

$$= 2 \tfrac{1}{4}^{\frac{1}{3}} (\bar{u})^{\frac{1}{3}} \left(\frac{p_1}{p_2} \right)^{\frac{1}{3}}$$

EXAMPLE: The Cobb-Douglas utility function

Find the expenditure function

$$u(x_1, x_2) = x_1 x_2^2 \qquad x_1 = \tfrac{1}{4}^{\frac{1}{3}} (\bar{u})^{\frac{1}{3}} \left(\frac{p_2}{p_1} \right)^{\frac{2}{3}} \qquad x_2 = 2 \cdot \tfrac{1}{4}^{\frac{1}{3}} (\bar{u})^{\frac{1}{3}} \left(\frac{p_1}{p_2} \right)^{\frac{1}{3}}$$

$$e[p_1, p_2, \bar{u}] = \underset{x_1, x_2}{MIN} \; p_1 x_1 + p_2 x_2 \quad \text{s.t. } x_1 x_2^2 = \bar{u}$$

$$e[p_1, p_2, \bar{u}] = p_1 \tfrac{1}{4}^{\frac{1}{3}} (\bar{u})^{\frac{1}{3}} \left(\frac{p_2}{p_1} \right)^{\frac{2}{3}} + p_2 2 \cdot \tfrac{1}{4}^{\frac{1}{3}} (\bar{u})^{\frac{1}{3}} \left(\frac{p_1}{p_2} \right)^{\frac{1}{3}}$$

$$= p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} \tfrac{1}{4}^{\frac{1}{3}} (\bar{u})^{\frac{1}{3}} + 2 \cdot \tfrac{1}{4}^{\frac{1}{3}} p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} (\bar{u})^{\frac{1}{3}}$$

$$\boxed{e[p_1, p_2, \bar{u}] = p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} \tfrac{1}{4}^{\frac{1}{3}} (\bar{u})^{\frac{1}{3}} \cdot 3}$$

Check Shepards Lemma:

$$\frac{de[p_1, p_2, \bar{u}]}{dp_1} = \tfrac{1}{3} p_1^{-\frac{2}{3}} p_2^{\frac{2}{3}} \tfrac{1}{4}^{\frac{1}{3}} (\bar{u})^{\frac{1}{3}} 3$$

$$= p_1^{-\frac{2}{3}} p_2^{\frac{2}{3}} \tfrac{1}{4}^{\frac{1}{3}} (\bar{u})^{\frac{1}{3}}$$

$$= x_1$$

EXAMPLE: The Cobb-Douglas utility function

Find the expenditure function the faster way

First get the indirect utility function!

$$u(x_1, x_2) = x_1 x_2^2 \quad \Leftrightarrow u(x_1, x_2) = \ln[x_1 x_2^2] = \ln[x_1] + 2\ln[x_2]$$

$$v[p_1, p_2, m] = \underset{x_1, x_2}{MAX} \ \ln[x_1] + 2\ln[x_2] \quad \text{s.t. } p_1 x_1 + p_2 x_2 = m$$

$$= \underset{x_1, x_2}{MAX} \ L[x_1 x_2, \lambda] = MAX_{x_1, x_2} \ \ln[x_1] + 2\ln[x_2] + \lambda\left(m - p_1 x_1 - p_2 x_2\right)$$

$FOC:$

$$x_1^{-1} = \lambda p_1$$

$$2x_2^{-1} = \lambda p_2$$

$$p_1 x_1 + p_2 x_2 = m$$

$$p_1 = x_1^{-1}\lambda^{-1}$$

$$p_2 = 2x_2^{-1}\lambda^{-1}$$

$$x_1^{-1}\lambda^{-1}x_1 + 2x_2^{-1}\lambda^{-1}x_2 = m$$

$$\lambda^{-1} + 2\lambda^{-1} = m$$

$$3\lambda^{-1} = m$$

$$\lambda = 3m^{-1}$$

$$x_1^{-1} = 3m^{-1}p_1$$

$$x_1 = \frac{m}{3p_1}$$

$$2x_2^{-1} = 3m^{-1}p_2$$

$$x_2 = \frac{2m}{3p_2}$$

EXAMPLE: The Cobb-Douglas utility function

Find the expenditure function   First get the indirect utility function!

$$u(x_1, x_2) = x_1 x_2^2 \qquad \Leftrightarrow u(x_1, x_2) = \ln[x_1 x_2^2] = \ln[x_1] + 2\ln[x_2]$$

$$v[p_1, p_2, m] = u[x_1^*, x_2^*] = x_1^* \, x_2^{*2}$$

$$= \frac{m}{3p_1}\left(\frac{2m}{3p_2}\right)^2$$

$$v[p_1, p_2, m] = \frac{4m^3}{27 p_1 p_2^2} \Leftrightarrow u = \frac{4(e[p_1, p_2, u])^3}{27 p_1 p_2^2}$$

$$\Leftrightarrow \left(\frac{27 p_1 p_1^2 u}{4}\right)^{\frac{1}{3}} = e[p_1, p_2, u]$$

$$x_1 = \frac{m}{3p_1}$$

$$x_2 = \frac{2m}{3p_2}$$

Compare with: $e[p_1, p_2, \bar{u}] = p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} \left(\frac{1}{4}\right)^{\frac{1}{3}} (\bar{u})^{\frac{1}{3}} \cdot 3$

- General example

EXAMPLE: The Cobb-Douglas utility function, min costs

$$u(x_1, x_2) = x_1^a x_2^{1-a}$$

$$\underset{x_1,x_2}{MIN}\ p_1 x_1 + p_2 x_2 \quad \text{s.t.}\ x_1^a x_2^{1-a} = \bar{u}$$

$$\underset{x_1,x_2,\lambda}{MIN}\ L(x_1, x_2, \mu) = \underset{x_1,x_2}{MIN}\ p_1 x_1 + p_2 x_2 + \mu(\bar{u} - x_1^a x_2^{1-a})$$

$FOC:$

$$\left.\begin{aligned} p_1 &= \mu a x_1^{a-1} x_2^{1-a} \\ p_2 &= \mu(1-a) x_1^a x_2^{-a} \end{aligned}\right\}$$

$$x_1^a x_2^{1-a} = \bar{u}$$

$$x_2 = \bar{u}^{\frac{1}{1-a}} x_1^{\frac{-a}{1-a}}$$

$$\frac{a x_1^{a-1} x_2^{1-a}}{(1-a) x_1^a x_2^{-a}} = \frac{p_1}{p_2}$$

$$\frac{a}{(1-a)} x_1^{-1} x_2 = \frac{p_1}{p_2}$$

$$a x_2 p_2 = (1-a) p_1 x_1$$

$$a \bar{u}^{\frac{1}{1-a}} x_1^{\frac{-a}{1-a}} p_2 = (1-a) p_1 x_1$$

$$a \bar{u}^{\frac{1}{1-a}} p_2 = (1-a) p_1 x_1^{\frac{1-a}{1-a}} x_1^{\frac{a}{1-a}}$$

$$\frac{a}{(1-a)} \bar{u}^{\frac{1}{1-a}} \frac{p_2}{p_1} = x_1^{\frac{1}{1-a}}$$

$$x_1 = \left(\frac{a}{(1-a)} \frac{p_2}{p_1} \right)^{1-a} \bar{u}$$

$$x_2 = \left(\frac{1-a}{a} \frac{p_1}{p_2} \right)^{a} \bar{u}$$

EXAMPLE: The Cobb-Douglas utility function, min costs

$$u(x_1, x_2) = x_1 x_2^2$$

$$x_1 = \left(\frac{a}{(1-a)} \frac{p_2}{p_1} \right)^{1-a} \bar{u} \qquad x_2 = \left(\frac{1-a}{a} \frac{p_1}{p_2} \right)^{a} \bar{u}$$

$$e[p_1, p_2, u] = p_1 x_1 + p_2 x_2$$

$$= p_1 \left(\frac{a}{(1-a)} \frac{p_2}{p_1} \right)^{1-a} \bar{u} + p_2 \left(\frac{1-a}{a} \frac{p_1}{p_2} \right)^{a} \bar{u}$$

$$= \left(p_1 a^{1-a} p_2^{1-a} (1-a)^{a-1} p_1^{a-1} + p_2 p_1^{a} (1-a)^{a} a^{-a} p_2^{-a} \right) \bar{u}$$

$$= p_1^{a} p_2^{1-a} \left(a^{1-a} (1-a)^{a-1} + (1-a)^{a} a^{-a} \right) \bar{u}$$

$$= p_1^{a} p_2^{1-a} \left(\frac{1-a}{a} \right)^{a} \left(\frac{a}{1-a} + 1 \right) \bar{u}$$

$$= p_1^{a} p_2^{1-a} \left(\frac{1-a}{a} \right)^{a} \left(\frac{a}{1-a} + \frac{1-a}{1-a} \right) \bar{u}$$

$$= p_1^{a} p_2^{1-a} \left(\frac{1-a}{a} \right)^{a} \left(\frac{1}{1-a} \right) \bar{u}$$

EXAMPLE: The Cobb-Douglas utility function, min costs

$$u(x_1, x_2) = x_1^a x_2^{1-a}$$

$$x_1 = \left(\frac{a}{(1-a)} \frac{p_2}{p_1} \right)^{1-a} \bar{u} \qquad x_2 = \left(\frac{1-a}{a} \frac{p_1}{p_2} \right)^{a} \bar{u}$$

$$e[p_1, p_2, u] = p_1^a p_2^{1-a} \left(\frac{1-a}{a} \right)^a \left(\frac{1}{1-a} \right) \bar{u}$$

$$\frac{de[p_1, p_2, u]}{dp_1} = a p_1^{a-1} p_2^{1-a} \left(\frac{1-a}{a} \right)^a \left(\frac{1}{1-a} \right) \bar{u}$$

$$= \left(\frac{p_2}{p_1} \right)^{1-a} \left(\frac{1-a}{a} \right)^a \left(\frac{a}{1-a} \right) \bar{u}$$

$$= \left(\frac{p_2}{p_1} \right)^{1-a} \left(\frac{a}{1-a} \right)^{-a} \left(\frac{a}{1-a} \right)^{1} \bar{u}$$

