Microeconomics

EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MŠMT
MINISTERSTVO ŠKOLSTVÍ,
MLÁDEŽE A TĚLOVÝCHOVY

Lecture 3

Silvester van Koten

- The presentation is based on the following sources:

- Snyder Christopher M Walter Nicholson and Robert Stewart. 2015. Microeconomic Theory : Basic Principles and Extensions EMEA ed. Andover: Cengage Learning. +
- Varian Hal R. 2020. Intermediate Microeconomics : A Modern Approach Ninth ed. New York: W.W. Norton & Company. +
- Excerpts from Gravelle H O Hart and R Rees. 2004. Advanced Microeconomics.

CONSUMER’S SURPLUS

Varian Intermediate Micro

Chap 14

Quasi-linear utility

- Varian, Chap 14, Appendix (p.268)

$$U[x,y] = x + v[y] \quad s.t. \quad p_x x + p_y y = M$$

Let x represents money: $p_x = 1$

$$L[x,y,\lambda] = x + v[y] + \lambda\left(M - x - p_y y\right)$$

FOCs:

$$\left.\begin{aligned} 0 &= L_1 = 1 - \lambda \\ 0 &= L_2 = v'[y] - \lambda p_y \end{aligned}\right\} \qquad v'[y] = p_y$$

$$0 = L_3 = M - p_x x - p_y y$$

$$U[x,y]=x+v[y] \quad s.t. \quad p_x x + p_y y = M$$

Let x represents money: $p_x = 1$

$$L[x,y,\lambda]=x+v[y]+\lambda\left(M-x-p_y y\right)$$

FOCs:

$$\left.\begin{aligned} 0 &= L_1 = 1-\lambda \\ 0 &= L_2 = v'[y]-\lambda p_y \end{aligned}\right\} \qquad v'[y]=p_y$$

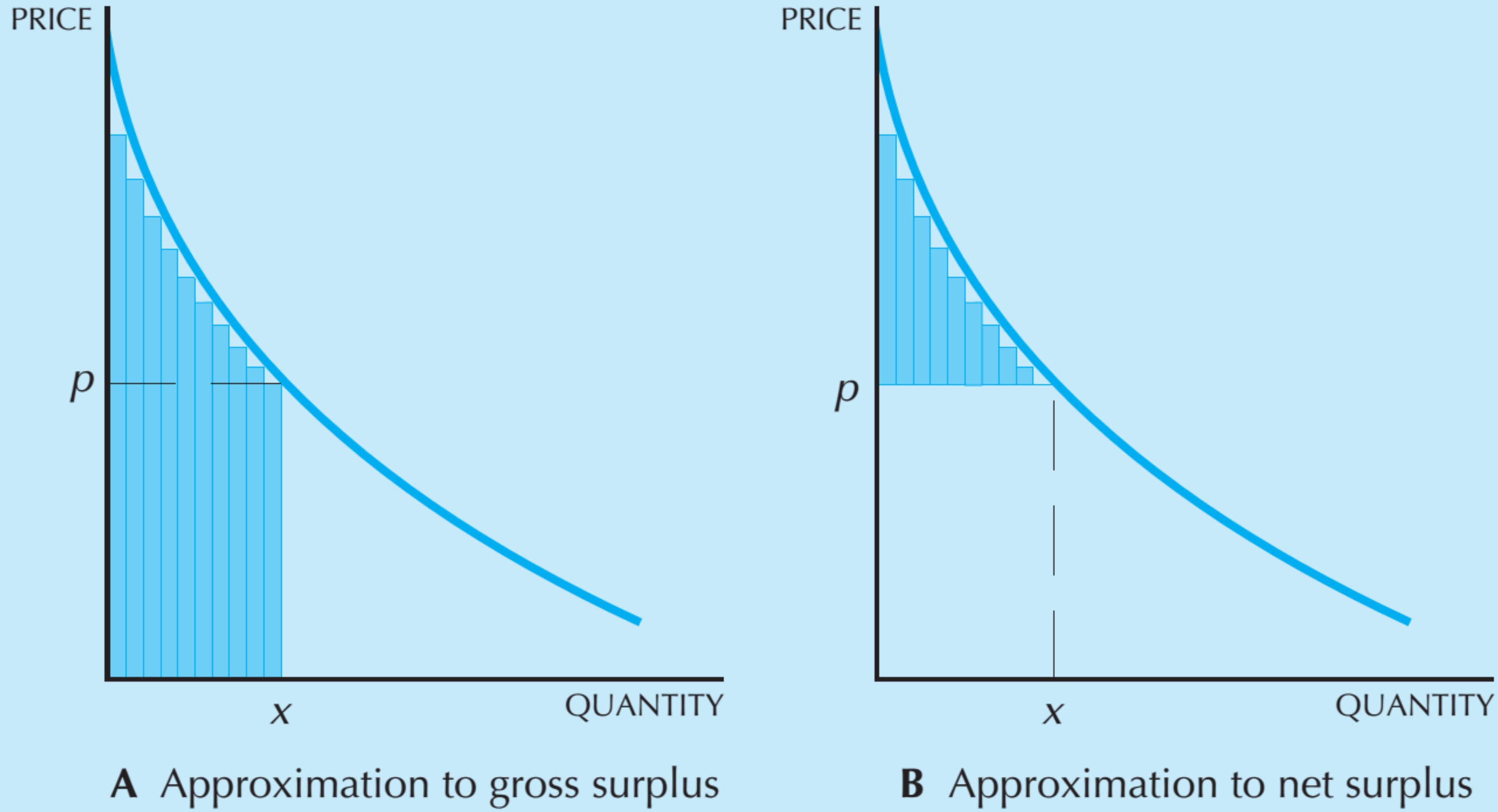
$$0 = L_3 = M - p_x x - p_y y$$

Assume that consuming zero units bring zero utility, then $v(0) = 0$

$$v[y]=v[y]-0$$

$$=v[y]-v[0]$$

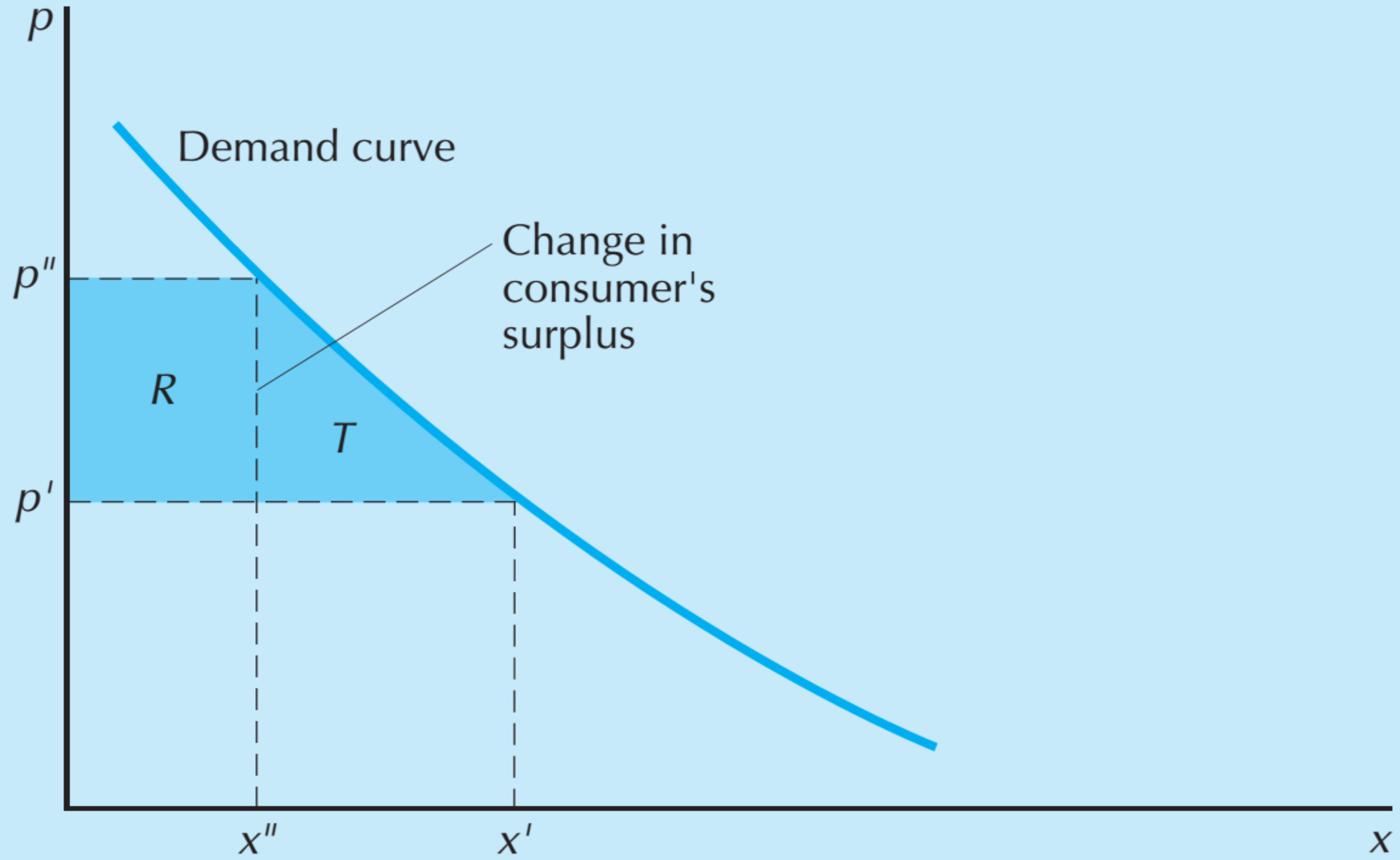
$$=\int_0^y v'[j]dj \quad =\int_0^y p[j]dj$$



Approximating a continuous demand. The consumer's surplus associated with a continuous demand curve can be approximated by the consumer's surplus associated with a discrete approximation to it.

- It is worth thinking about the role that quasilinear utility plays in this analysis.
- In general the price at which a consumer is willing to purchase some amount of good 1 will depend on how much money he has for consuming other goods.
- This means that in general the reservation prices for good 1 will depend on how much good 2 is being consumed.

- But in the special case of quasilinear utility the reservation prices are independent of the amount of money the consumer has to spend on other goods.
- With quasilinear utility there is “no income effect” since changes in income don’t affect demand.
- This is what allows us to calculate utility in such a simple way.
- Using the area under the demand curve to measure utility will only beexactly correct when the utility function is quasilinear.



Change in consumer’s surplus. The change in consumer’s surplus will be the difference between two roughly triangular areas, and thus will have a roughly trapezoidal shape.

Question: Consider the linear demand curve $D(p) = 20 - 2p$. When the price changes from 2 to 3 what is the associated change in consumer's surplus?

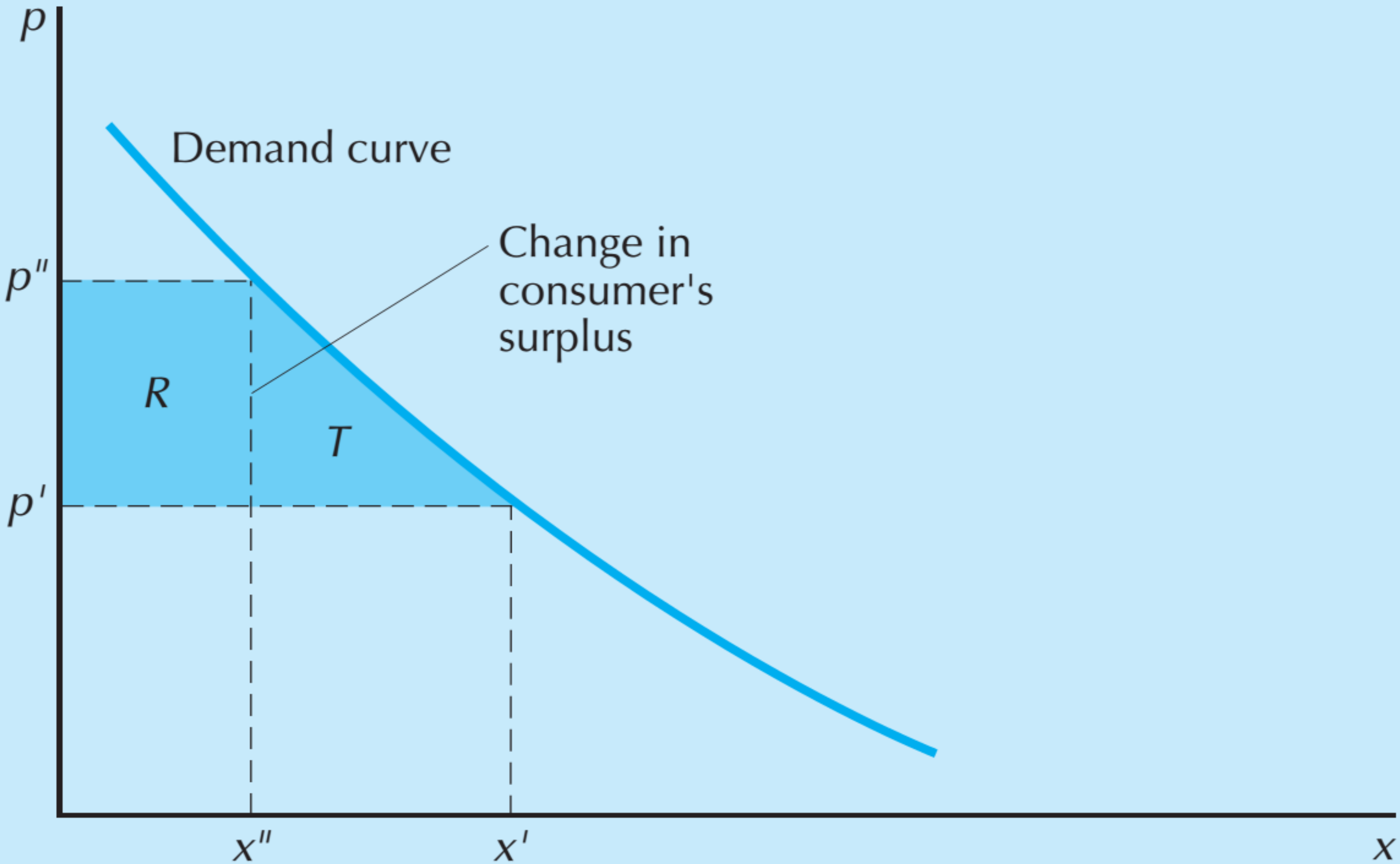
Question: Consider the linear demand curve $D(p) = 20 - 2p$. When the price changes from 2 to 3 what is the associated change in consumer's surplus?

Answer: When $p = 2$, $D(2) = 16$, and when $p = 3$, $D(3) = 14$. Thus we want to compute the area of a trapezoid with a height of 1 and bases of 14 and 16. This is equivalent to a rectangle with height 1 and base 14 (having an area of 14), plus a triangle of height 1 and base 2 (having an area of 1). The total area will therefore be 15.

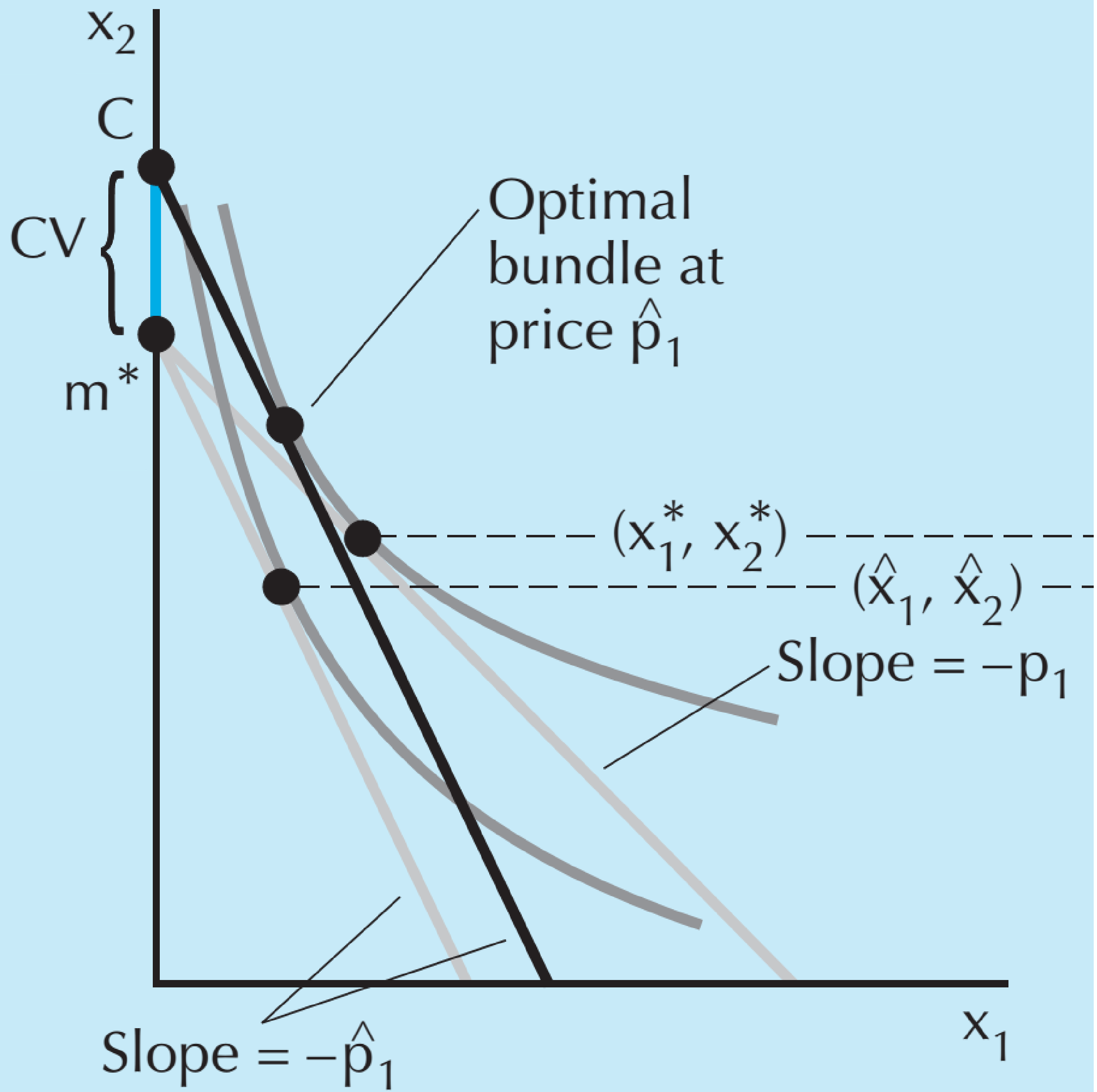
Compensation

- how much money we would have to give a consumer to compensate him for a change in prices?
  - Consumer’s surplus (CS)
  - Compensating variations (CV)
  - Equivalent variations (EV)

    - Varian, p.258-262
    - Nicholson, p.145-149

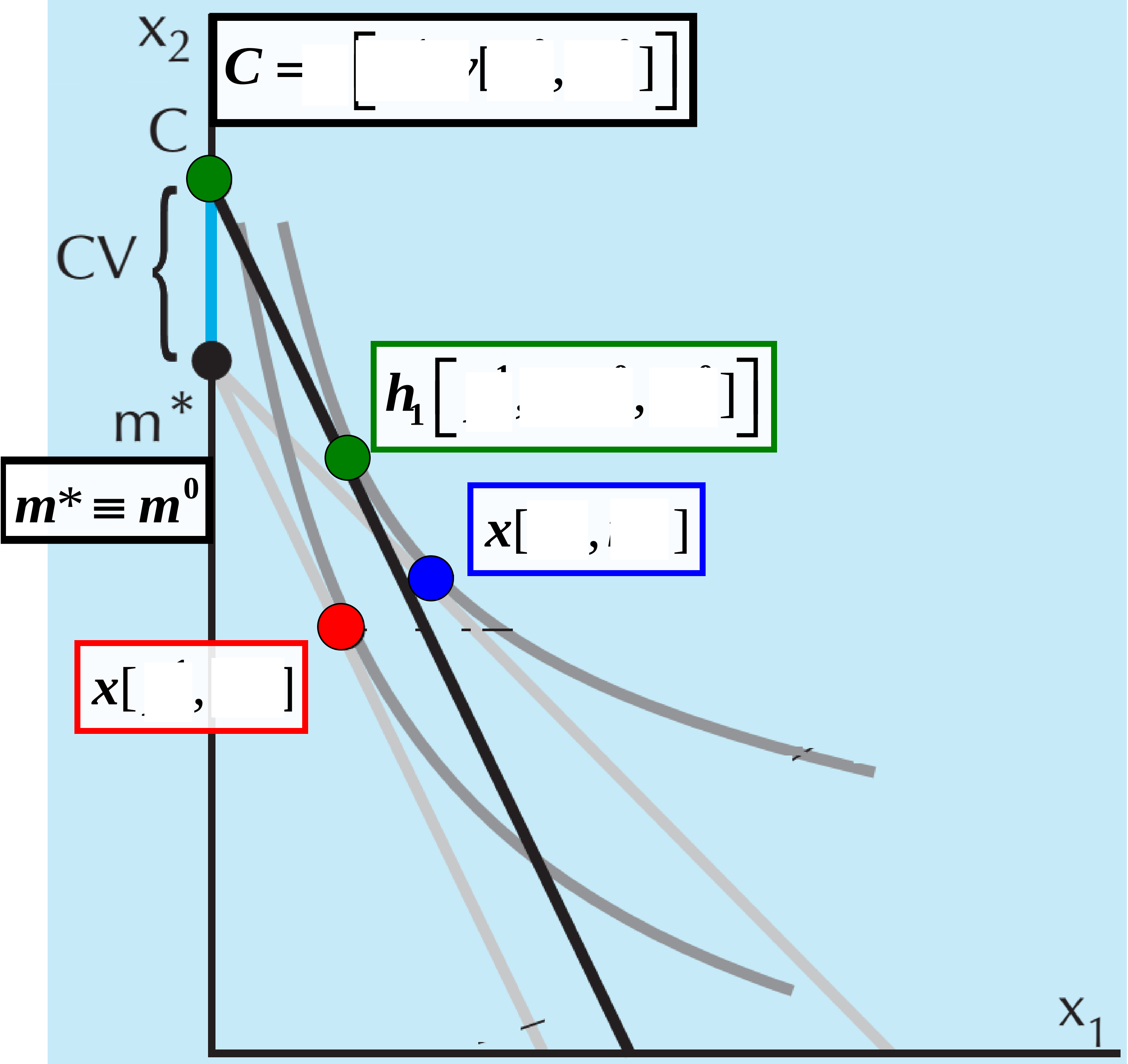


Change in consumer’s surplus. The change in consumer’s surplus will be the difference between two roughly triangular areas, and thus will have a roughly trapezoidal shape.

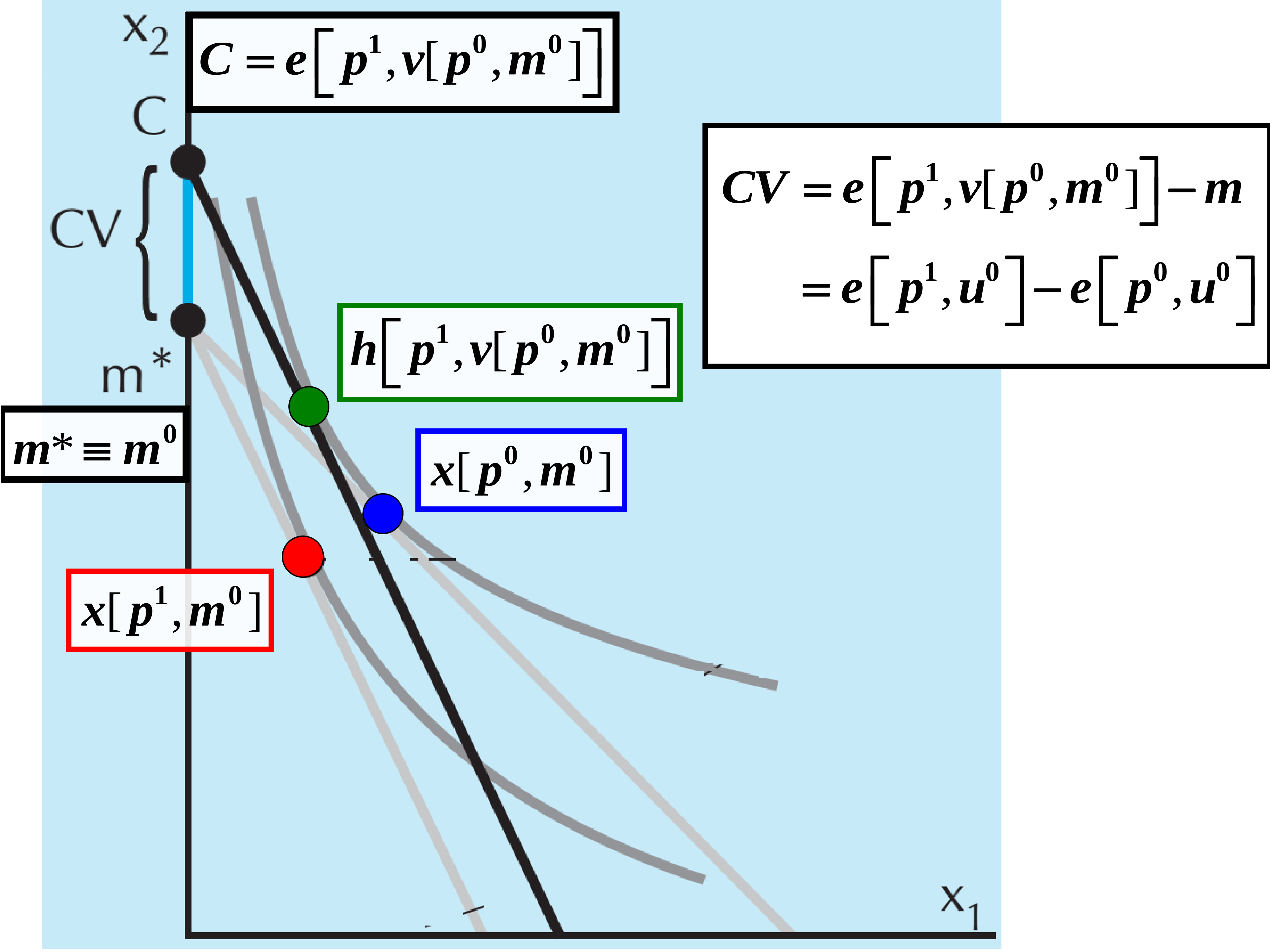


x_2
C
CV
m^*
Optimal bundle at price $\hat{p}_1$
(x_1^*, x_2^*)
$(\hat{x}_1, \hat{x}_2)$
Slope = $-p_1$
Slope = $-\hat{p}_1$
x_1