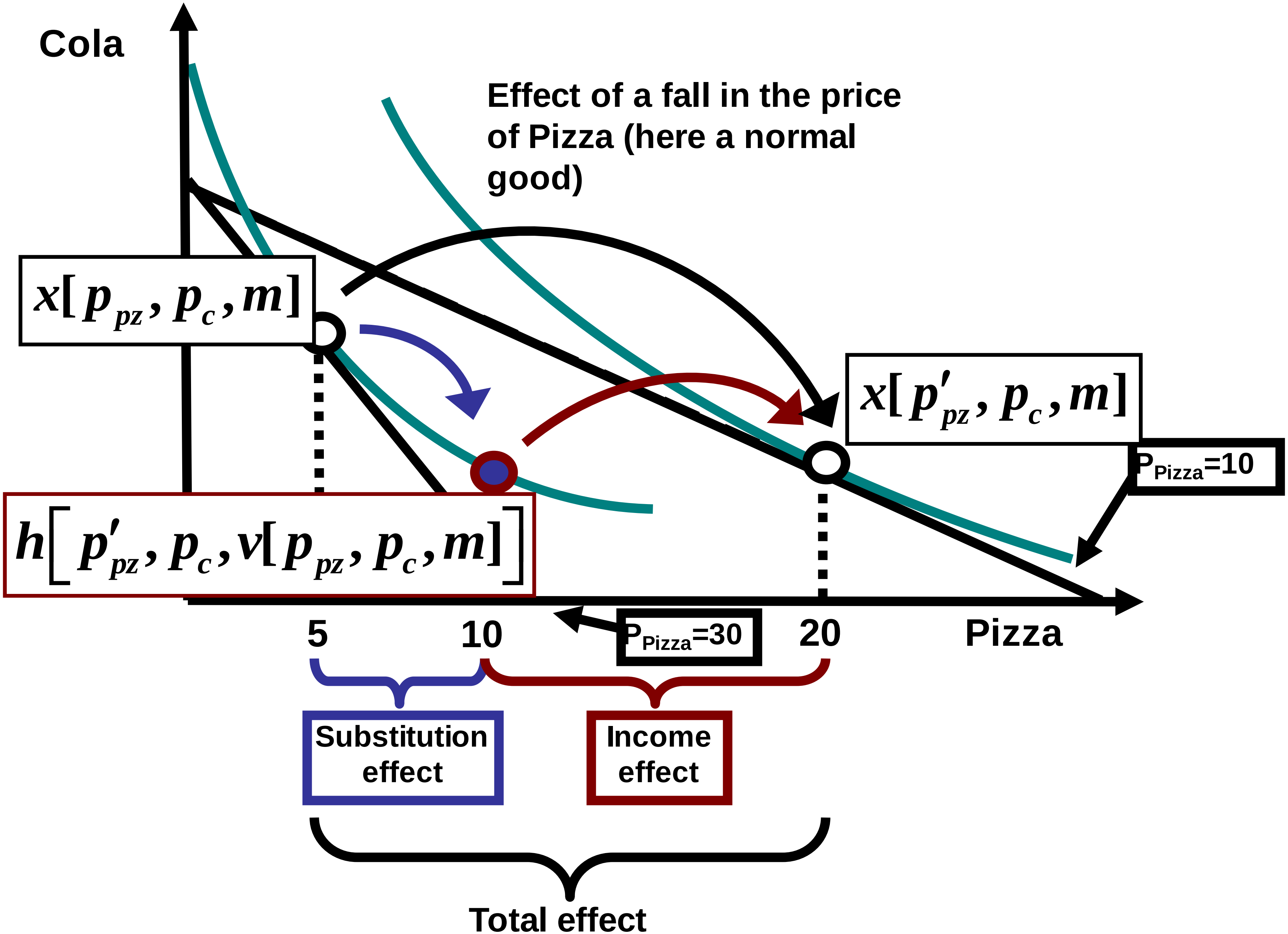
$$= \left(\frac{p_2}{p_1} \right)^{1-a} \left(\frac{a}{1-a} \right)^{1-a} \bar{u}$$

- Let’s now talk about Slutsky

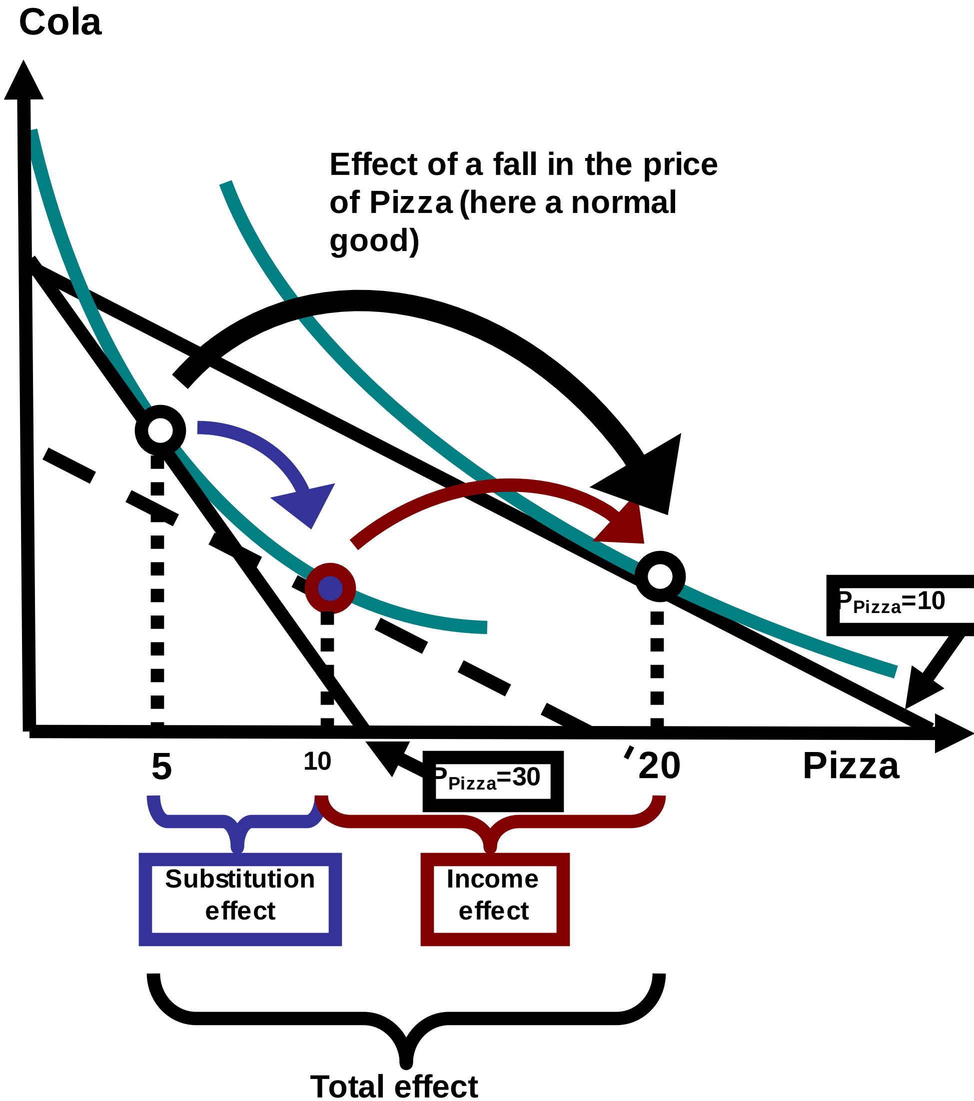
- Remember from Slide_set L6

- It is natural to think that when the price of a good rises the demand for it will fall.
- Giffen good.
  - But possible to construct examples where the optimal demand for a good decreases when its price falls.

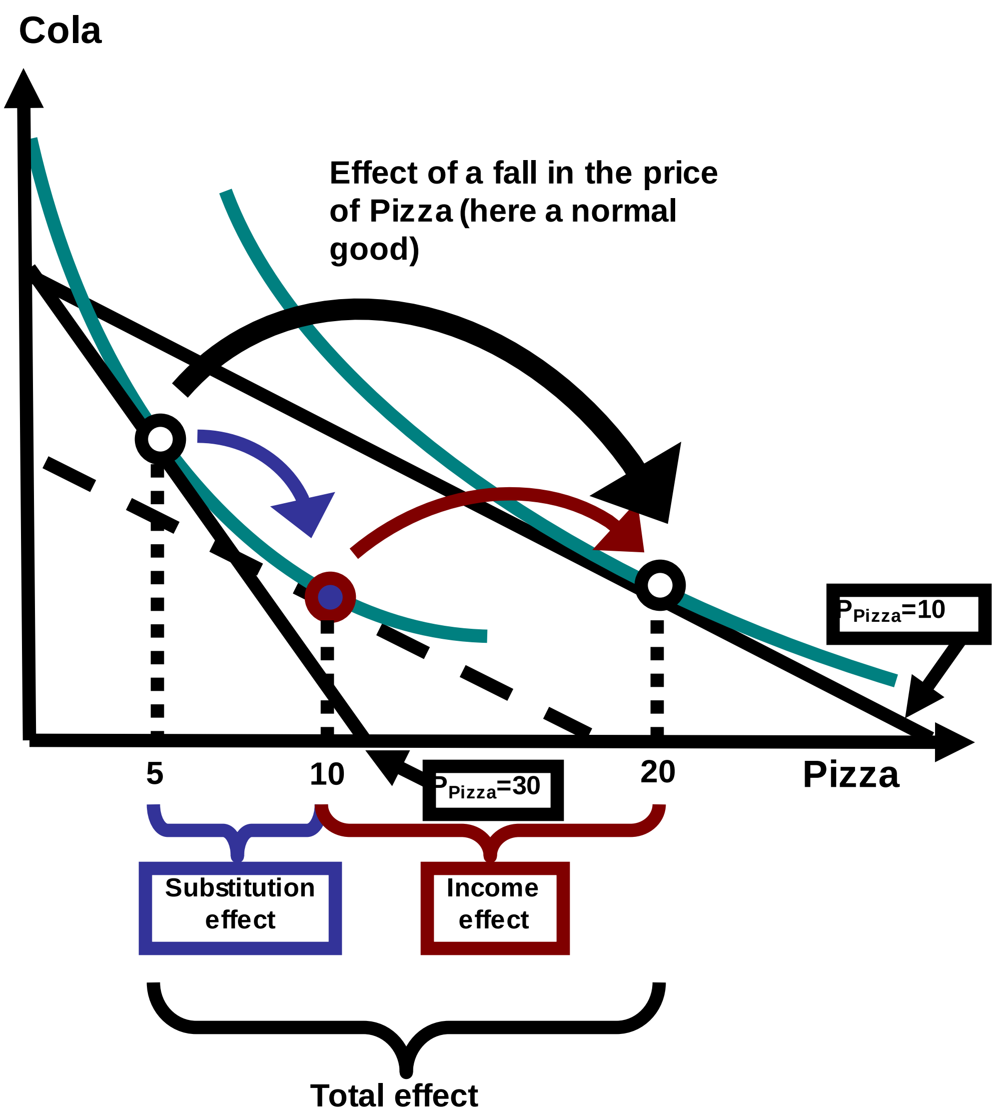
- What is going on here? How is it that changes in price can have these ambiguous effects on demand?



Cola
Effect of a fall in the price of Pizza (here a normal good)
$x[p_{pz}, p_c, m]$
$x[p'_{pz}, p_c, m]$
$h\left[p'_{pz}, p_c, v[p_{pz}, p_c, m]\right]$
$P_{Pizza}=10$
$P_{Pizza}=30$
5
10
20
Pizza
Substitution effect
Income effect
Total effect