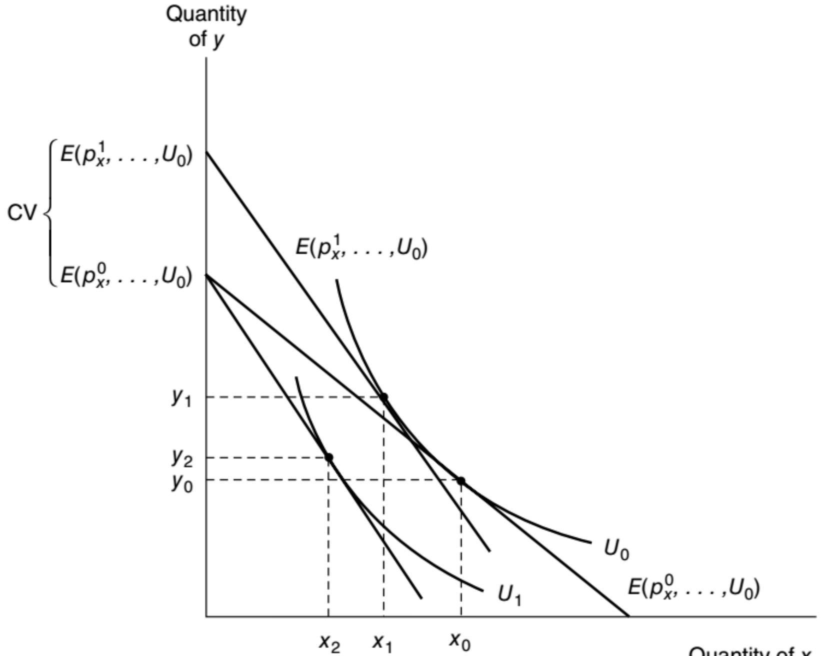
A



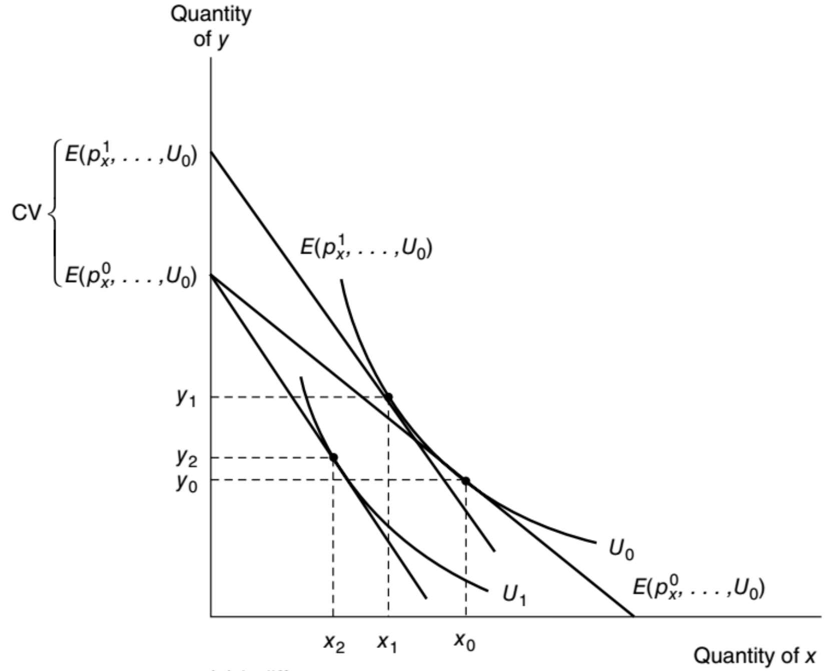
x2
C =
C
CV
m*
m* ≡ m0
x1



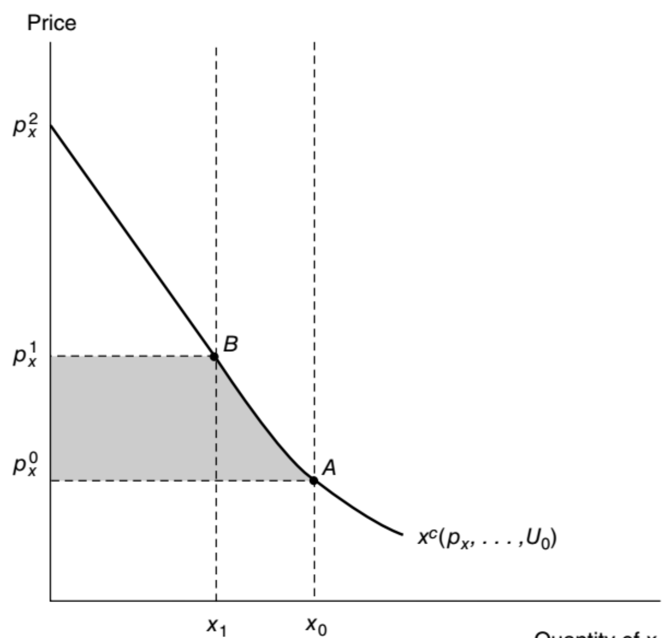
x_2
$C = e\left[p^1, v[p^0, m^0]\right]$
C
CV
m^*
$m^* \equiv m^0$
$CV = e\left[p^1, v[p^0, m^0]\right] - m$
$= e\left[p^1, u^0\right] - e\left[p^0, u^0\right]$
$h\left[p^1, v[p^0, m^0]\right]$
$x[p^0, m^0]$
$x[p^1, m^0]$
x_1