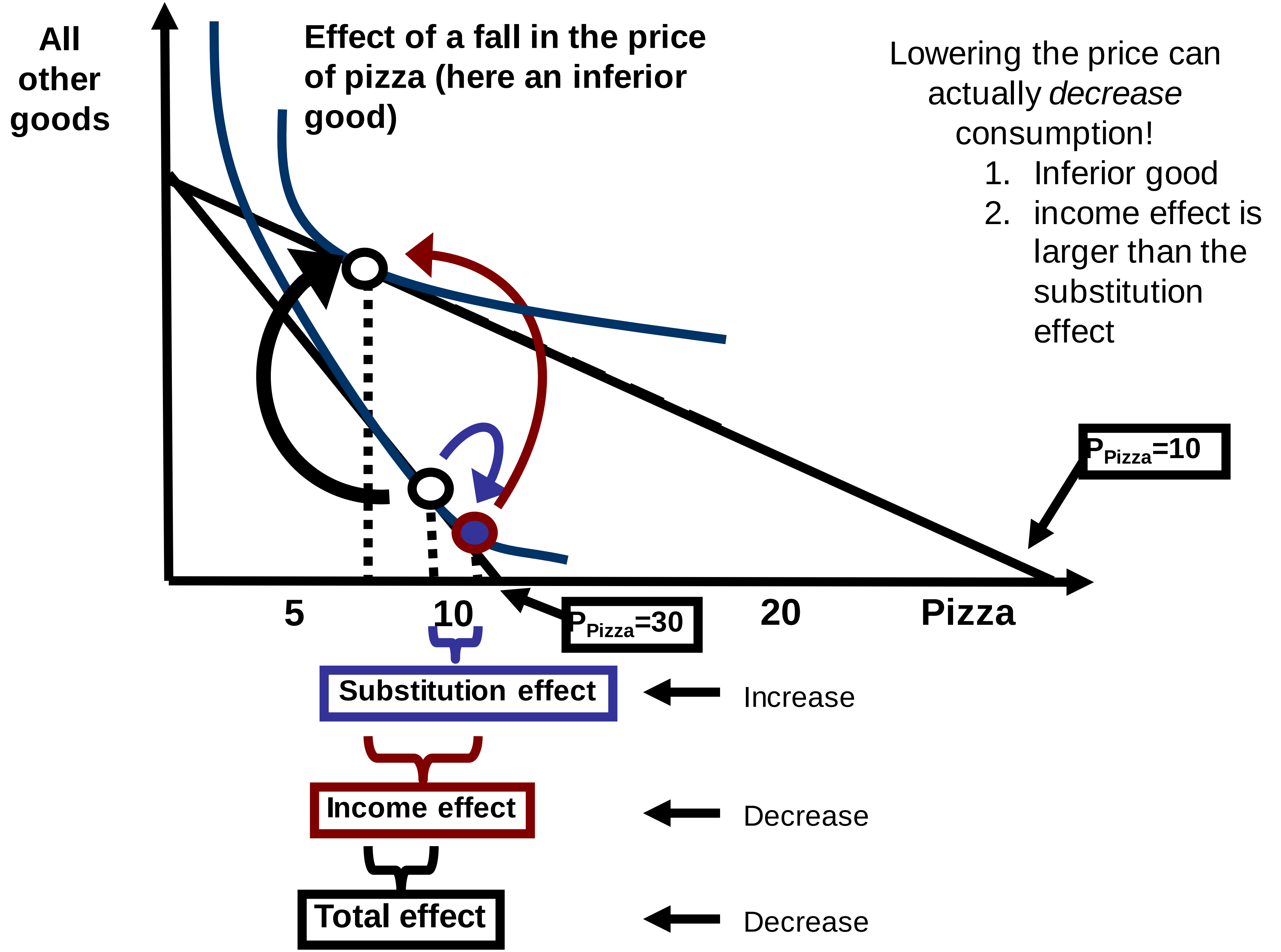


1. Draw the new price line (P_{Pizza}=10)
2. Find the point where a IC touches the new price line
3. The difference between the original point and the new point is **the total effect.**
4. Move the new price line, until it touches the old IC.
5. Mark the point where it touches with a special "in-between point".
6. Now:
   - The difference between the original point and the "in-between" point is the substitution effect. (a movement along the IC)
   - The difference between the "in-between" point and new point the income effect (a movement between the two parallel budged lines).

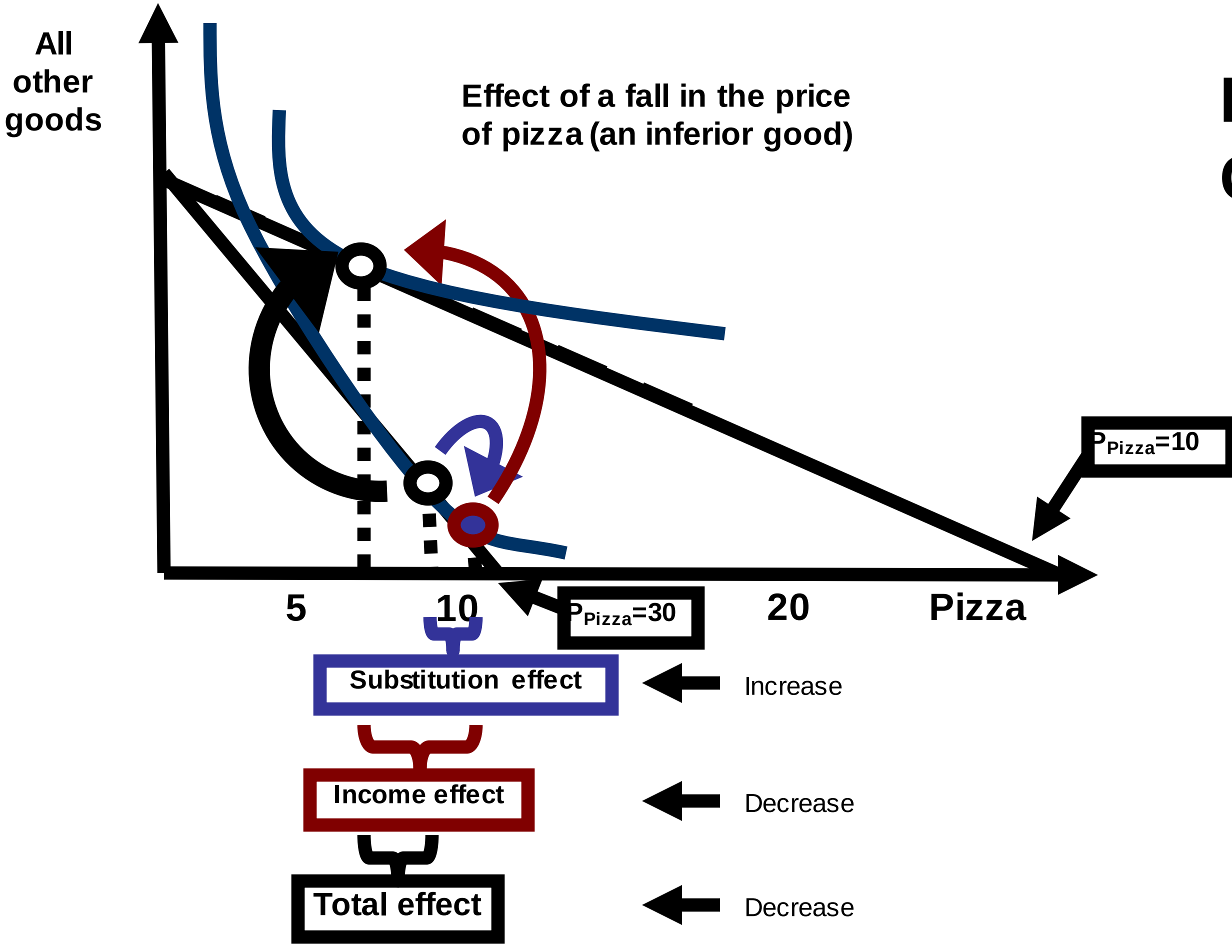


When lowering the price:

- The substitution effect always gives an increase
- What about the income effect?
  - Normal goods: increase
  - Inferior good: decrease

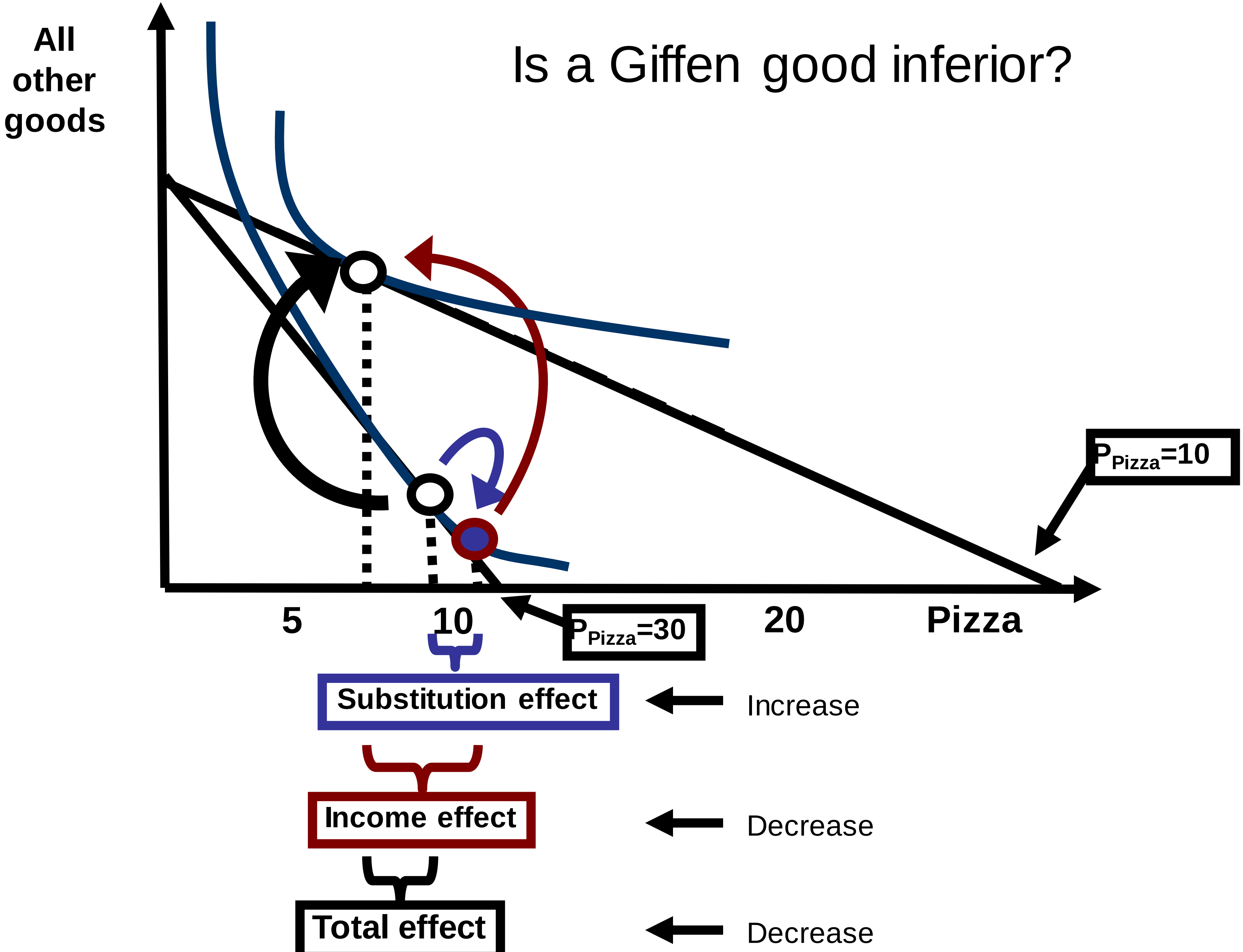

All other goods
Effect of a fall in the price of pizza (here an inferior good)
Lowering the price can actually *decrease* consumption!
1. Inferior good
2. income effect is larger than the substitution effect
P_{Pizza}=10
5
10
P_{Pizza}=30
20
Pizza
Substitution effect
Increase
Income effect
Decrease
Total effect
Decrease

Pizza is here a Giffen good



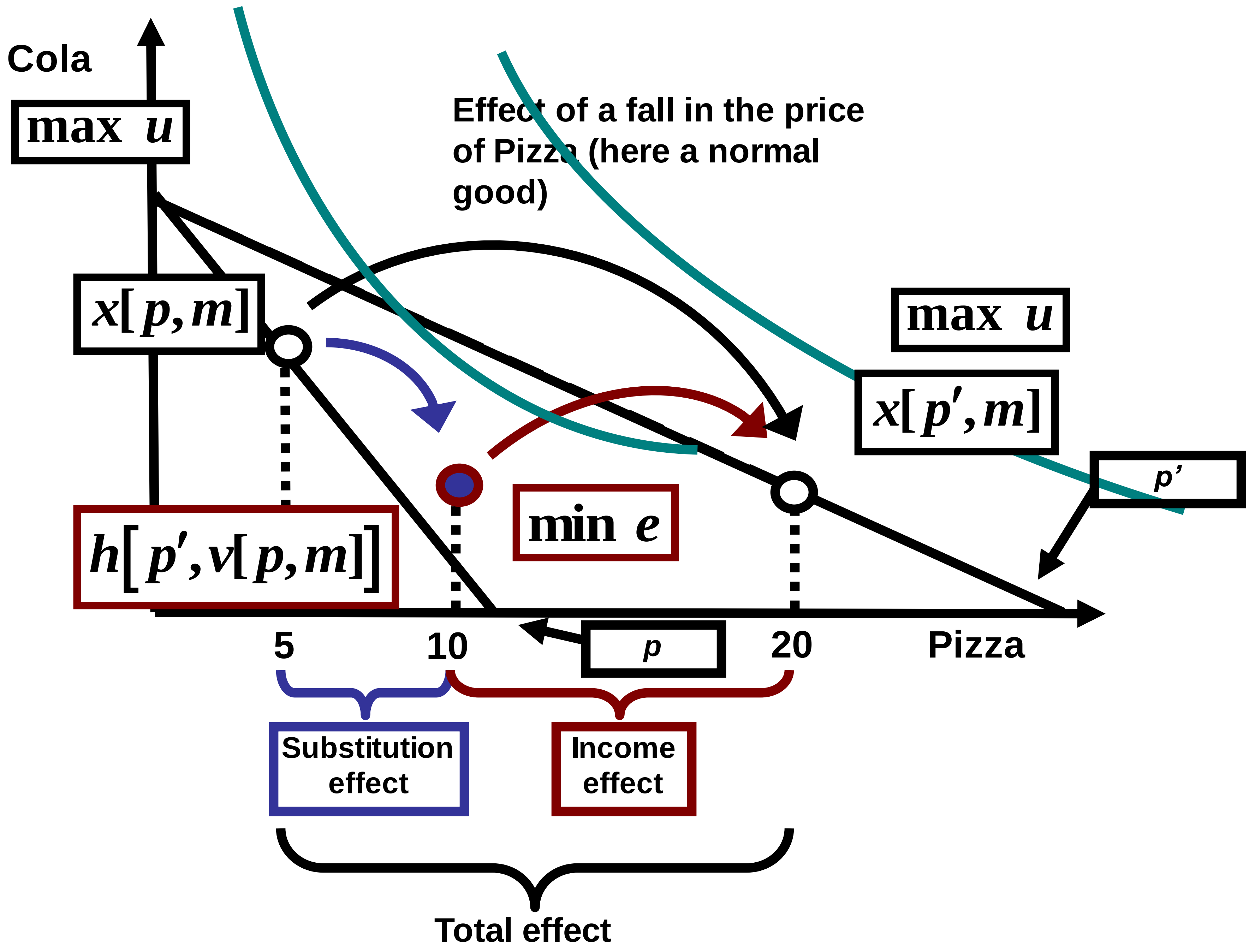
- The income effect is negative
  - (inferior good)
- The income effect is larger than the substitution effect
- Thus, a decrease in price *decreases* consumption
- Does an increase in price thus increase consumption?
  - Yes!
- Such goods are called “Giffen goods”