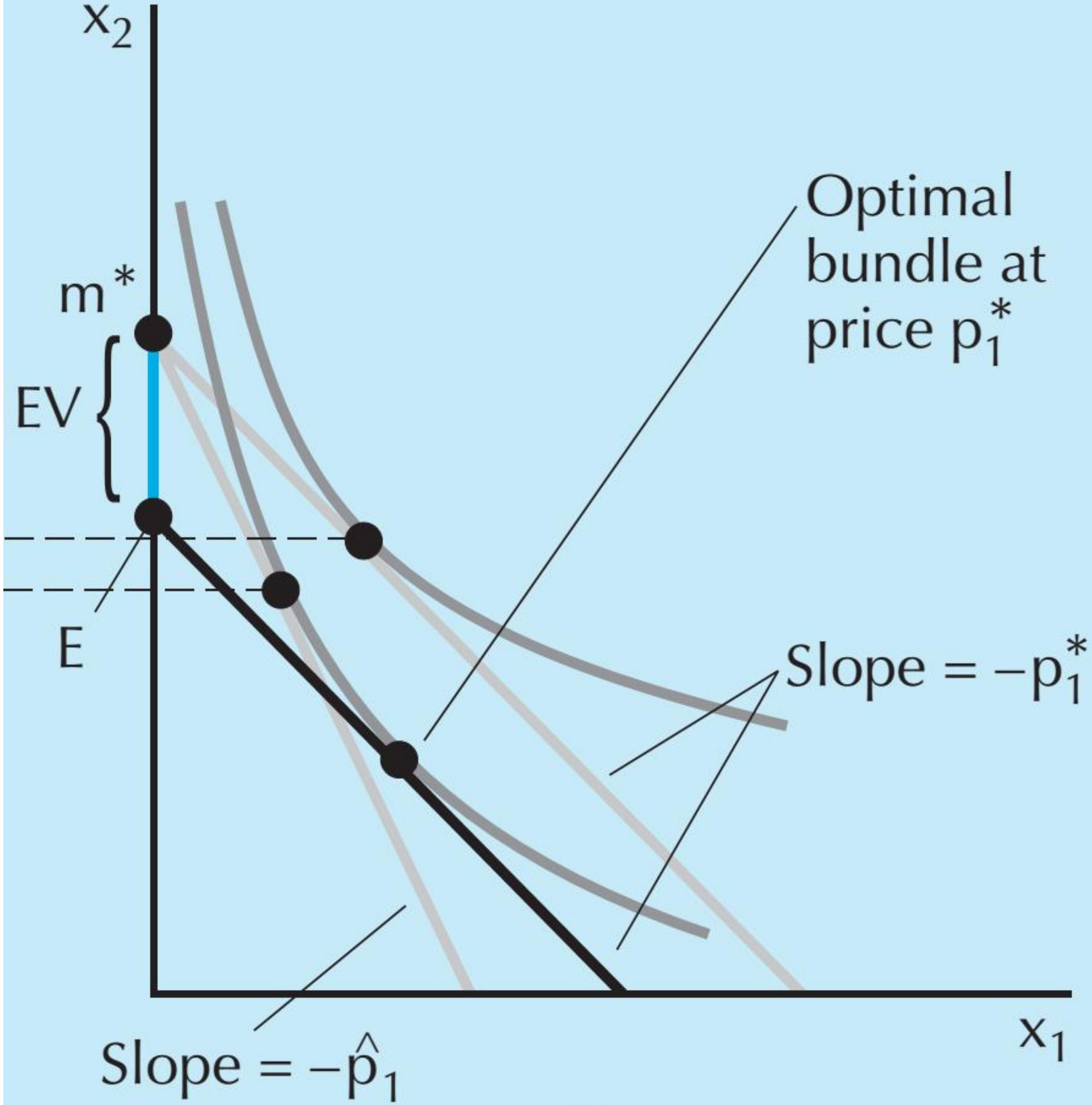
(a) Indifference curve map



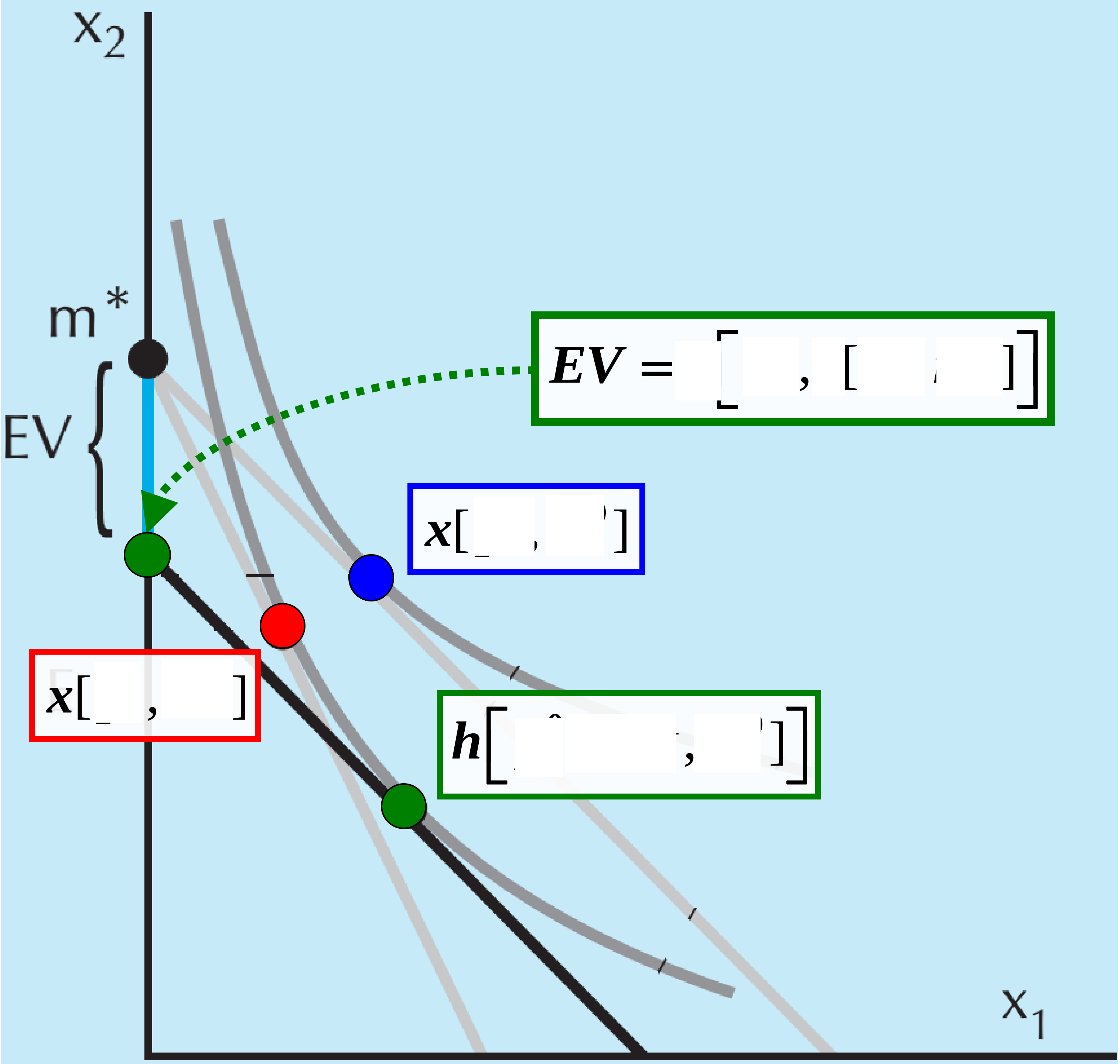
(a) Indifference curve map



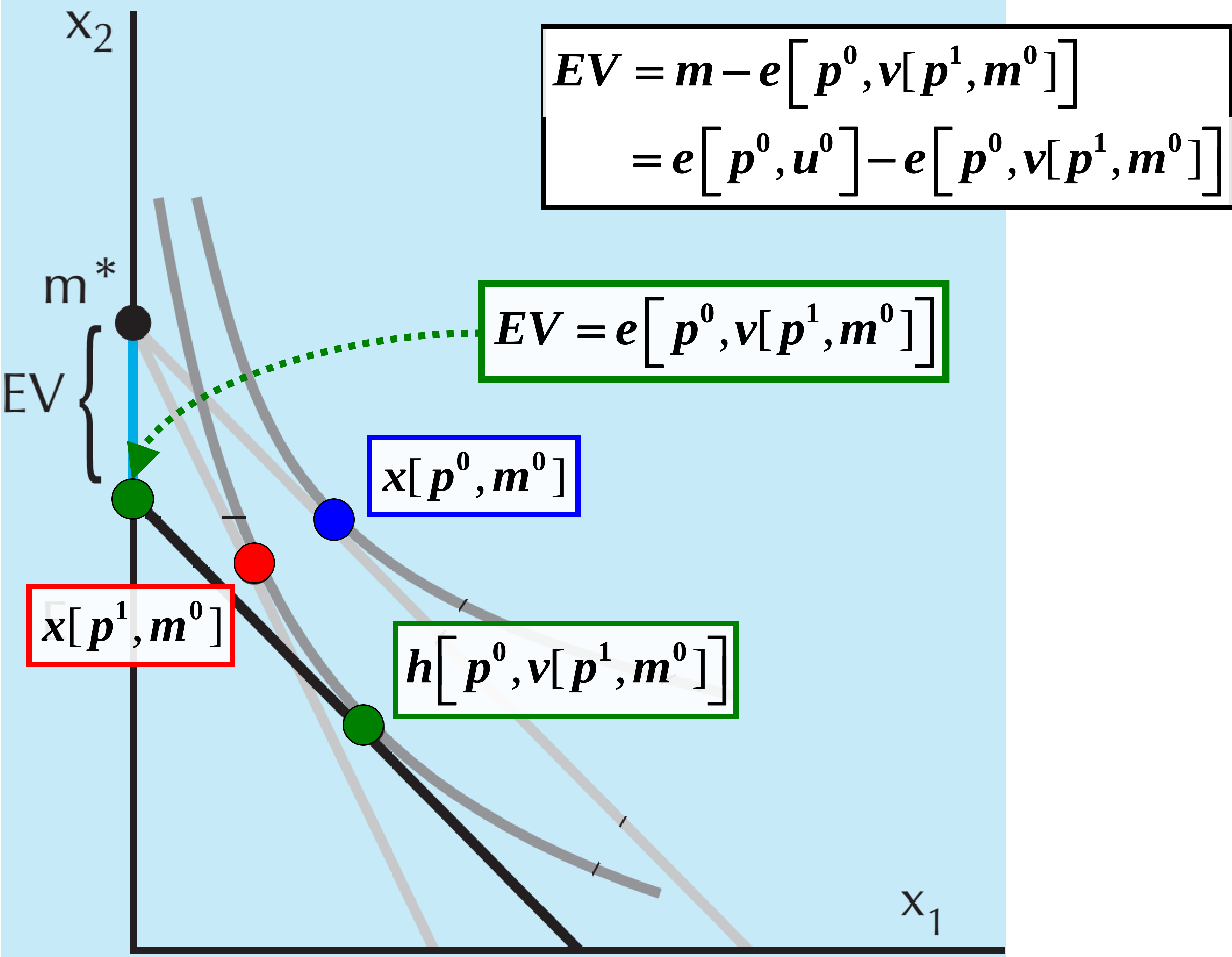
(b) Compensated demand curve



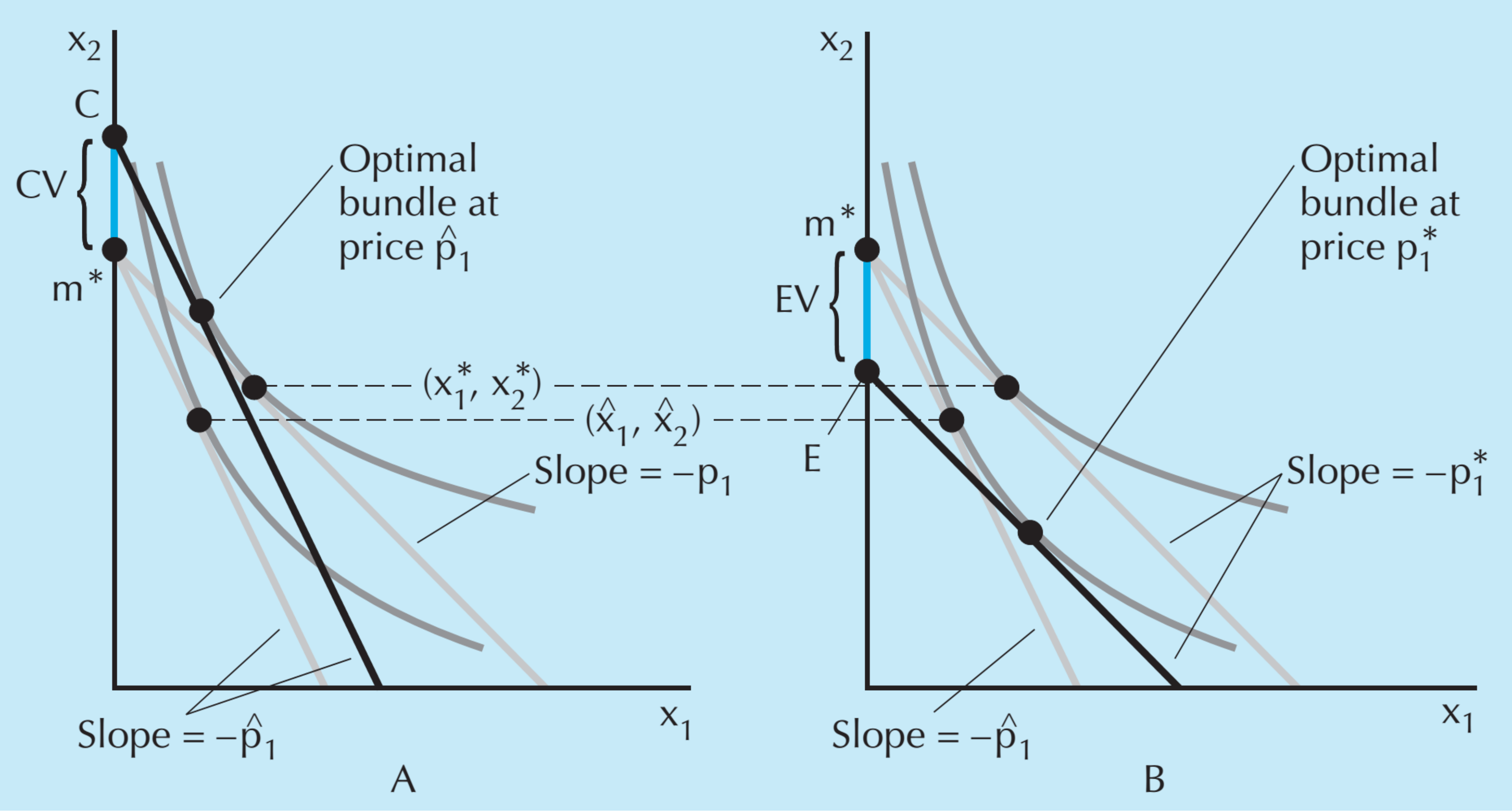
B



x2
m*
EV
x1
EV = [, [ ]]
x[,]
x[,]
h[,]



$EV = m - e\left[p^0, v[p^1, m^0]\right]$
$= e\left[p^0, u^0\right] - e\left[p^0, v[p^1, m^0]\right]$
x_2
m^*
EV
$EV = e\left[p^0, v[p^1, m^0]\right]$
$x[p^0, m^0]$
$x[p^1, m^0]$
$h\left[p^0, v[p^1, m^0]\right]$
x_1



A shows the compensating variation (CV), and panel B shows the equivalent variation (EV).

- Example
- (see Varian intermediair, p. 260-261)

utility function $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$

$$x_1 = \frac{m}{2p_1}$$

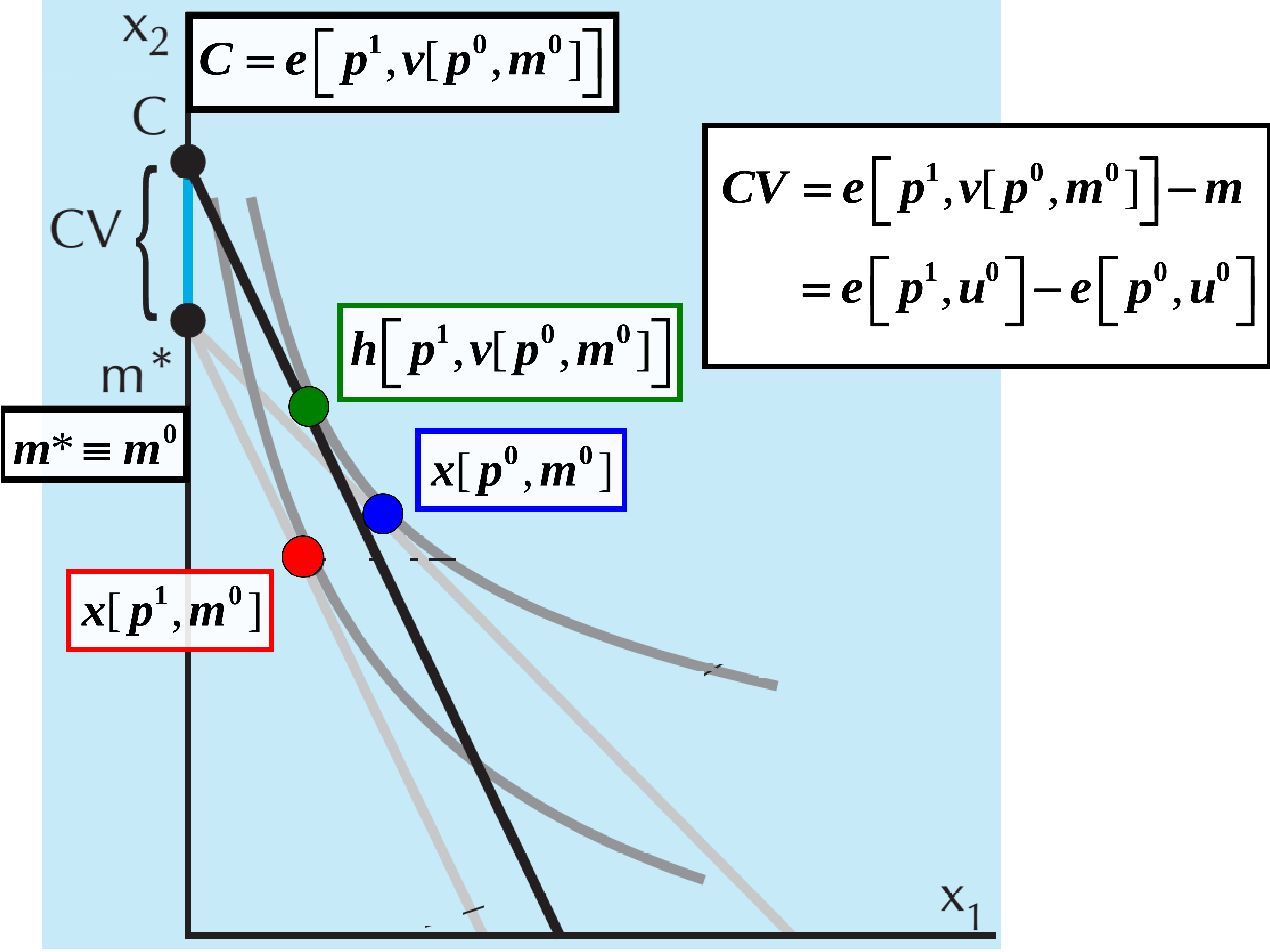
$$x_2 = \frac{m}{2p_2}$$

$$p_1^0 = 1 \qquad p_1^1 = 1$$

$$p_2^0 = 1 \qquad p_2^1 = 2$$

$$M^0 = 100 \qquad M^1 = 100$$

how much money would be necessary at prices (2,1) to make the consumer as well off as at prices (1,1)?



x_2
$C = e[p^1, v[p^0, m^0]]$
C
CV
m^*
$m^* \equiv m^0$
$h[p^1, v[p^0, m^0]]$
$x[p^0, m^0]$
$x[p^1, m^0]$
x_1
$CV = e[p^1, v[p^0, m^0]] - m$
$= e[p^1, u^0] - e[p^0, u^0]$

utility function $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$

$$x_1 = \frac{m}{2p_1}$$

$$x_2 = \frac{m}{2p_2}$$

$p_1 = 1$, $p_2 = 1$, $M = 100$ $\longrightarrow$ $p_1' = 1$, $p_2' = 2$, $M' = 100$

how much money would be necessary at prices (2,1) to make the consumer as well off as at prices (1,1)?

what is the CV?

$$CV = e\left[p', v[p,m]\right] - m = e\left[p', u\right] - e\left[p, u\right]$$

$$v[p,m] = \sqrt{\frac{m}{2p_1}} \cdot \sqrt{\frac{m}{2p_2}} = \tfrac{1}{2} m p_1^{-\frac{1}{2}} p_2^{-\frac{1}{2}} \Leftrightarrow u = \tfrac{1}{2} e p_1^{-\frac{1}{2}} p_2^{-\frac{1}{2}}$$

$$\Leftrightarrow e = 2u p_1^{\frac{1}{2}} p_2^{\frac{1}{2}}$$

$$v[p;m] = \tfrac{1}{2} m p_1^{-\frac{1}{2}} p_2^{-\frac{1}{2}}$$

$$e[p;u] = 2u p_1^{\frac{1}{2}} p_2^{\frac{1}{2}}$$

$$v[p;m] = \tfrac{1}{2} m p_1^{-\frac{1}{2}} p_2^{-\frac{1}{2}}$$

$$e[p;u] = 2 u p_1^{\frac{1}{2}} p_2^{\frac{1}{2}}$$

$$CV = 41.42$$

$$p_1 = 1 \quad p_2 = 1 \quad M = 100 \longrightarrow p_1' = 1 \quad p_2' = 2 \quad M' = 100$$

$$CV = e\left[p', v[p,m]\right] - m = e\left[p', u\right] - e\left[p, u\right]$$

$$= e\left[1,2; v[1,1;100]\right] - 100$$

$$= 2 \cdot \left(v[1,1;100]\right) \cdot 1^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} - 100$$