Is a Giffen good inferior?



- A bit more formal…
  - Using infinitesimal changes (dp_i)
- We need to look at
  - the indirect utility function
  - the expenditure function

- End lecture 2016.10.31



Cola
max u
Effect of a fall in the price of Pizza (here a normal good)
x[p,m]
max u
x[p′,m]
p'
min e
h[p′,v[p,m]]
5
10
20
p
Pizza
Substitution effect
Income effect
Total effect

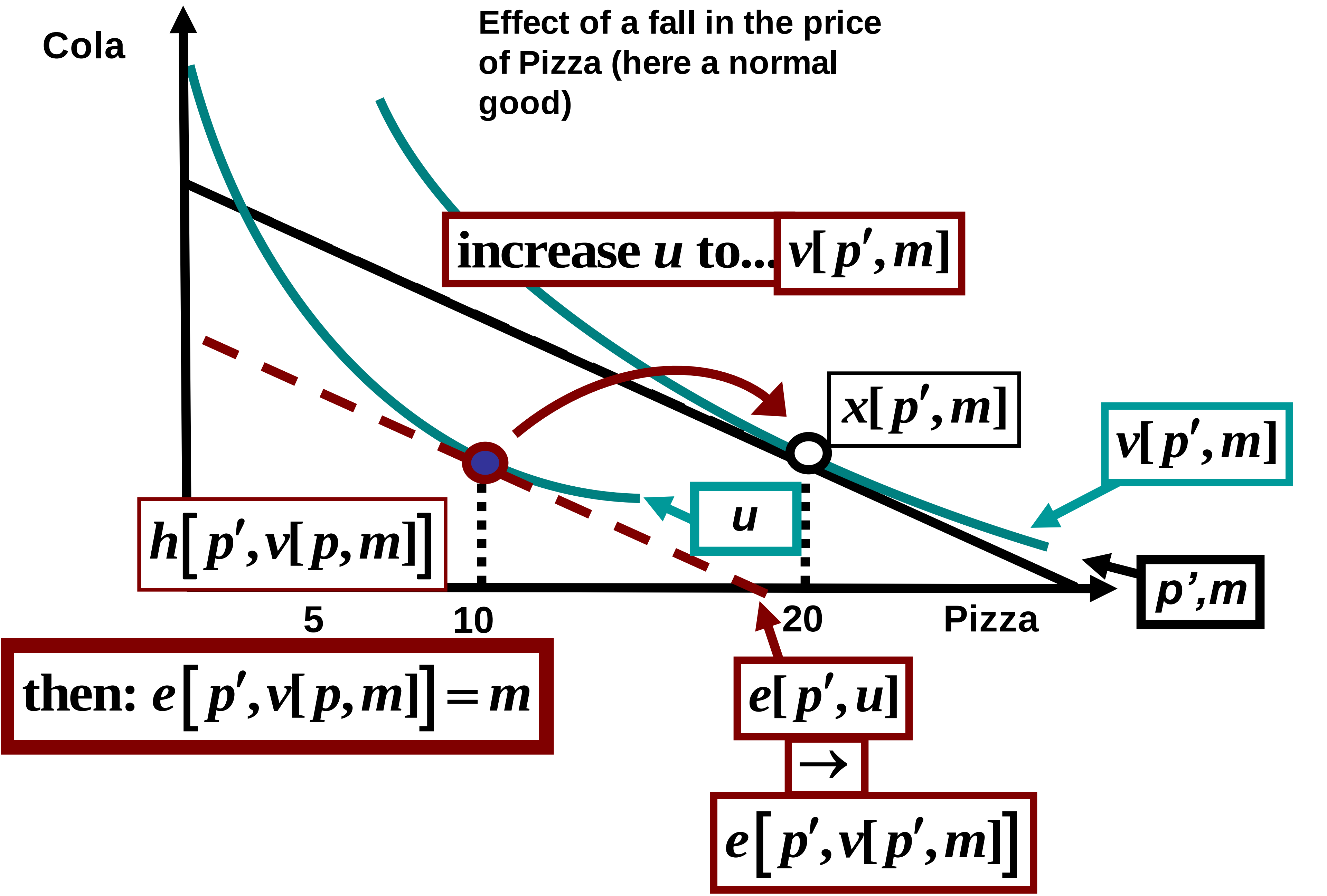
Memorize and understand these 4 identities!

$$m \equiv e[p, ?]$$

$$\boxed{m \equiv e\left[p, v[p, m]\right]}$$

Varian, Microeconomic Analysis, p.106

Minimum expenditure to reach utility *v[p,m]* is *m*



Effect of a fall in the price of Pizza (here a normal good)
Cola
Pizza
5
10
20
increase u to...
v[p′,m]
x[p′,m]
v[p′,m]
u
p',m
h[p′,v[p,m]]
then: e[p′,v[p,m]]=m
e[p′,u]
→
e[p′,v[p′,m]]

Memorize and understand these 4 identities!

$$m \equiv e[p, ?]$$

$$\boxed{m \equiv e\left[p, v[p,m]\right]}$$

Varian, Microeconomic Analysis, p.106

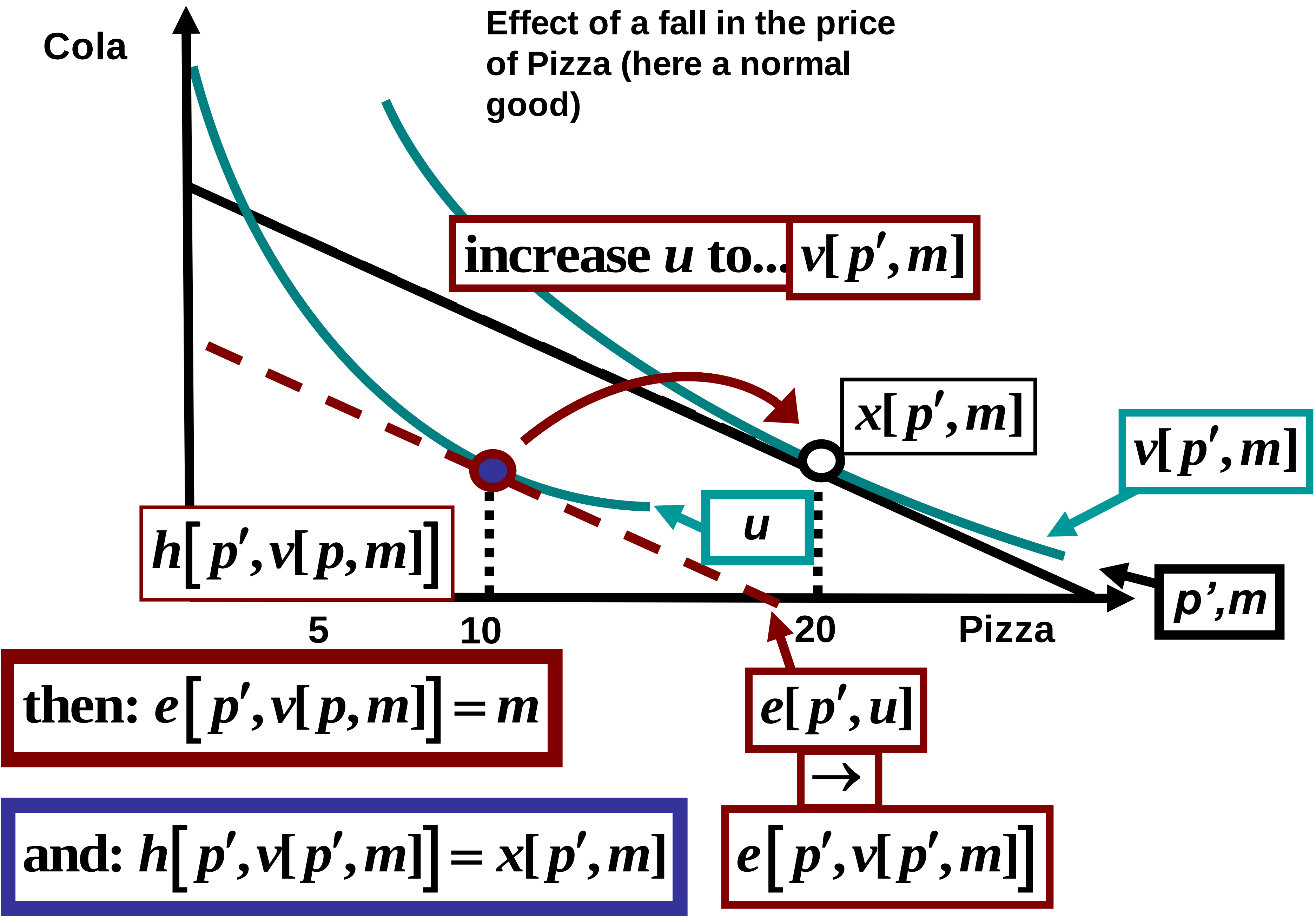
Minimum expenditure to reach utility *v[p,m]* is *m*

$$x_1[p, ?] \equiv h_1[p, ?]$$

$$x_1[p, m] \equiv h_1[p, ?]$$

$$\boxed{x_1[p, m] \equiv h_1\left[p, v[p,m]\right]}$$

Marshalian demand at income *m* = Hicksian demand at utility *v[p,m]*


Effect of a fall in the price of Pizza (here a normal good)
Cola
Pizza
5
10
20
increase u to... $v[p',m]$
$x[p',m]$
$v[p',m]$
u
p',m
$h[p',v[p,m]]$
then: $e[p',v[p,m]]=m$
and: $h[p',v[p',m]]=x[p',m]$
$e[p',u]$
$\rightarrow$
$e[p',v[p',m]]$

Memorize and understand these 4 identities!