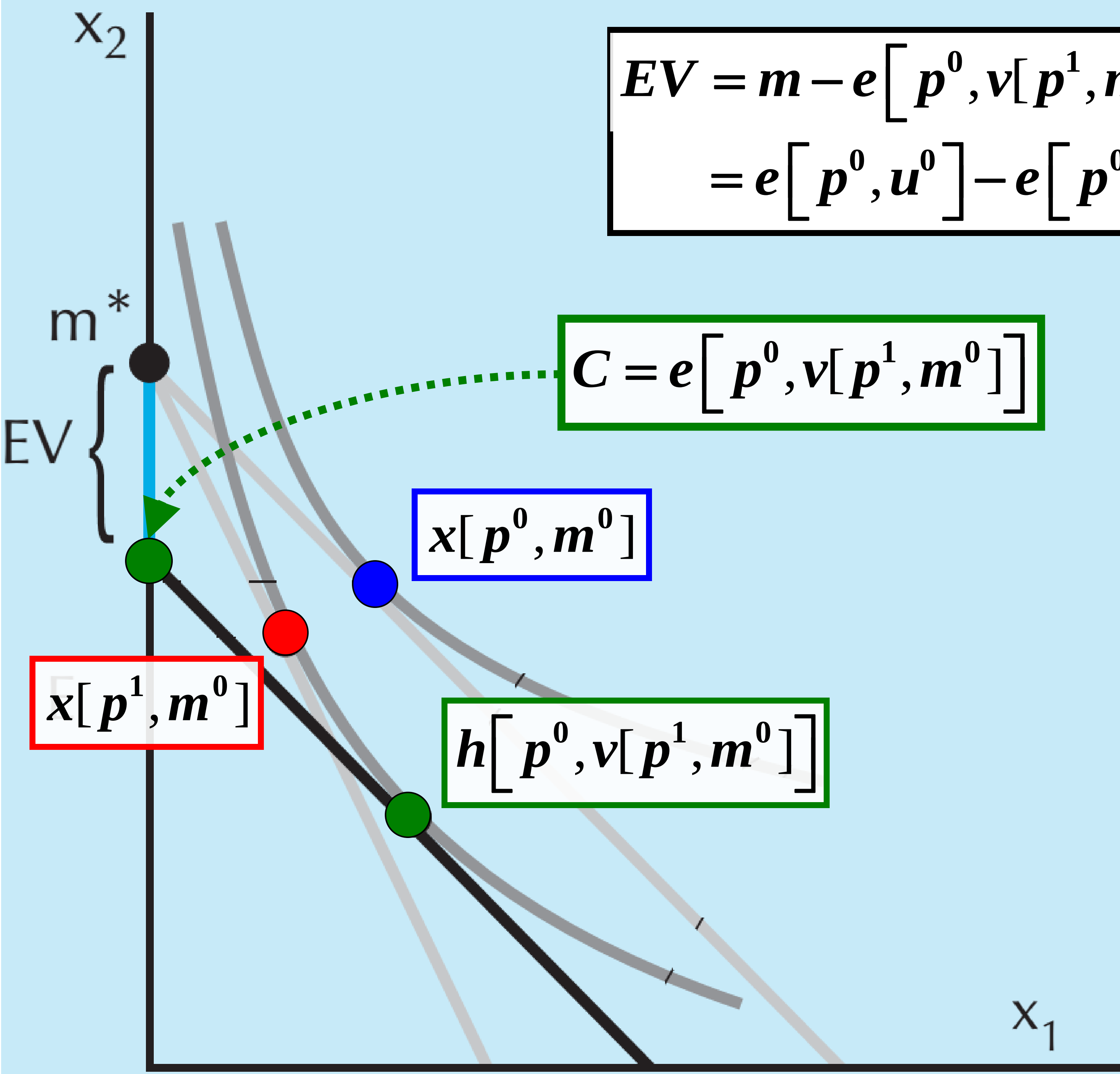
$$= 2 \cdot \left(\tfrac{1}{2} \cdot 100 \cdot 1^{-\frac{1}{2}} \cdot 1^{-\frac{1}{2}}\right) \cdot 1^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} - 100$$

$$= 2 \cdot (50) \cdot 2^{\frac{1}{2}} - 100$$

$$= 100\left(\sqrt{2} - 1\right) = 41.42$$

how much money should be taken away from the consumer at the original prices (1,1) to make the consumer as bad off as at prices (2,1)?
what is the EV?

$$EV = m - e\left[p^0, v[p^1, m^0]\right]$$
$$= e\left[p^0, u^0\right] - e\left[p^0, v[p^1, m^0]\right]$$



$$v[p;m]=\tfrac{1}{2}mp_1^{-\frac{1}{2}}p_2^{-\frac{1}{2}}$$

$$e[p;u]=2up_1^{\frac{1}{2}}p_2^{\frac{1}{2}}$$

$$CV=41.42$$

$$EV=29.3$$

$$p_1=1 \quad p_2=1 \quad M=100 \longrightarrow p_1'=1 \quad p_2'=2 \quad M'=100$$

$$EV=m-e\left[p;v[p';m]\right]$$

$$=100-e\left[1,1;v[1,2;100]\right]$$

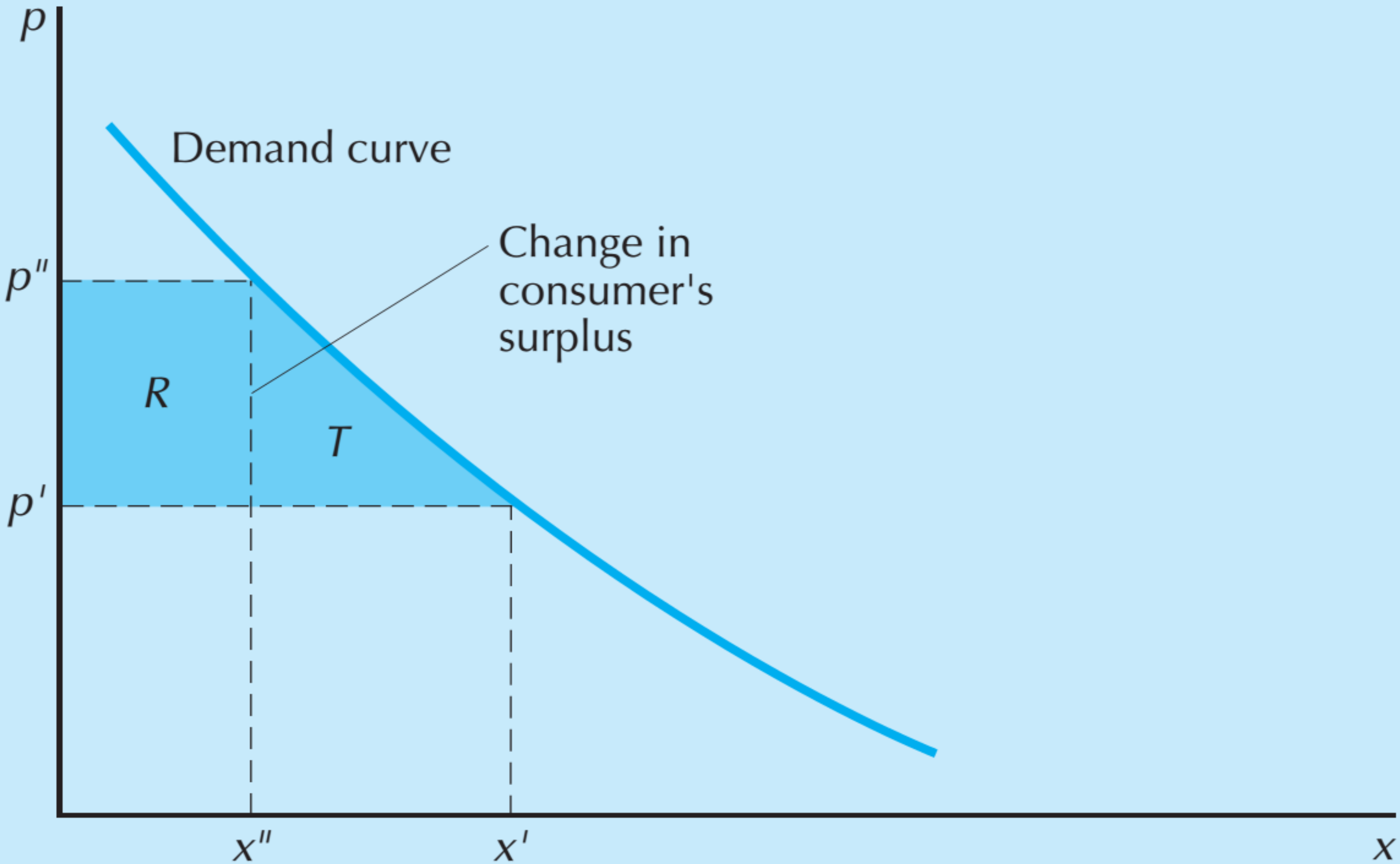
$$=100-2\cdot\left(v[1,2;100]\right)\cdot 1^{\frac{1}{2}}\cdot 1^{\frac{1}{2}}$$

$$=100-2\cdot\left(\tfrac{1}{2}\cdot 100\cdot 1^{-\frac{1}{2}}\cdot 2^{-\frac{1}{2}}\right)$$

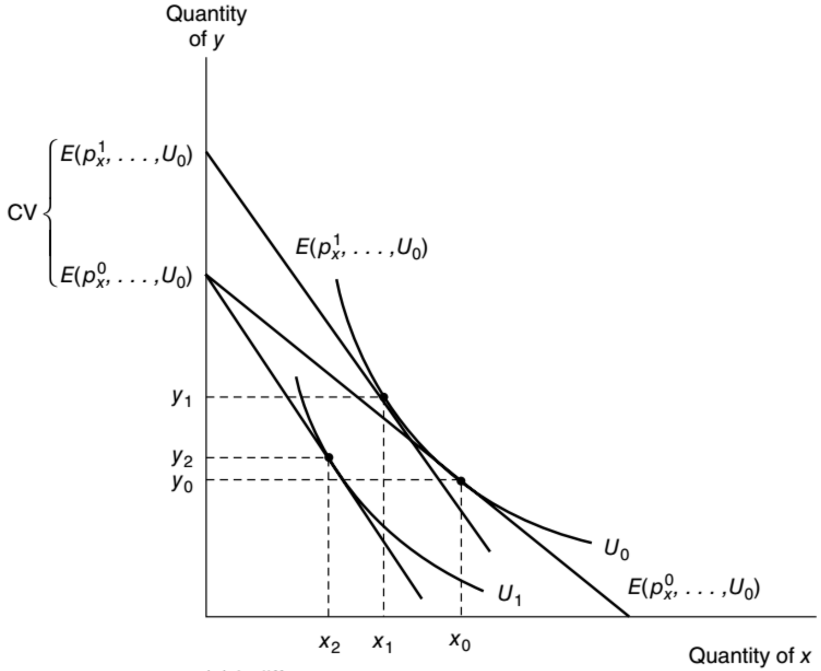
$$=100\left(1-2^{-\frac{1}{2}}\right)$$

$$=100\left(1-\frac{1}{\sqrt{2}}\right)=29.3$$

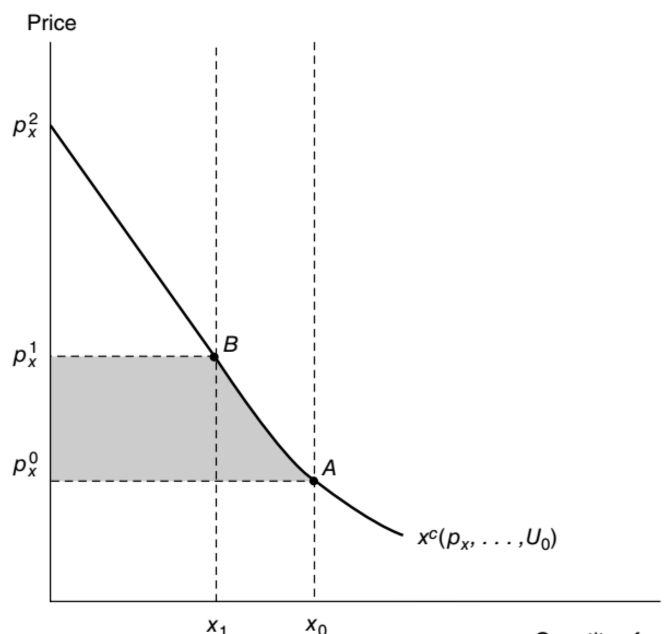
- What is best measure for loss/compensation?
  - Consumer surplus
  - CV
  - EV
- All three are connected