$$m \equiv e[p,?]$$

$$\boxed{m \equiv e\left[p, v[p,m]\right]}$$

Varian, Microeconomic Analysis, p.106

Minimum expenditure to reach utility *v[p,m]* is *m*

$$u \equiv v[p,?]$$

$$\boxed{u \equiv v\left[p, e[p,u]\right]}$$

Maximum utility from income *e[p,u]* is *u*

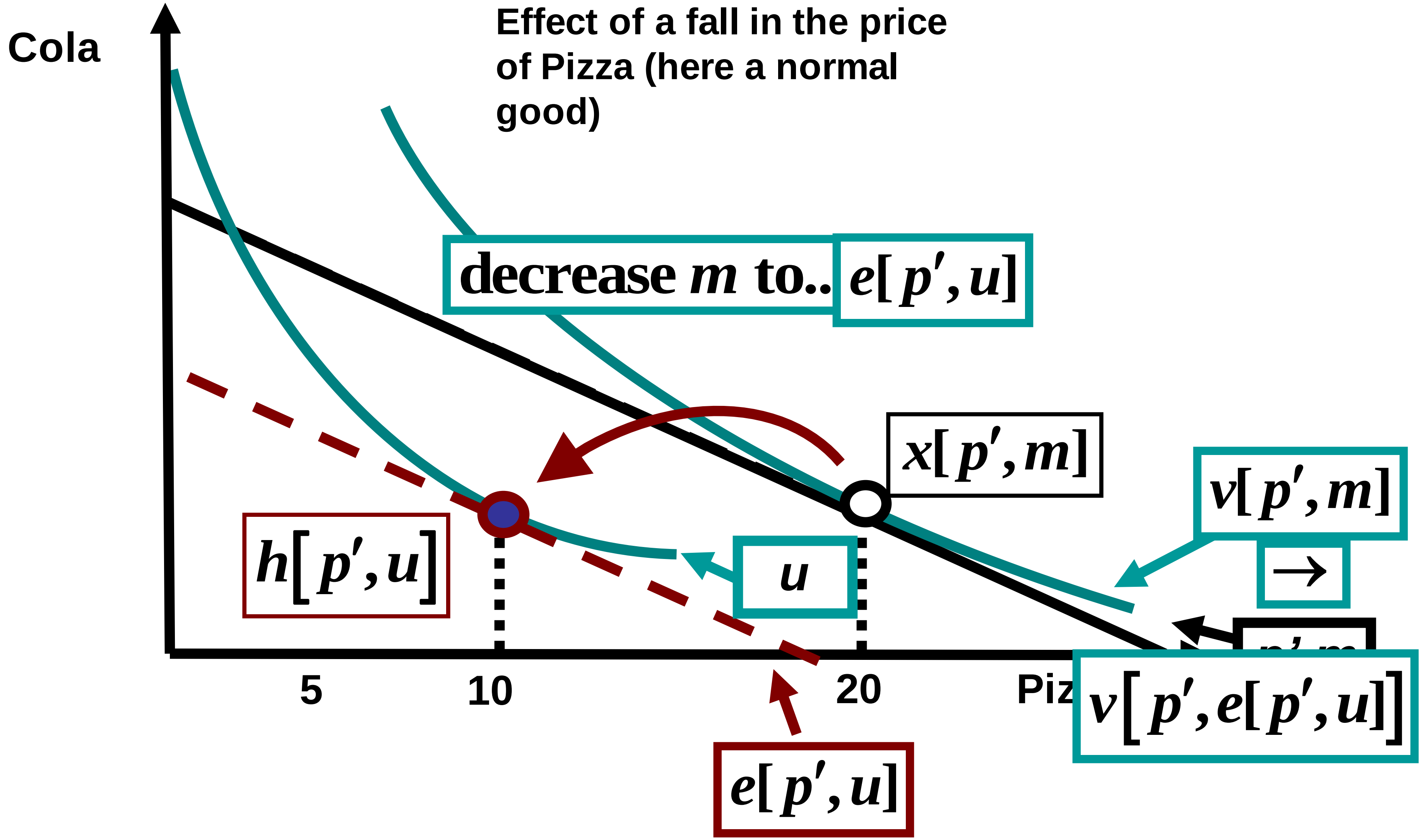
$$x_1[p,?] \equiv h_1[p,?]$$

$$x_1[p,m] \equiv h_1[p,?]$$

$$\boxed{x_1[p,m] \equiv h_1\left[p, v[p,m]\right]}$$

Marshalian demand at income *m* = Hicksian demand at utility *v[p,m]*

Effect of a fall in the price of Pizza (here a normal good)



then: $v[p',e[p',u]]=u$

Memorize and understand these 4 identities!

$$m \equiv e[p,?]$$

$$\boxed{m \equiv e\left[p, v[p,m]\right]}$$

Varian, Microeconomic Analysis, p.106

Minimum expenditure to reach utility *v[p,m]* is *m*

$$u \equiv v[p,?]$$

$$\boxed{u \equiv v\left[p, e[p,u]\right]}$$

Maximum utility from income *e[p,u]* is *u*

$$x_1[p,?] \equiv h_1[p,?]$$

$$x_1[p,m] \equiv h_1[p,?]$$

$$\boxed{x_1[p,m] \equiv h_1\left[p, v[p,m]\right]}$$

Marshalian demand at income *m* = Hicksian demand at utility *v[p,m]*

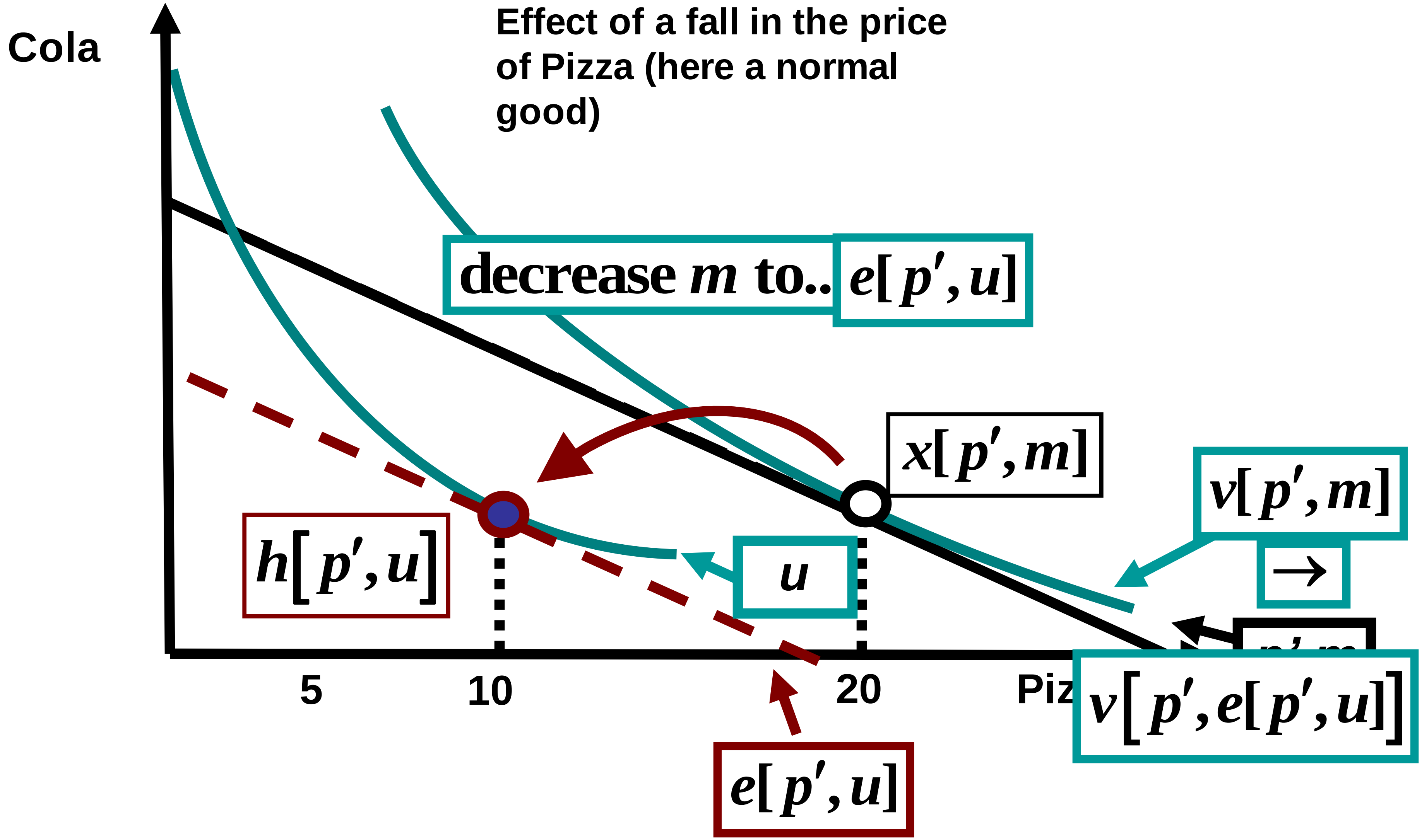
$$h_1[p,?] \equiv x_1[p,?]$$

$$h_1[p,u] \equiv x_1[p,?]$$

$$\boxed{h_1[p,u] \equiv x_1\left[p, e[p,u]\right]}$$

Hicksian demand at utility *u*= Marshalian demand at income *e[p,u]*

Effect of a fall in the price of Pizza (here a normal good)



then: $v\left[p', e[p', u]\right] = u$ and: $x\left[p', e[p', u]\right] = h\left[p', u\right]$

Memorize and understand these 4 identities!