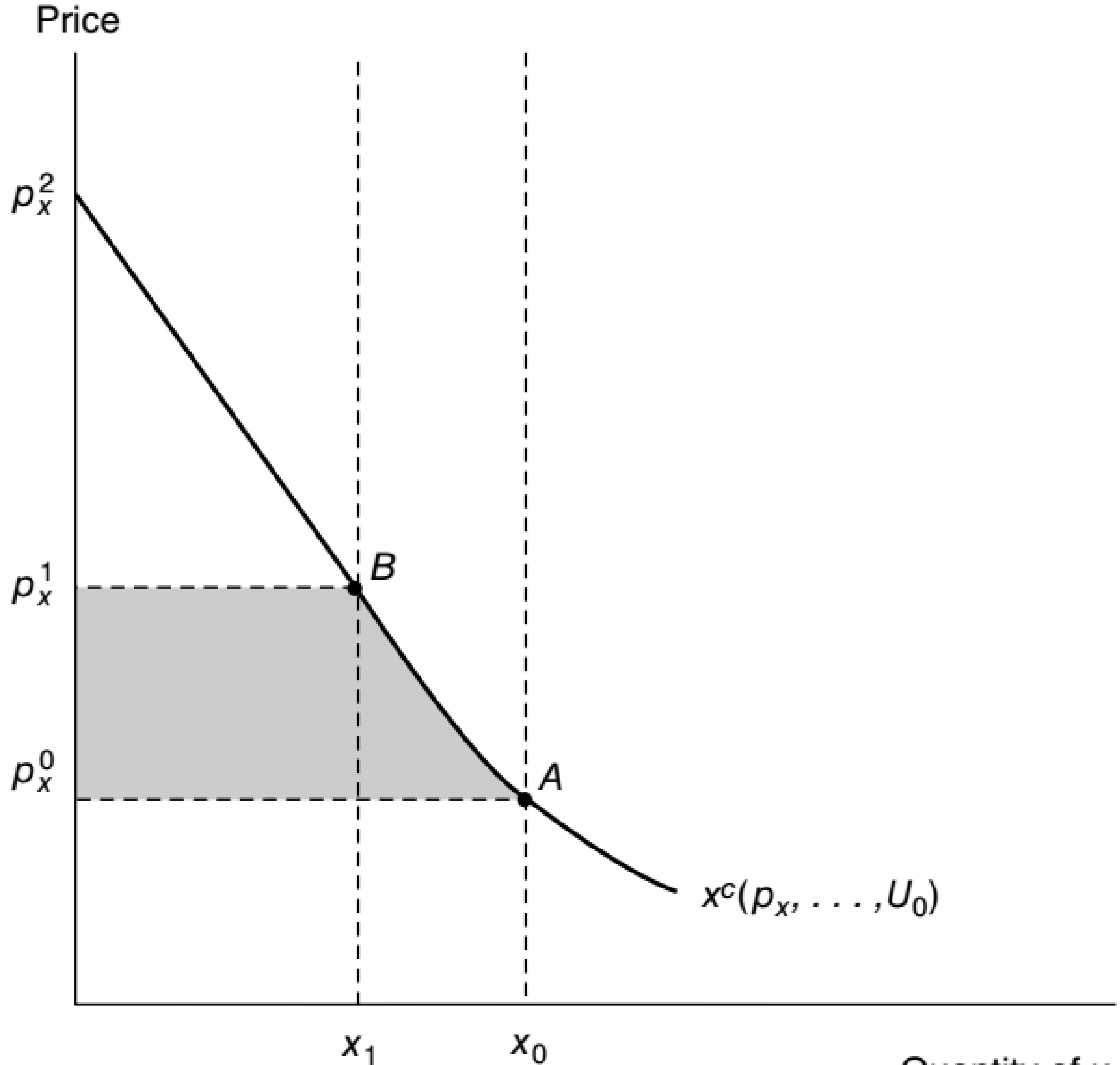
Change in consumer’s surplus. The change in consumer’s surplus will be the difference between two roughly triangular areas, and thus will have a roughly trapezoidal shape.



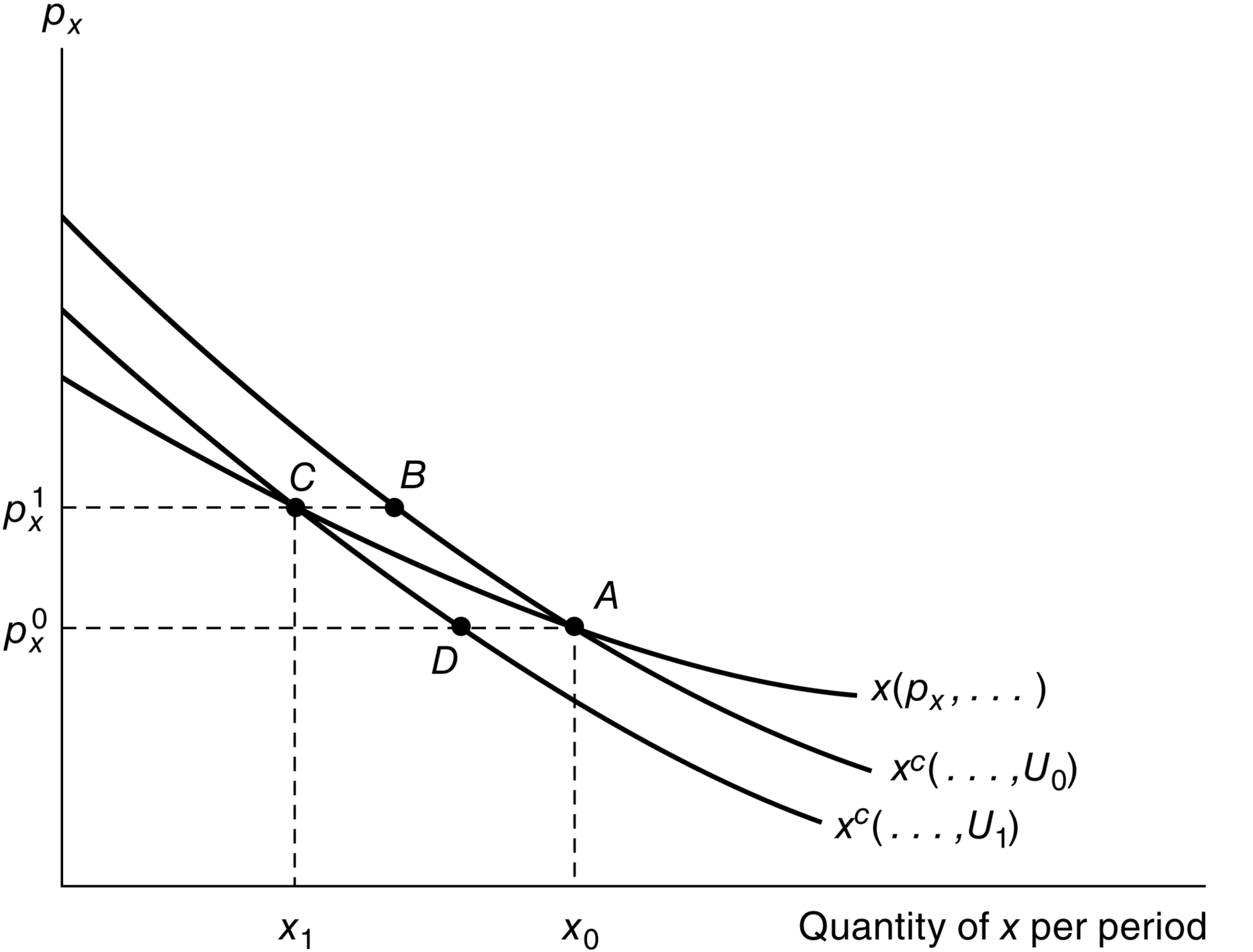
(a) Indifference curve map



(b) Compensated demand curve



(b) Compensated demand curve



p_x
p_x^1
p_x^0
C
B
A
D
$x(p_x, \ldots)$
$x^c(\ldots, U_0)$
$x^c(\ldots, U_1)$
x_1
x_0
Quantity of x per period

utility function $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$

$$x_1 = \frac{m}{2p_1}$$

$$x_2 = \frac{m}{2p_2}$$

$$\begin{aligned} p_1 &= 1 \\ p_2 &= 1 \\ M &= 100 \end{aligned} \longrightarrow \begin{aligned} p_1' &= 1 \\ p_2' &= 2 \\ M' &= 100 \end{aligned}$$

$$CV = 41.42$$

$$CS = 34.66$$

$$EV = 29.3$$

Consumer surplus lost?

$$CS = \int_1^2 \left(\tfrac{1}{2} m \rho_2^{\;-1} \right) d\rho$$

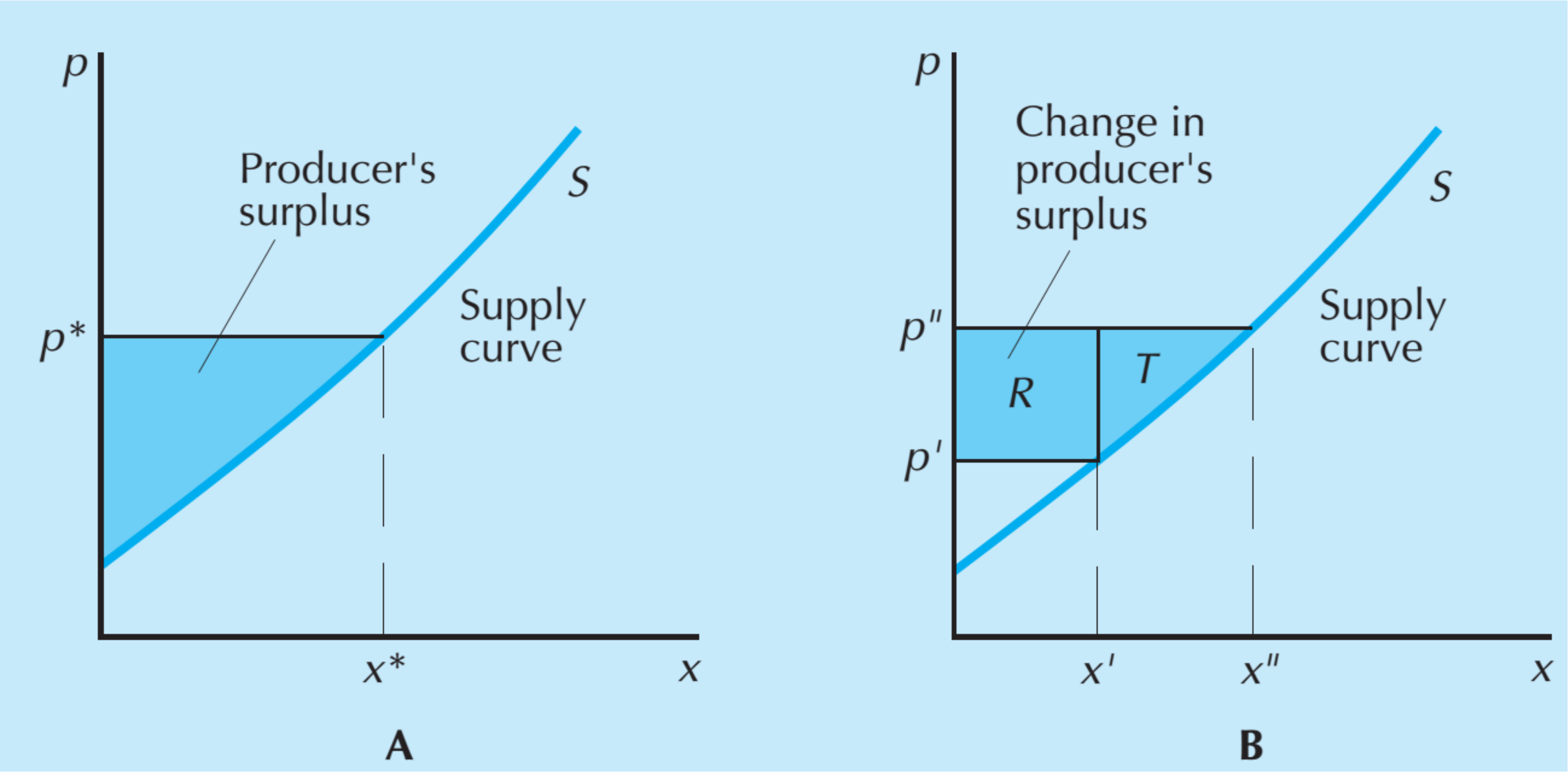
$$= \left[\tfrac{1}{2} m \ln[\rho] \right]_1^2$$

$$= \tfrac{1}{2} m \left(\ln[2] - \ln[1] \right)$$

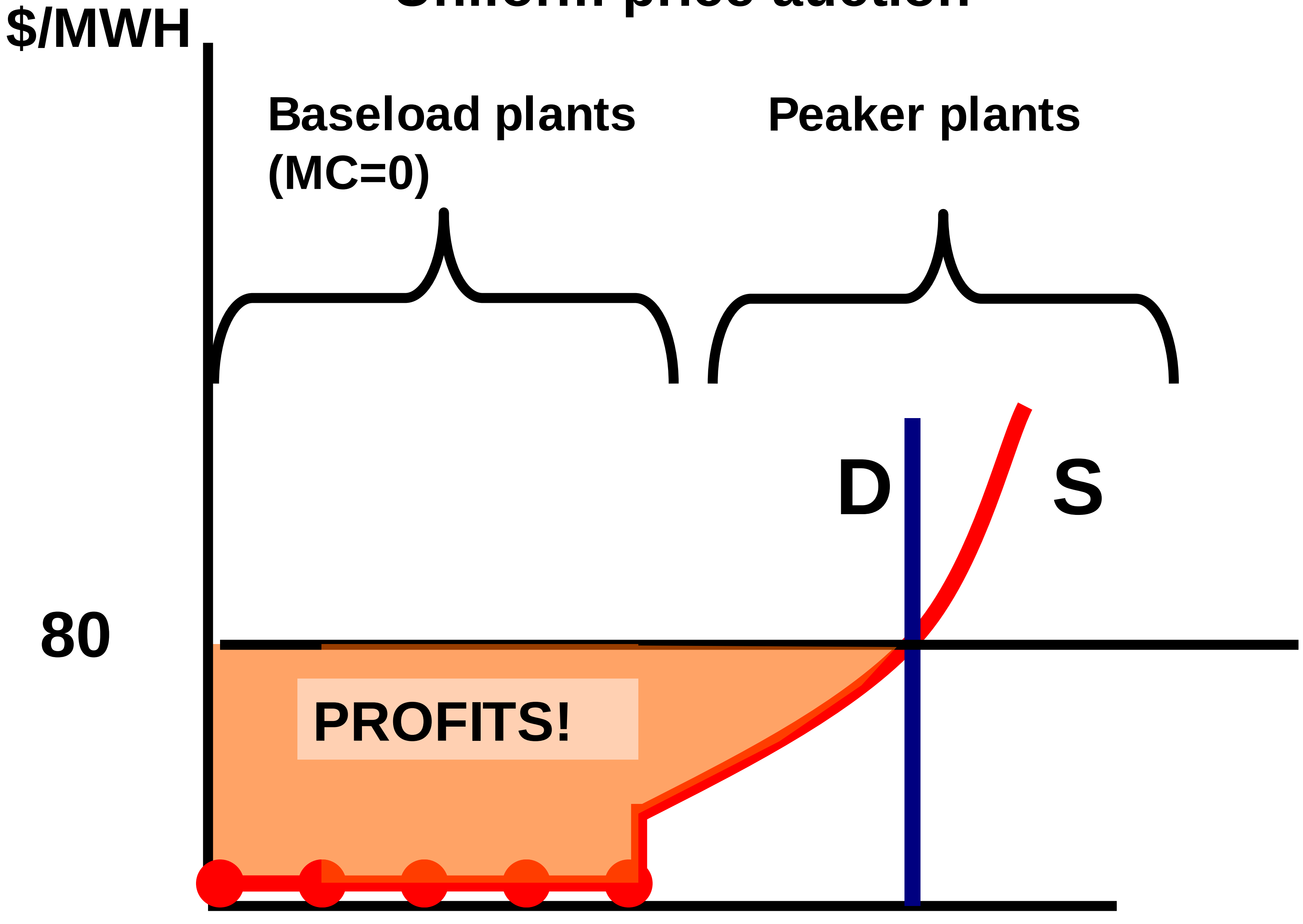
$$= \tfrac{1}{2} 100 \left(\ln[2] - 0 \right)$$

$$= 50 \ln[2] \quad \approx 34.66$$

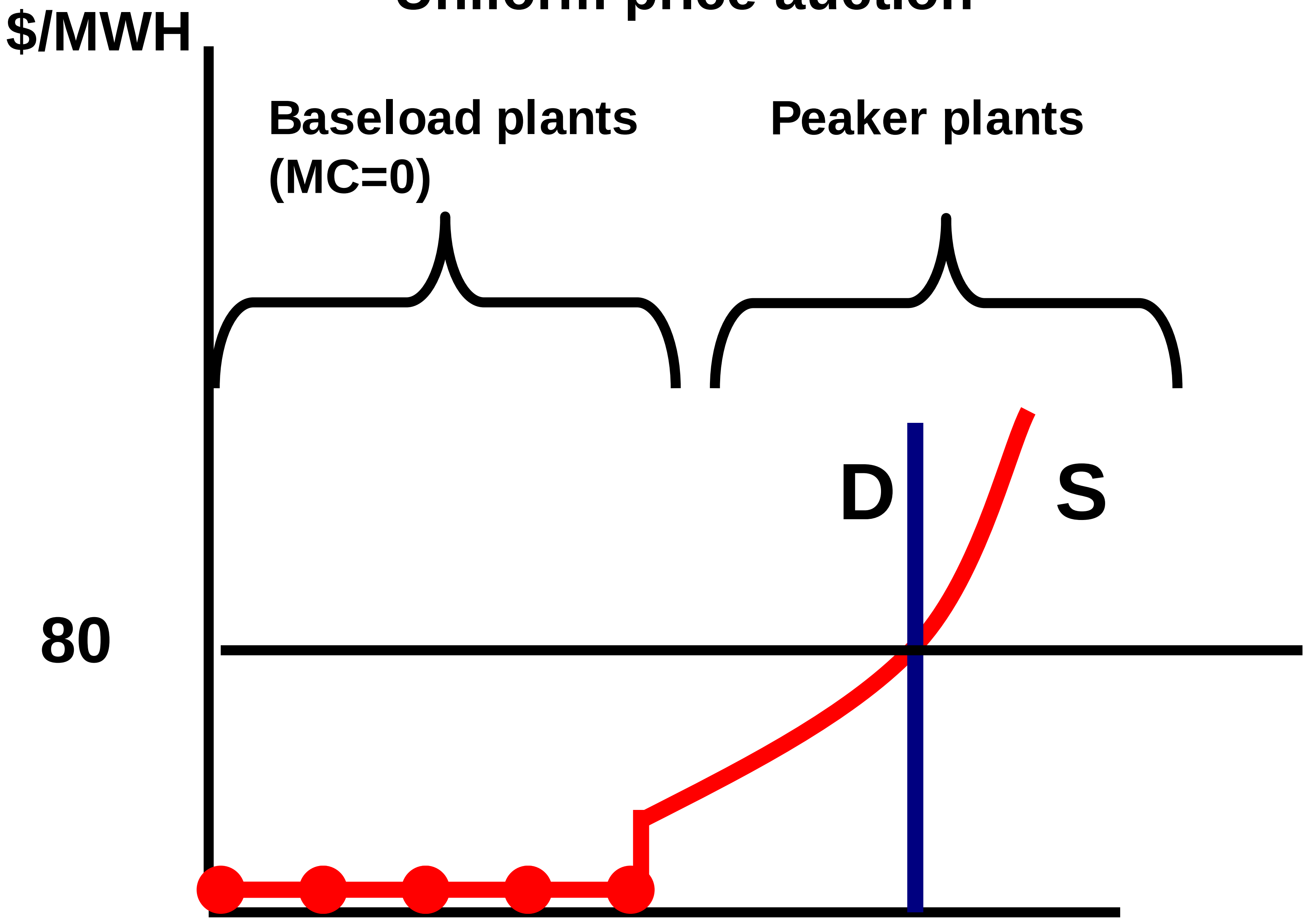
- Producer’s surplus


p
Producer's surplus
S
Supply curve
p*
x*
x
A
p
Change in producer's surplus
S
p''
Supply curve
T
R
p'
x'
x''
x
B