$$m \equiv e[p, ?]$$

$$\boxed{m \equiv e\left[p, v[p, m]\right]}$$

Varian, Microeconomic Analysis, p.106

Minimum expenditure to reach utility *v[p,m]* is *m*

$$u \equiv v[p, ?]$$

$$\boxed{u \equiv v\left[p, e[p, u]\right]}$$

Maximum utility from income *e[p,u]* is *u*

$$x_1[p, ?] \equiv h_1[p, ?]$$

$$x_1[p, m] \equiv h_1[p, ?]$$

$$\boxed{x_1[p, m] \equiv h_1\left[p, v[p, m]\right]}$$

Marshalian demand at income *m* = Hicksian demand at utility *v[p,m]*

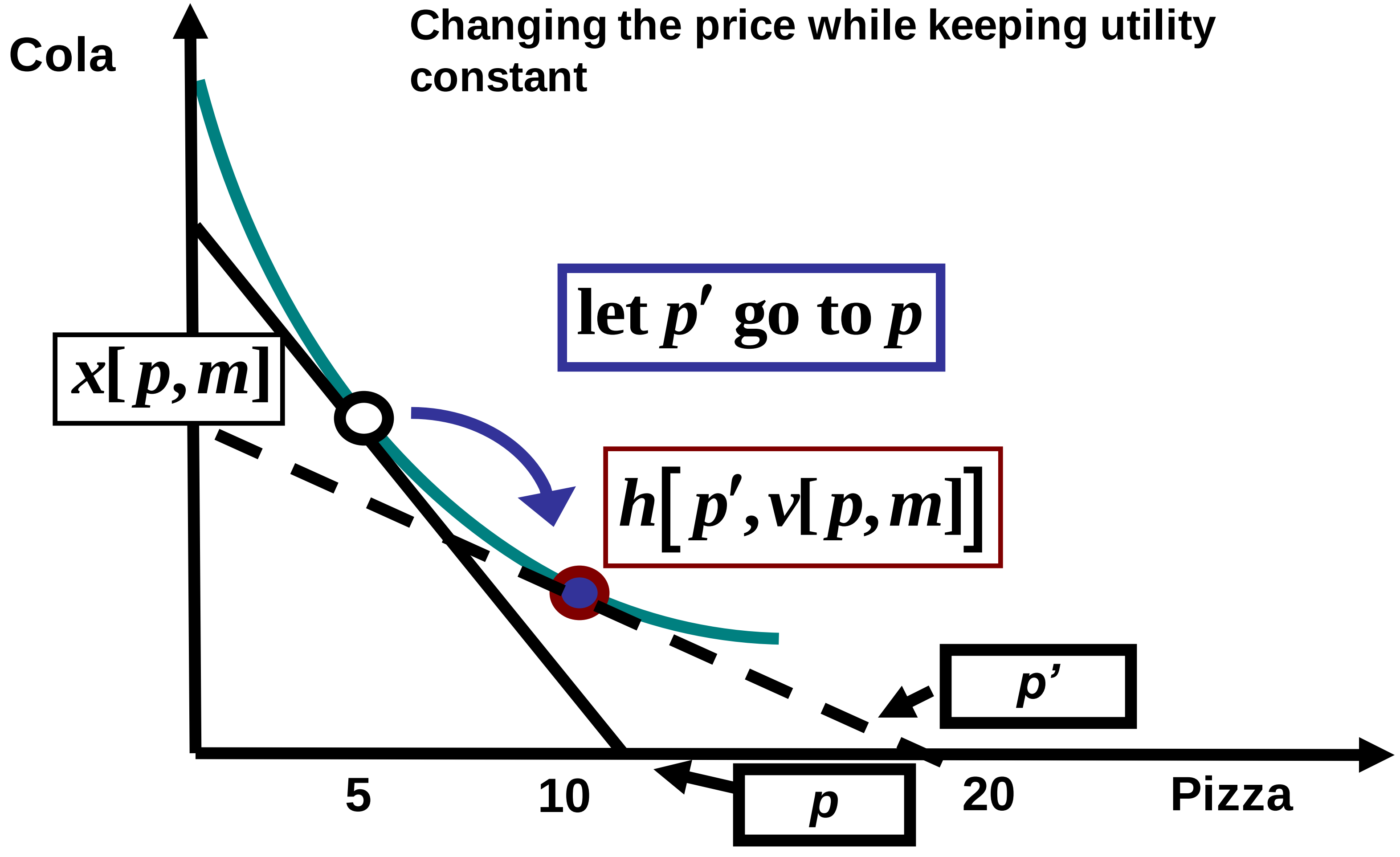
$$h_1[p, ?] \equiv x_1[p, ?]$$

$$h_1[p, u] \equiv x_1[p, ?]$$

$$\boxed{h_1[p, u] \equiv x_1\left[p, e[p, u]\right]}$$

Hicksian demand at utility *u*= Marshalian demand at income *e[p,u]*

Changing the price while keeping utility constant



$$\text{then: } x[p,m] = h\left[p, v[p,m]\right]$$

Memorize and understand these 4 identities!

$$m \equiv e[p,?]$$

$$\boxed{m \equiv e\big[p, v[p,m]\big]}$$

Varian, Microeconomic Analysis, p.106

Minimum expenditure to reach utility *v[p,m]* is *m*

$$u \equiv v[p,?]$$

$$\boxed{u \equiv v\big[p, e[p,u]\big]}$$

Maximum utility from income *e[p,u]* is *u*

$$x_1[p,?] \equiv h_1[p,?]$$

$$x_1[p,m] \equiv h_1[p,?]$$

$$\boxed{x_1[p,m] \equiv h_1\big[p, v[p,m]\big]}$$

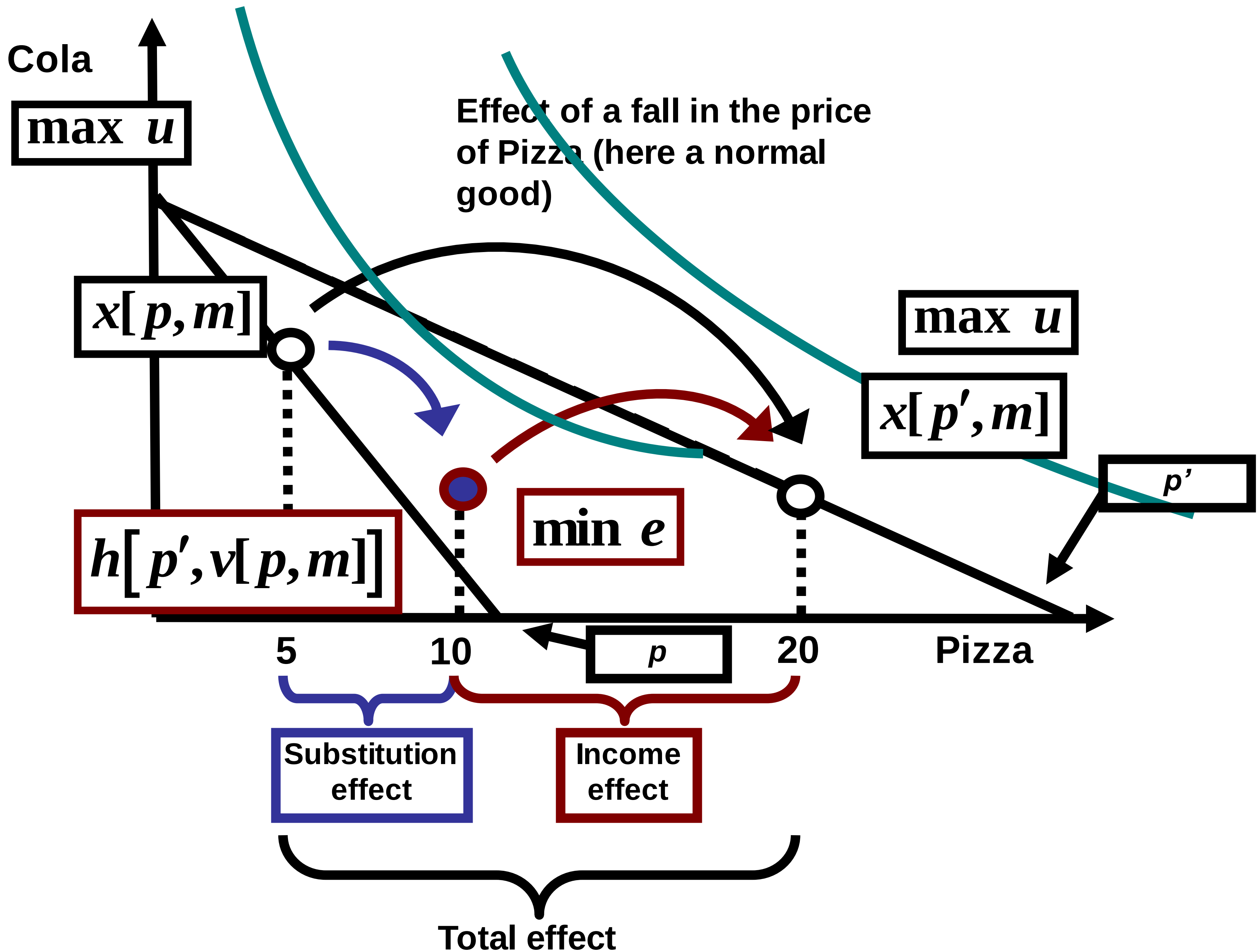
Marshalian demand at income *m* = Hicksian demand at utility *v[p,m]*

$$h_1[p,?] \equiv x_1[p,?]$$

$$h_1[p,u] \equiv x_1[p,?]$$

$$\boxed{h_1[p,u] \equiv x_1\big[p, e[p,u]\big]}$$

Hicksian demand at utility *u*= Marshalian demand at income *e[p,u]*



Cola
max u
Effect of a fall in the price of Pizza (here a normal good)
x[p,m]
max u
x[p′,m]
p'
min e
h[p′,v[p,m]]
5
10
p
20
Pizza
Substitution effect
Income effect
Total effect

Memorize and understand these 4 identities!

$$m \equiv e[p, ?]$$

$$\boxed{m \equiv e\big[p, v[p, m]\big]}$$

Varian, Microeconomic Analysis, p.106

Minimum expenditure to reach utility *v[p,m]* is *m*

$$u \equiv v[p, ?]$$

$$\boxed{u \equiv v\big[p, e[p, u]\big]}$$

Maximum utility from income *e[p,u]* is *u*

$$x_1[p, ?] \equiv h_1[p, ?]$$

$$x_1[p, m] \equiv h_1[p, ?]$$

$$\boxed{x_1[p, m] \equiv h_1\big[p, v[p, m]\big]}$$

Marshalian demand at income *m* = Hicksian demand at utility *v[p,m]*

$$h_1[p, ?] \equiv x_1[p, ?]$$

$$h_1[p, u] \equiv x_1[p, ?]$$

$$\boxed{h_1[p, u] \equiv x_1\big[p, e[p, u]\big]}$$

Hicksian demand at utility *u*= Marshalian demand at income *e[p,u]*

- Now let’s do some magic with these 4 identities!

- Can be used to derive (once more) **Roy's Identity**

Memorize and understand these 4 identities!

$$m \equiv e[p, ?]$$

$$\boxed{m \equiv e\left[p, v[p, m]\right]}$$

Varian, Microeconomic Analysis, p.106

Minimum expenditure to reach utility *v[p,m]* is *m*

$$u \equiv v[p, ?]$$

$$\boxed{u \equiv v\left[p, e[p, u]\right]}$$

Maximum utility from income *e[p,u]* is *u*

$$x_1[p, ?] \equiv h_1[p, ?]$$

$$x_1[p, m] \equiv h_1[p, ?]$$

$$\boxed{x_1[p, m] \equiv h_1\left[p, v[p, m]\right]}$$

Marshalian demand at income *m* = Hicksian demand at utility *v[p,m]*

$$h_1[p, ?] \equiv x_1[p, ?]$$

$$h_1[p, u] \equiv x_1[p, ?]$$

$$\boxed{h_1[p, u] \equiv x_1\left[p, e[p, u]\right]}$$

Hicksian demand at utility *u*= Marshalian demand at income *e[p,u]*

$$x_1[p,m] \equiv h_1\left[p, v[p,m]\right]$$

$$v[p, e[p,u]] \equiv u$$

x^* gives maximum u^* at (p^*, m^*)

$$m^* \equiv e[p, u^*]$$

then $x[p^*, m^*] \equiv h[p^*, u^*]$

$$x[p^*, m^*] \equiv h\left[p^*, u^*\right]$$

$$\frac{dv[p^*, e[p^*, u^*]]}{dp_1^*} = \frac{du^*}{dp_1^*} = 0$$

$$\frac{\partial v[p^*, m^*]}{\partial p_1^*} + \frac{\partial v[p^*, m^*]}{\partial m^*}\frac{\partial e[p^*, u^*]}{\partial p_1^*} = 0$$

$$\frac{\partial v[p^*, m^*]}{\partial p_1^*} + \frac{\partial v[p^*, m^*]}{\partial m^*} h_1[p^*, u^*] = 0$$

$$\Leftrightarrow h_1[p^*, u^*] = \frac{-\dfrac{\partial v[p^*, m^*]}{\partial p_1^*}}{\dfrac{\partial v[p^*, m^*]}{\partial m^*}}$$

$$\Leftrightarrow x_1[p^*, m^*] = \frac{-\dfrac{\partial v[p^*, m^*]}{\partial p_1^*}}{\dfrac{\partial v[p^*, m^*]}{\partial m^*}}$$

- Use to derive **Slutsky equation**

Memorize and understand these 4 identities!

$$m \equiv e[p,?]$$

$$\boxed{m \equiv e\left[p, v[p,m]\right]}$$

Varian, Microeconomic Analysis, p.106

Minimum expenditure to reach utility *v[p,m]* is *m*

$$u \equiv v[p,?]$$

$$\boxed{u \equiv v\left[p, e[p,u]\right]}$$

Maximum utility from income *e[p,u]* is *u*

$$x_1[p,?] \equiv h_1[p,?]$$

$$x_1[p,m] \equiv h_1[p,?]$$

$$\boxed{x_1[p,m] \equiv h_1\left[p, v[p,m]\right]}$$

Marshalian demand at income *m* = Hicksian demand at utility *v[p,m]*

$$h_1[p,?] \equiv x_1[p,?]$$

$$h_1[p,u] \equiv x_1[p,?]$$

$$\boxed{h_1[p,u] \equiv x_1\left[p, e[p,u]\right]}$$

Hicksian demand at utility *u*= Marshalian demand at income *e[p,u]*

$$h_i[p,u] \equiv x_i\left[p, e[p,u]\right]$$

$$x_i[p,m] \equiv h_i\left[p, v[p,m]\right]$$

x^* gives maximum u^* at (p^*, m^*)

$$m^* \equiv e[p, u^*]$$

then $h_i[p^*, u^*] \equiv x_i[p^*, m^*]$

$$x_i[p^*, m^*] \equiv h_i\left[p^*, u^*\right]$$

$$\frac{dh_i[p^*,u^*]}{dp_j} = \frac{dx_i[p^*,m^*]}{dp_j}$$

$$\frac{dh_i[p^*,u^*]}{dp_j} = \frac{\partial x_i[p^*,m^*]}{\partial p_j} + \frac{\partial x_i[p^*,m^*]}{\partial m}\frac{\partial e[p^*,u^*]}{\partial p_j}$$