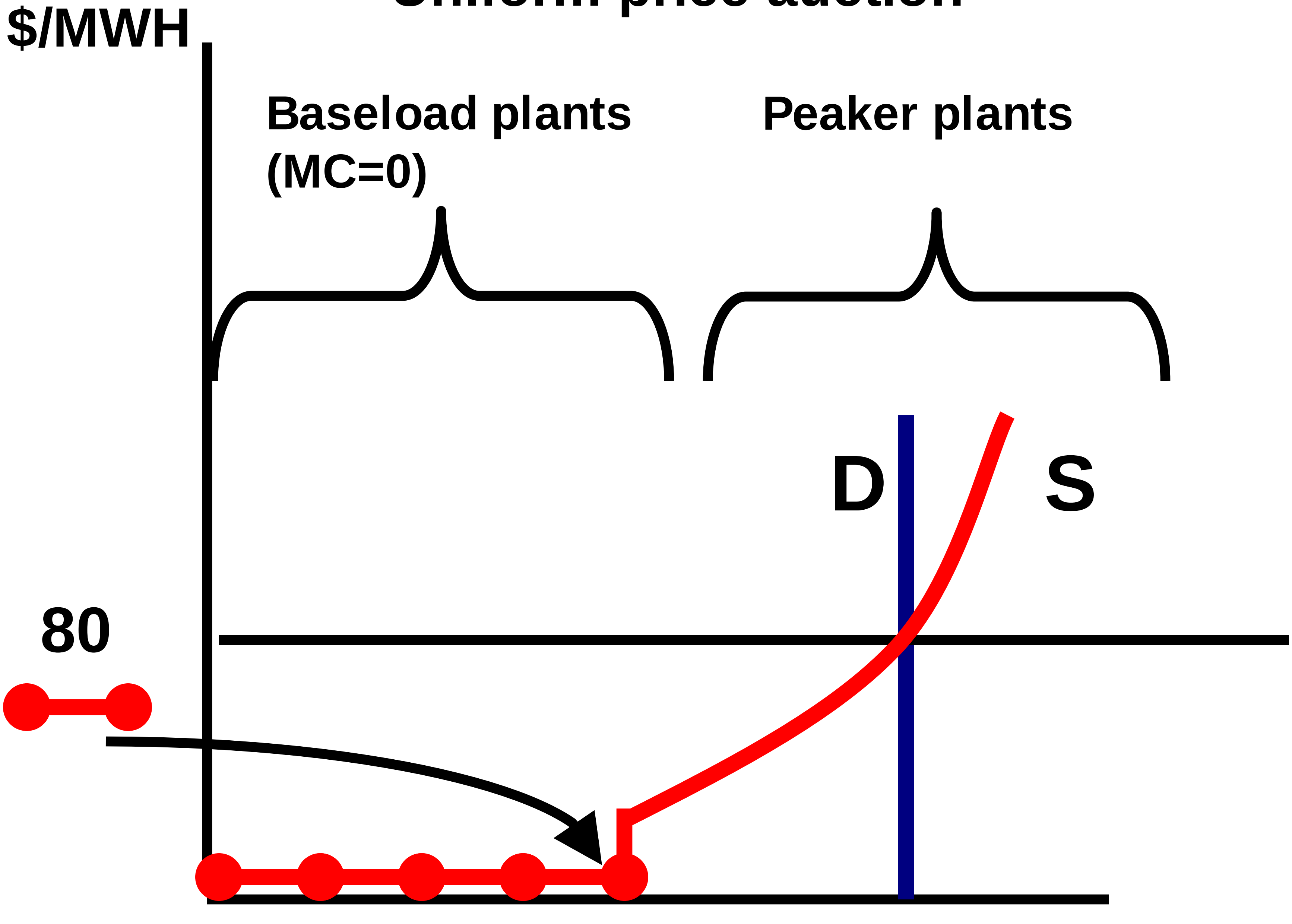
Uniform price auction



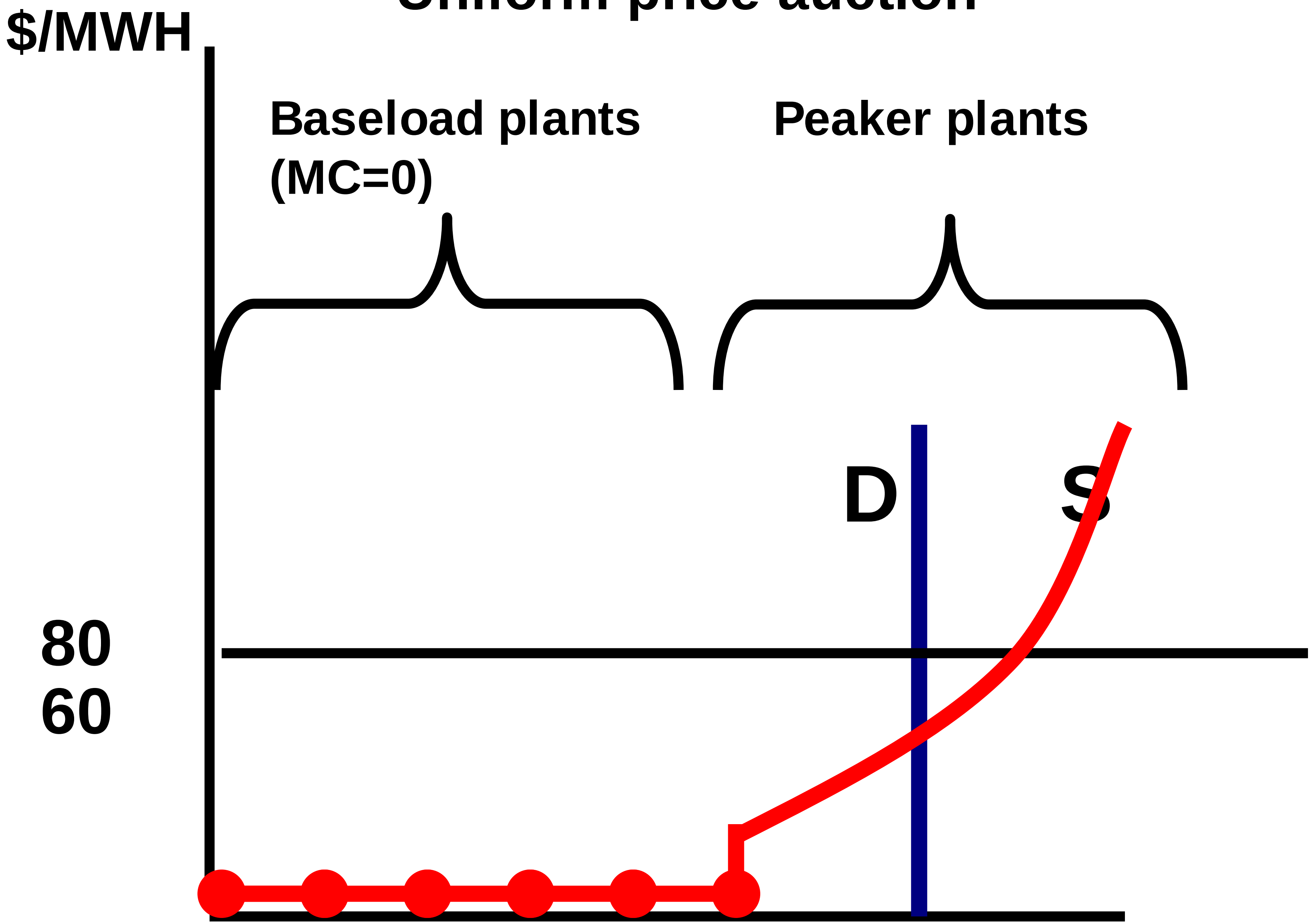
Uniform price auction



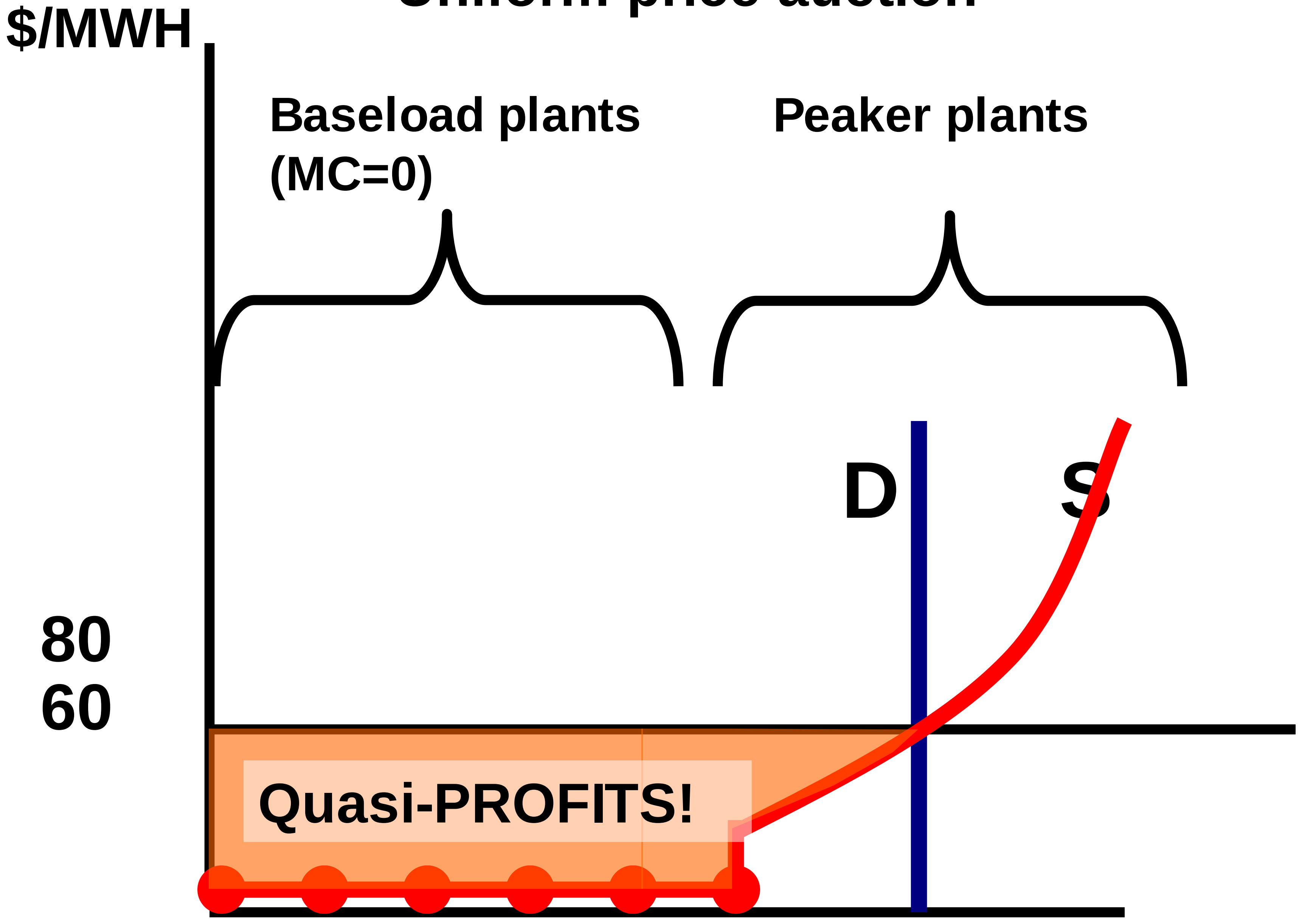
Uniform price auction



Uniform price auction

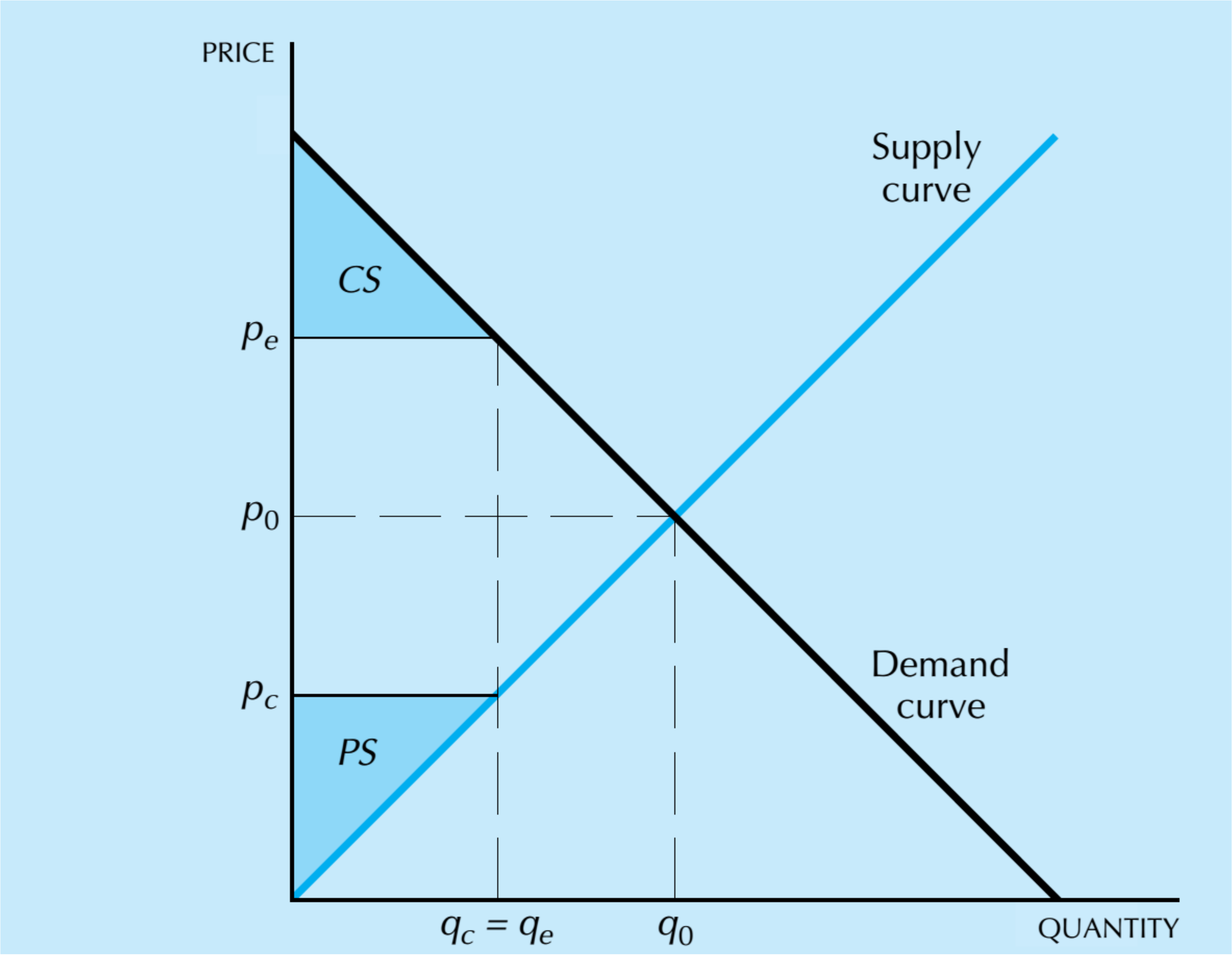


Uniform price auction

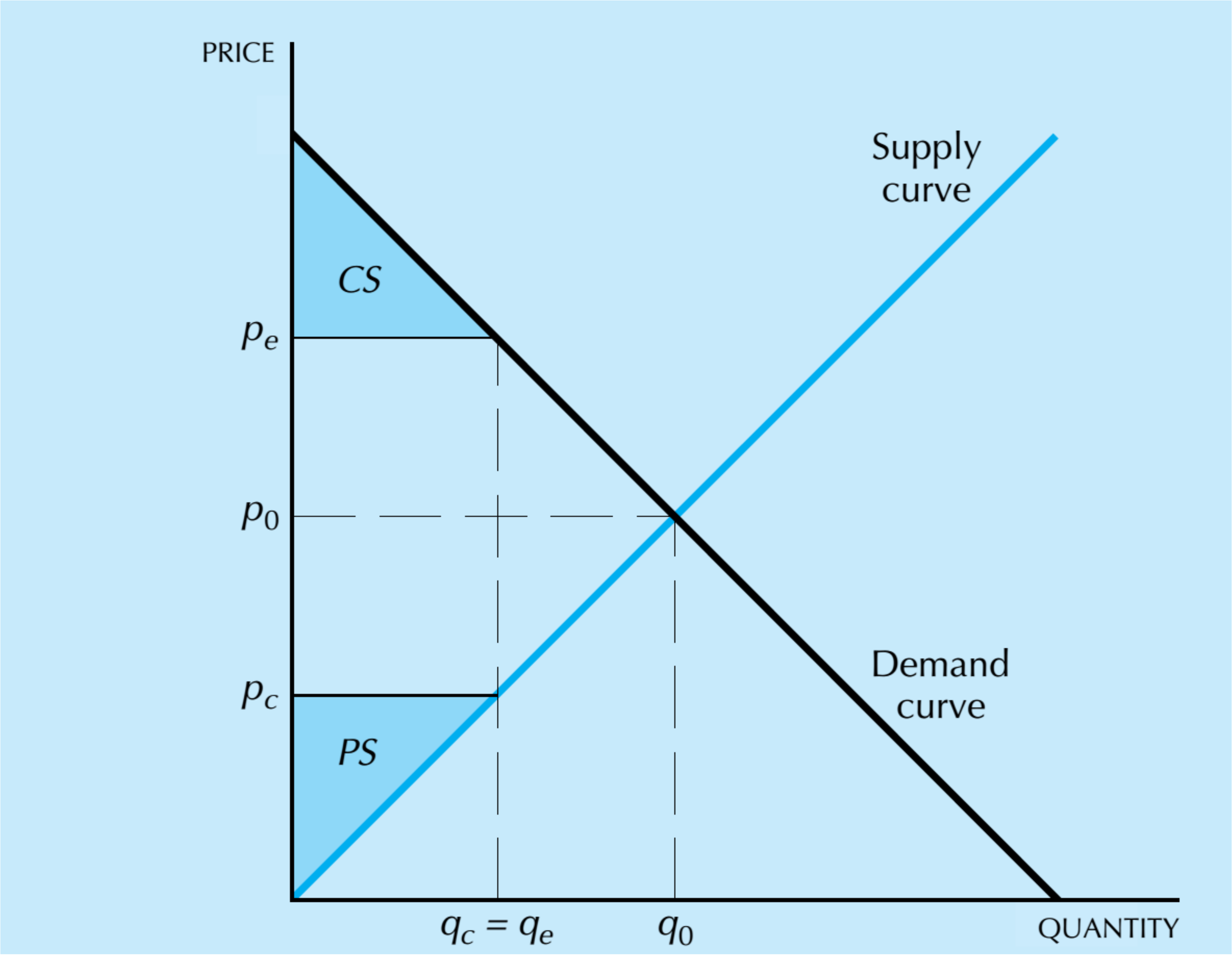


Cost-benefit analysis

A price ceiling.



Rationing



PRICE
Supply
curve
CS
p_e
p_0
p_c
PS
Demand
curve
$q_c = q_e$
q_0
QUANTITY

1. A good can be produced in a competitive industry at a cost of \$10 per unit. There are 100 consumers are each willing to pay \$12 each to consume a single unit of the good (additional units have no value to them.) What is the equilibrium price and quantity sold? The government imposes a tax of \$1 on the good. What is the deadweight loss of this tax?

14.1. The equilibrium price is \$10 and the quantity sold is 100 units. If the tax is imposed, the price rises to \$11, but 100 units of the good will still be sold, so there is no deadweight loss.

2. Suppose that the demand curve is given by $D(p) = 10 - p$. What is the gross benefit from consuming 6 units of the good?

14.2. We want to compute the area under the demand curve to the left of the quantity 6. Break this up into the area of a triangle with a base of 6 and a height of 6 and a rectangle with base 6 and height 4. Applying the formulas from high school geometry, the triangle has area 18 and the rectangle has area 24. Thus gross benefit is 42.

3. In the above example, if the price changes from 4 to 6, what is the change in consumer's surplus?

14.3. When the price is 4, the consumer's surplus is given by the area of a triangle with a base of 6 and a height of 6; i.e., the consumer's surplus is 18. When the price is 6, the triangle has a base of 4 and a height of 4, giving an area of 8. Thus the price change has reduced consumer's surplus by $10.

4. Suppose that a consumer is consuming 10 units of a discrete good and the price increases from $5 per unit to $6. However, after the price change the consumer continues to consume 10 units of the discrete good. What is the loss in the consumer's surplus from this price change?

14.4. Ten dollars. Since the demand for the discrete good hasn't changed, all that has happend is that the consumer has had to reduce his expenditure on other goods by ten dollars.

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MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

Národohospodářská fakulta VŠE v Praze