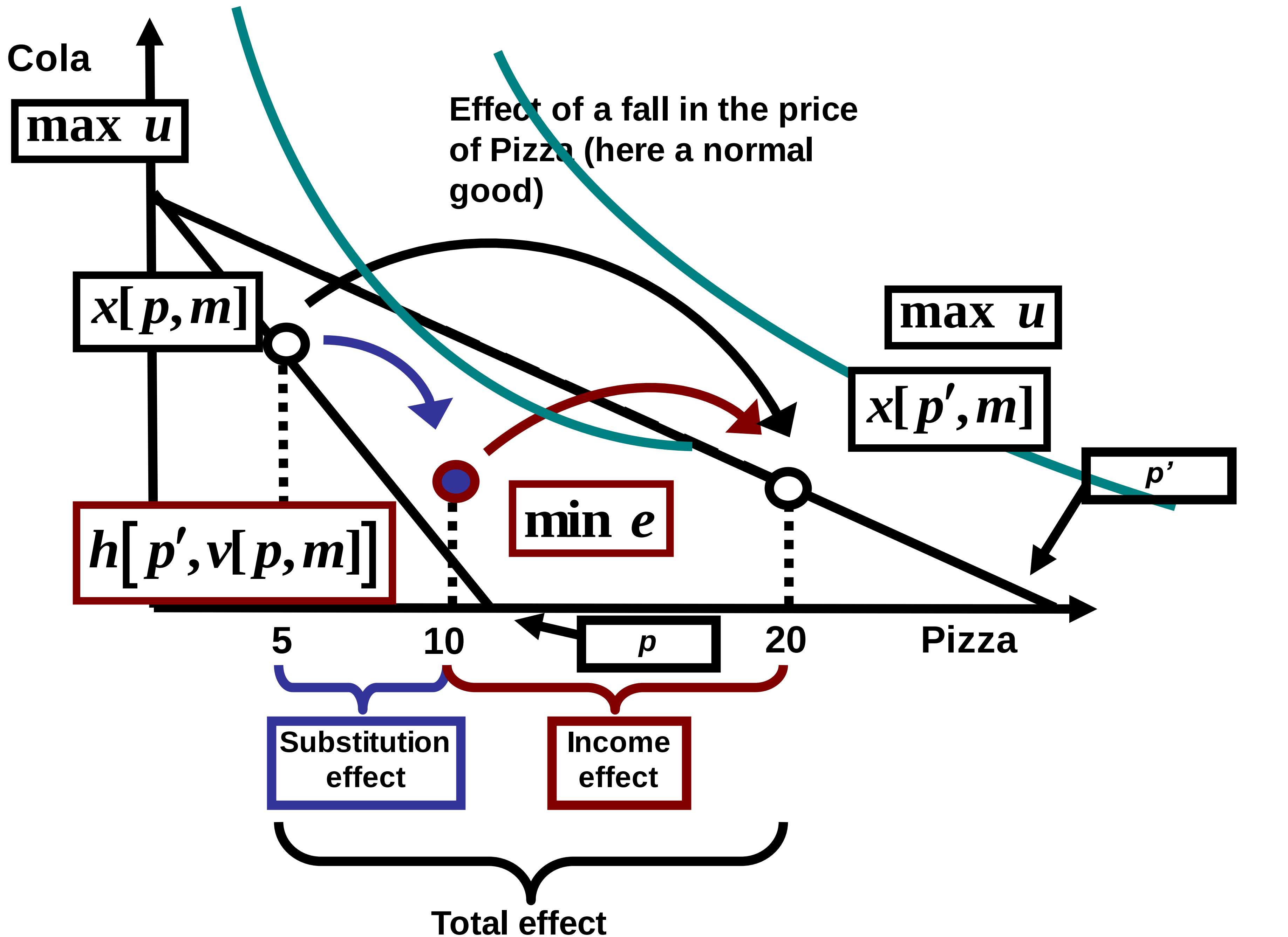
$$\Leftrightarrow \frac{\partial x_i[p^*,m^*]}{\partial p_j} = \frac{dh_i[p^*,u^*]}{dp_j} - \frac{\partial x_i[p^*,m^*]}{\partial m}\frac{\partial e[p^*,u^*]}{\partial p_j}$$

$$\Leftrightarrow \frac{\partial x_i[p^*,m^*]}{\partial p_j} = \frac{dh_i[p^*,u^*]}{dp_j} - \frac{\partial x_i[p^*,m^*]}{\partial m} h_j[p^*,u^*]$$

$$\Leftrightarrow \frac{\partial x_i[p^*,m^*]}{\partial p_j} = \frac{dh_i[p^*,u^*]}{dp_j} - \frac{\partial x_i[p^*,m^*]}{\partial m} x_j[p^*,u^*]$$

Slutsky equation

$$\Leftrightarrow \frac{\partial x_i[p,m]}{\partial p_j} = \frac{dh_i[p,u]}{dp_j} - \frac{\partial x_i[p,m]}{\partial m} x_j[p,u]$$


Cola
max u
Effect of a fall in the price of Pizza (here a normal good)
x[p,m]
max u
x[p′,m]
p’
min e
h[p′,v[p,m]]
p
5
10
20
Pizza
Substitution effect
Income effect
Total effect

- Examples

- **Good old CD**

$$\frac{\partial x_i[p,m]}{\partial p_j} = \frac{dh_i[p,u]}{dp_j} - \frac{\partial x_i[p,m]}{\partial m} x_j[p,u]$$

Slutsky equation

$u = x_1 x_2$, **determine the slutsky equation for** x_1 **wrt** p_1

$h_1 = ?$

$x_1 = \frac{1}{2} m p_1^{-1}, x_2 = \frac{1}{2} m p_2^{-1}$

$$v = \frac{1}{4} m^2 (p_1 p_2)^{-1}$$

$v = \frac{1}{2} m p_1^{-1} \cdot \frac{1}{2} m p_2^{-1} = \frac{1}{4} m^2 p_1^{-1} p_2^{-1}$

$$e = 2(u p_1 p_2)^{.5}$$

Use Shepards Lemma $h_1 = \frac{\partial e}{\partial p_1} = \frac{\partial 2(u p_1 p_2)^{\frac{1}{2}}}{\partial p_1} = (u p_2)^{\frac{1}{2}} p_1^{-\frac{1}{2}}$

$$\frac{\partial x_i[p,m]}{\partial p_j} = \frac{dh_i[p,u]}{dp_j} - \frac{\partial x_i[p,m]}{\partial m} x_j[p,u]$$

Slutsky equation

$u = x_1 x_2$, **determine the slutsky equation for** x_1 **wrt** p_1

$$h_1 = \left(up_2\right)^{\frac{1}{2}} p_1^{-\frac{1}{2}} \qquad x_1 = \tfrac{1}{2} m p_1^{-1},\; x_2 = \tfrac{1}{2} m p_2^{-1} \qquad \boxed{v = \tfrac{1}{4} m^2 \left(p_1 p_2\right)^{-1}}$$

$$\frac{\partial x[p,m]}{\partial p_1} = \frac{d\left(up_2\right)^{\frac{1}{2}} p_1^{-\frac{1}{2}}}{dp_1} - \frac{d\,\tfrac{1}{2} m p_1^{-1}}{dm} \cdot \tfrac{1}{2} m p_1^{-1}$$

$$= -\tfrac{1}{2} \cdot \left(up_2\right)^{\frac{1}{2}} p_1^{-\frac{3}{2}} - \tfrac{1}{2} p_1^{-1} \cdot \tfrac{1}{2} m p_1^{-1}$$

$$= -\tfrac{1}{2} \cdot \left(up_2\right)^{\frac{1}{2}} p_1^{-\frac{3}{2}} - \tfrac{1}{4} m p_1^{-2}$$

$$= -\tfrac{1}{2} \cdot \left(\tfrac{1}{4} m^2 \left(p_1 p_2\right)^{-1} p_2\right)^{\frac{1}{2}} p_1^{-\frac{3}{2}} - \tfrac{1}{4} m p_1^{-2}$$

$$= -\tfrac{1}{4} \cdot m p_1^{-2} - \tfrac{1}{4} m p_1^{-2}$$

$$= -\tfrac{1}{2} \cdot m p_1^{-2}$$

$$\frac{\partial x_i[p,m]}{\partial p_j} = \frac{dh_i[p,u]}{dp_j} - \frac{\partial x_i[p,m]}{\partial m} x_j[p,u]$$

Slutsky equation

$u = x_1 x_2,$ **determine the slutsky equation for** x_1 **wrt** p_2

$$h_1 = (up_2)^{\frac{1}{2}} p_1^{-\frac{1}{2}} \qquad x_1 = \tfrac{1}{2} m p_1^{-1}, x_2 = \tfrac{1}{2} m p_2^{-1} \qquad v = \tfrac{1}{4} m^2 (p_1 p_2)^{-1}$$

$$\frac{\partial x[p,m]}{\partial p_2} = \frac{d(up_2)^{\frac{1}{2}} p_1^{-\frac{1}{2}}}{dp_2} - \frac{d\,\frac{1}{2} m p_1^{-1}}{dm} \cdot \tfrac{1}{2} m p_2^{-1}$$

$$= \tfrac{1}{2} \cdot u^{\frac{1}{2}} (p_2 p_1)^{-\frac{1}{2}} - \tfrac{1}{2} p_1^{-1} \cdot \tfrac{1}{2} m p_2^{-1}$$

$$= \tfrac{1}{2} \cdot \left(\tfrac{1}{4} m^2 (p_1 p_2)^{-1}\right)^{\frac{1}{2}} (p_2 p_1)^{-\frac{1}{2}} - \tfrac{1}{4} m (p_1 p_2)^{-1}$$

$$= \tfrac{1}{4} m \cdot (p_1 p_2)^{-1} - \tfrac{1}{4} m (p_1 p_2)^{-1}$$

$$= 0$$

- **Perfect substitutes**

$$\frac{\partial x_i[p,m]}{\partial p_j} = \frac{dh_i[p,u]}{dp_j} - \frac{\partial x_i[p,m]}{\partial m} x_j[p,u]$$

Slutsky equation

$u = x_1 + x_2$, **determine the slutsky equation for** x_1 **wrt** p_1, p_2 **with** $p_1 << p_2$

$h_1 = ?$

$x_1 = mp_1^{-1}, x_2 = 0$ $\quad v = mp_1^{-1}$ $\quad \boxed{e = up_1}$

Use Shepards Lemma

$$h_1 = \frac{\partial e}{\partial p_1} = \frac{\partial (up_1)}{\partial p_1} = u$$

$$h_2 = \frac{\partial e}{\partial p_2} = \frac{\partial (up_1)}{\partial p_2} = 0$$

$$\frac{\partial x_i[p,m]}{\partial p_j} = \frac{dh_i[p,u]}{dp_j} - \frac{\partial x_i[p,m]}{\partial m} x_j[p,u]$$

Slutsky equation

$u = x_1 + x_2$, **determine the slutsky equation for** x_1 **wrt** p_1, p_2

with $p_1 << p_2$

$h_1 = ?$

$x_1 = mp_1^{-1}, x_2 = 0$

$v = mp_1^{-1}$

$e = up_1$

Use Shepards Lemma

$$h_1 = \frac{\partial e}{\partial p_1} = \frac{\partial (up_1)}{\partial p_1} = u$$

$$h_2 = \frac{\partial e}{\partial p_2} = \frac{\partial (up_1)}{\partial p_2} = 0$$

$$\boxed{\frac{\partial x_i[p,m]}{\partial p_j} = \frac{dh_i[p,u]}{dp_j} - \frac{\partial x_i[p,m]}{\partial m} x_j[p,u]}$$

Slutsky equation

$u = x_1 + x_2$, **determine the slutsky equation for** x_1 **wrt** p_1, p_2 **with** $p_1 << p_2$

$h_1 = u$ $\quad x_1 = mp_1^{-1}, x_2 = 0$ $\quad v = mp_1^{-1}$ $\quad \boxed{e = up_1}$

$$\frac{\partial x[p,m]}{\partial p_1} = \frac{du}{dp_1} - \frac{dmp_1^{-1}}{dm} \cdot mp_1^{-1}$$

$$= 0 - p_1^{-1} \cdot mp_1^{-1}$$

$$= -mp_1^{-2}$$

?

All income-effect, no substitution effect

$$\frac{\partial x_i[p,m]}{\partial p_j} = \frac{dh_i[p,u]}{dp_j} - \frac{\partial x_i[p,m]}{\partial m} x_j[p,u]$$

Slutsky equation

$u = x_1 + x_2$, **determine the slutsky equation for** x_1 **wrt** p_1, p_2 **with** $p_1 << p_2$

$h_1 = u$

$x_1 = mp_1^{-1}, x_2 = 0$

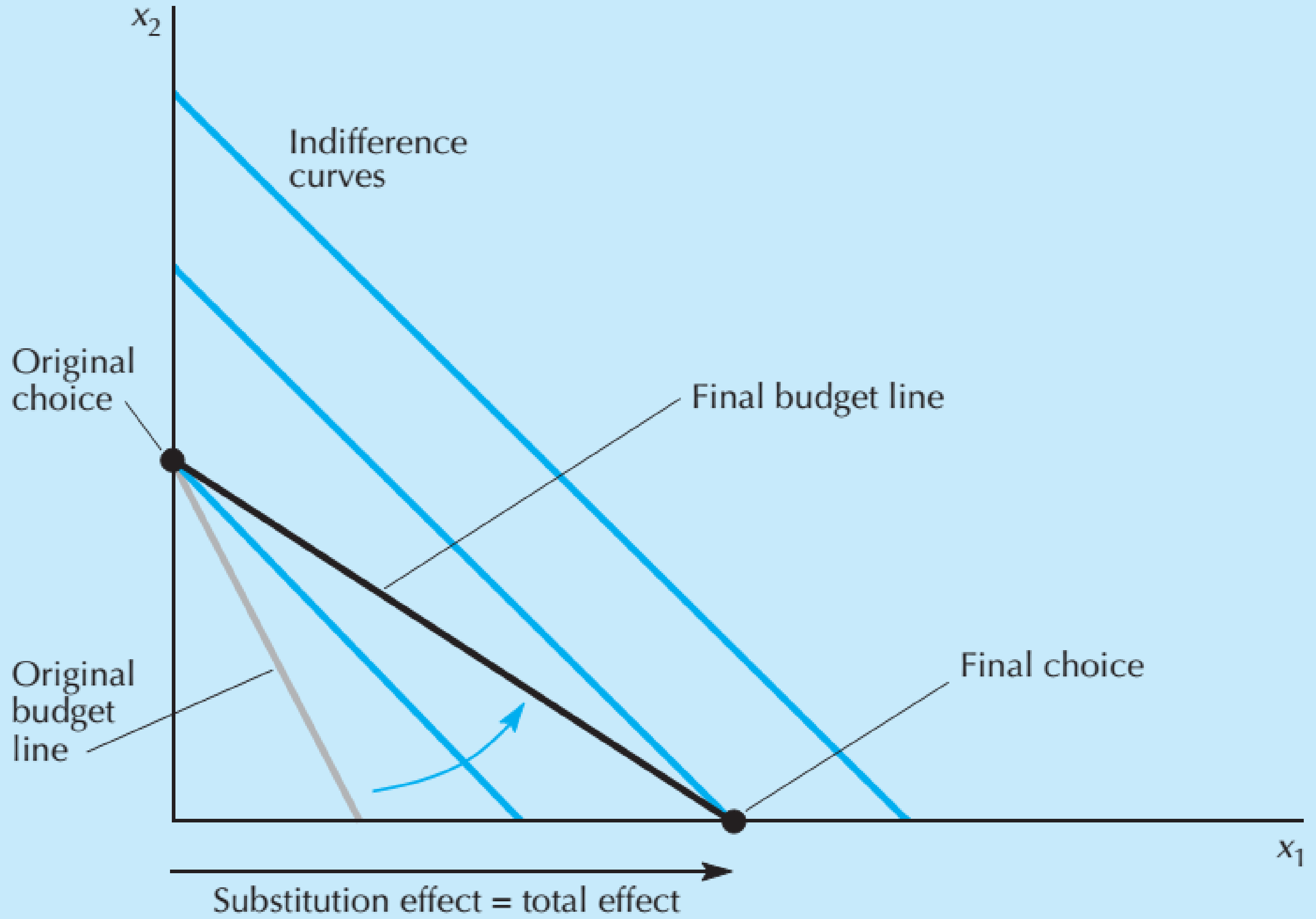
$v = mp_1^{-1}$

$$\frac{\partial x[p,m]}{\partial p_2} = \frac{du}{dp_2} - \frac{dmp_1^{-1}}{dm} \cdot 0$$

$$= 0 - 0$$

$$= 0$$

Examples of Income and Substitution Effects



Perfect substitutes. Slutsky decomposition with perfect substitutes.

- **Perfect complements**

$$\frac{\partial x_i[p,m]}{\partial p_j} = \frac{dh_i[p,u]}{dp_j} - \frac{\partial x_i[p,m]}{\partial m} x_j[p,u]$$

Slutsky equation

$u = \min[x_1, x_2]$, **determine the slutsky equation for** x_1 **wrt** p_1, p_2

$h_1 = ?$

$x_1 = x_2 = m\left(p_1 + p_2\right)^{-1}$ $\quad v = m\left(p_1 + p_2\right)^{-1}$ $\quad e = u\left(p_1 + p_2\right)$

Use Shepards Lemma

$$h_1 = \frac{\partial e}{\partial p_1} = \frac{\partial u\left(p_1 + p_2\right)}{\partial p_1} = u$$

$$h_2 = \frac{\partial e}{\partial p_2} = \frac{\partial u\left(p_1 + p_2\right)}{\partial p_2} = u$$

$$\boxed{\frac{\partial x_i[p,m]}{\partial p_j} = \frac{dh_i[p,u]}{dp_j} - \frac{\partial x_i[p,m]}{\partial m} x_j[p,u]}$$

Slutsky equation

$u = \min[x_1, x_2]$, **determine the slutsky equation for** x_1 **wrt** p_1, p_2

$h_1 = up_2$ $\quad x_1 = x_2 = m\left(p_1 p_2\right)^{-1}$ $\quad v = m\left(p_1 p_2\right)^{-1}$ $\quad \boxed{e = up_1 p_2}$

$$\frac{\partial x[p,m]}{\partial p_1} = \frac{du}{dp_1} - \frac{dm\left(p_1 p_2\right)^{-1}}{dm} \cdot m\left(p_1 p_2\right)^{-1}$$

$$= 0 - \left(p_1 p_2\right)^{-1} \cdot m\left(p_1 p_2\right)^{-1}$$

$$= -m\left(p_1 p_2\right)^{-2}$$

?

All income-effect, no substitution effect

$$\boxed{\frac{\partial x_i[p,m]}{\partial p_j} = \frac{dh_i[p,u]}{dp_j} - \frac{\partial x_i[p,m]}{\partial m} x_j[p,u]}$$

Slutsky equation

$u = \min[x_1, x_2]$, **determine the slutsky equation for x_1 wrt p_1, p_2**

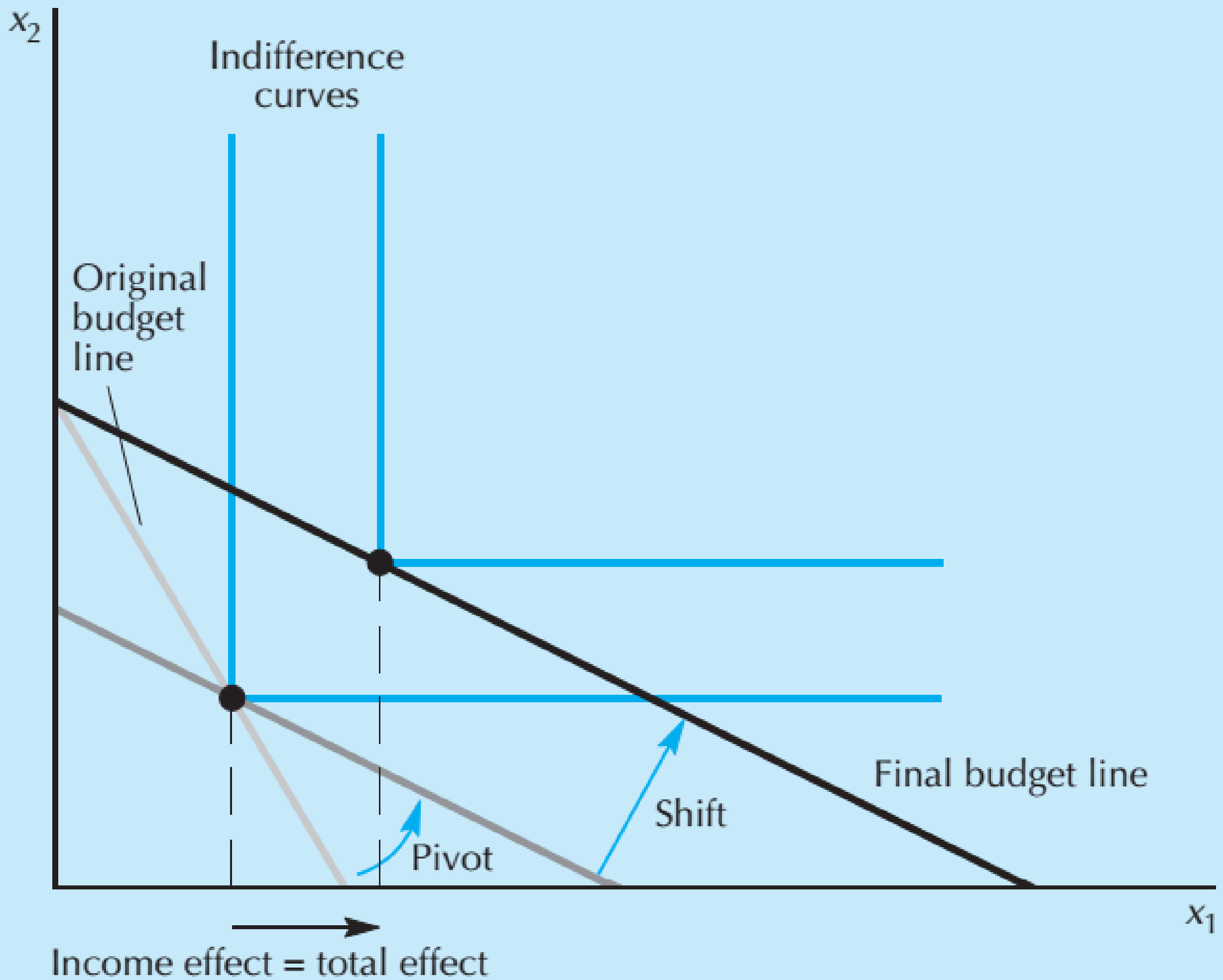
$h_1 = up_2$ $\quad x_1 = x_2 = m(p_1 p_2)^{-1}$ $\quad v = m(p_1 p_2)^{-1}$ $\quad \boxed{e = up_1 p_2}$

$$\frac{\partial x[p,m]}{\partial p_2} = \frac{du}{dp_2} - \frac{dm(p_1 p_2)^{-1}}{dm} \cdot m(p_1 p_2)^{-1}$$

$$= 0 - (p_1 p_2)^{-1} \cdot m(p_1 p_2)^{-1}$$

$$= -m(p_1 p_2)^{-2}$$

Examples of Income and Substitution Effects



Perfect complements. Slutsky decomposition with perfect complements.

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

Národohospodářská fakulta VŠE v Praze